University of Guelph
Department of Mathematics and Statistics
MATH*1080 Lab Assignment 1
Friday September 27th t 2019
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Lab ASSIGNMENT 1

Version 4

Instructions

1. Answer the questions in the spaces provided.

2. DO NOT WRITE ON THIS COVER PAGE

Marking Scheme

Question	Out of
Participation	1
Q1	2
Q2	2
Total	**5**

Participation (1 point)

Question (1) [2 points]

Use the closed form formulas to evaluate the following series (Do not calculate the final answer):

a) **[0.1 point]** $\sum_{n=1}^{100} 3 = 100\,(3)$

0.05 0.05

0.1 0.1

b) $[\mathbf{0.3\ point}]$ $\sum_{k=3}^{45} k = \sum_{k=1}^{45} k - \sum_{k=1}^{3-1} k$

$$Note: \sum_{k=1}^{N} k = \frac{N(N+1)}{2}$$

$$\sum_{k=3}^{45} k = \frac{45(45+1)}{2} - \frac{2(2+1)}{2}$$

0.1

c) $[\mathbf{1\ point}]$ $\sum_{n=1}^{15}(2n+4)^2 =$

$$\sum_{n=1}^{15}(2n+4)^2 = \sum_{n=1}^{15}(4n^2+16n+16)$$

$$= \sum_{n=1}^{15}(4n^2) + \sum_{n=1}^{15}(16n) + \sum_{n=1}^{15}(16) =$$

$$Note: \sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\sum_{n=1}^{N} n = \frac{N(N+1)}{2}$$

$$= 4\sum_{n=1}^{15} n^2 + 16\sum_{n=1}^{15} n + \sum_{n=1}^{15} 16$$

$$= 4\,\frac{15(15+1)(2(15)+1)}{6} + 16\,\frac{15(15+1)}{2} + 15(16)$$

0.2

d) **[0.6 point]** $\sum_{n=1}^{60} 4\left(\frac{1}{9}\right)^{n+1} = \sum_{n=1}^{60} 4\left(\frac{1}{9}\right)^{2}\left(\frac{1}{9}\right)^{n-1} = \frac{4}{81}\left[\frac{1-\frac{1}{9}^{60}}{1-\frac{1}{9}}\right]$

0.3 0.1

Question (2) [2 points]

a) **[0.5 point]** The exact value of the $\lim_{x\to 0} \frac{\sqrt{12+3x} - \sqrt{12}}{x}$ is

0.2

$$\lim_{x\to 0} \frac{\sqrt{12+3x} - \sqrt{12}}{x} \cdot \frac{\sqrt{12+3x} + \sqrt{12}}{\sqrt{12+3x} + \sqrt{12}} =$$

0.2

$$\lim_{x\to 0} \frac{12+3x-12}{x(\sqrt{12+3x} + \sqrt{12})} =$$

0.1

$$\lim_{x\to 0} \frac{3}{(\sqrt{12+3x} + \sqrt{12})} = \frac{3}{2\sqrt{12}}$$

b) **[1 point]** Is the function below discontinuous at x= 1? Show your work.

$$f(x) = \begin{cases} 2x+8 & \text{if } x \le -1 \\ 4x^2 & \text{if } -1 < x < 1 \\ 4-2x & \text{if } x \ge 1 \end{cases}$$

A. True

B. False

$\lim_{x\to 1^-} f(x) = 4(1)^2 = 4$ 0.2

$\lim_{x\to 1^+} f(x) = 4 - 2 = 2$ 0.2

$\rightarrow \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ 0.2

$\rightarrow \lim_{x\to 1} f(x)$ DNE 0.2

So the function above is discontinuous is True 0.2

c) **[0.5 point]** Let f be the function defined below.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 1, & x = 2 \end{cases}$$

Which of the following statements about f is/are true? Show your work.

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

A. I only

B. II only

C. I and II

D. None of the above.

$\lim_{x\to 2^-} f(x) = \frac{2^2-4}{2-2} = 0$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^-}(x+2) = 4$ 0.1

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^{\mp}} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^+}(x+2) = 4$ 0.1

$\rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \rightarrow \lim_{x\to 2} f(x) = 4$ 0.1

$\lim_{x\to 2} f(x)$ Exists.

$f(2) = 1$ 0.1

$\rightarrow \lim_{x\to 2} f(x) \neq f(2)$ f is not continuous at x = 2 0.1