PHY 1321 / 1331 - Assignment 3

Your best bud

First year course

University of Ottawa

Ottawa, ON, Canada

Fall 2019

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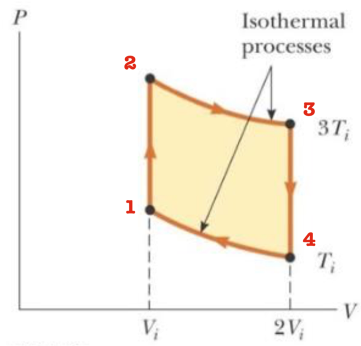
Assignment 3

Question 1

We have **n mol** of an ideal **monatomic** gas. We want: (a) the net energy transferred by heat to the gas. (b) the ratio of the work performed by the system to the heat absorbed by it in one cycle.

Number the points of the processes from 1 to 4.



The processes 1-2 and 3-4 are isovolumetric (V = Const). The processes 2-3 and 4-1 are isothermic (T = Const).

(a)

The net energy (Q) will be the sum of all the energies in the cycle.

$$Q_{net} = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$

For the isovolumetric processes:

$$Q_{12} = nC_v\Delta T = nC_v(3T_i - T_i) = 2nC_vT_i$$

$$Q_{34} = nC_v\Delta T = nC_v(T_i - 3T_i) = -2nC_vT_i$$

For the isothermic processes:

$$Q_{23} = nRT\ln\frac{V_f}{V_i} = nR(3T_i)\ln\frac{2V_i}{V_i} = 3nRT_i\ln 2$$

$$Q_{41} = nRT \ln \frac{V_f}{V_i} = nR(T_i) \ln \frac{V_i}{2V_i} = nRT_i \ln 2^{-1} = -nRT_i \ln 2$$

Therefore,

$$Q_{net} = Q_{12} + Q_{23} + Q_{34} + Q_{41} = 2nC_vT_i + 3nRT_i \ln 2 - 2nC_vT_i - nRT_i \ln 2$$

$$Q_{net} = 2nRT_i \ln 2$$

(b)

In order to find the ratio, we need to calculate the work performed **by** the system and the heat absorbed by it. The gas is absorbing energy in process 1-2 (temperature is becoming higher) and 2-3 (expansion). Hence,

$$Q_{absorbed} = Q_{12} + Q_{23} = 2nC_vT_i + 3nRT_i \ln 2$$

Since it's a monoatomic gas: $C_v = \frac{3R}{2}$. Therefore,

$$Q_{absorbed} = 2n(\frac{3R}{2})T_i + 3nRT_i \ln 2 = 3nRT_i + 3nRT_i \ln 2 = 3nRT_i(1 + \ln 2)$$

$$Q_{absorbed} = 3nRT_i(1 + \ln 2)$$

Now, let's find the work performed by the system in a cycle:

$$W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41}$$

For isovoluetric processes:

$$W_{12} = W_{34} = 0$$

For isothermic processes:

$$W_{23} = nRT\ln\frac{V_f}{V_i} = nR(3T_i)\ln\frac{2V_i}{V_i} = 3nRT_i\ln 2$$

$$W_{41} = nRT\ln\frac{V_f}{V_i} = nR(T_i)\ln\frac{V_i}{2V_i} = -nRT_i\ln 2$$

So,

$$W_{cycle} = 0 + 3nRT_i\ln 2 + 0 + -nRT_i\ln 2 = 2nRT_i\ln 2$$

$$ratio = \frac{W_{cycle}}{Qabsorbed} = \frac{2nRT_i\ln 2}{3nRT_i(1+\ln 2)} = \frac{2\ln 2}{3(1+\ln 2)}$$

$$ratio = 0.2729 = 27.29\%$$

Question 2

(a)

This is a discrete distribution.

(b)

Analysing the graph:

$$v_{mp} = 120km/h$$

$$v_{avg} = \frac{70\times 4 + 80\times 9 + 90\times 20 + 100\times 35 + 110\times 47 + 120\times 58 + 130\times 50 + 140\times 43 + 150\times 35 + 160\times 22 + 170\times 12 + 180\times 2}{4+9+20+35+47+58+50+43+35+22+12+2}$$

$$v_{avg} = \frac{42120}{337} = 125km/h$$

$$v_{rms}^2 = \frac{\begin{array}{c}70^2 \times 4 + 80^2 \times 9 + 90^2 \times 20 + 100^2 \times 35 + 110^2 \times 47 + 120^2 \times 58 + \\ 130^2 \times 50 + 140^2 \times 43 + 150^2 \times 35 + 160^2 \times 22 + 170^2 \times 12 + 180^2 \times 2\end{array}}{4 + 9 + 20 + 35 + 47 + 58 + 50 + 43 + 35 + 22 + 12 + 2}$$

$$v_{rms} = \sqrt{\frac{5443200}{337}} = 127km/h$$

(c)

$$P(v < 125) = \frac{N_{cars.with.v<125}}{N_{total}}$$

$$P(v < 125) = \frac{4 + 9 + 20 + 35 + 47 + 58}{337}$$

$$P(v < 125) = 0.5133 = 51.33\%$$

(d)

$$P(95 < v < 135) = \frac{N_{cars.with.95<v<135}}{N_{total}}$$

$$P(v < 125) = \frac{35 + 47 + 58 + 50}{337}$$

$$P(v < 125) = 0.5638 = 56.38\%$$

Question 3

	DOF	E_{kin}	C_v	C_p	γ
A	5	$\frac{5}{2}k_bT$	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5}$
B	1	$\frac{1}{2}k_bT$	$\frac{1}{2}R$	$\frac{3}{2}R$	3
C	9	$\frac{9}{2}k_bT$	$\frac{9}{2}R$	$\frac{11}{2}R$	$\frac{11}{9}$
D	11	$\frac{11}{2}k_bT$	$\frac{11}{2}R$	$\frac{13}{2}R$	$\frac{13}{11}$
E	6	$3k_bT$	$3R$	$4R$	$\frac{4}{3}$

Question 4

(a)

We have: $n = 1mol$, $W = 5000J$, $P_f = 2.00atm = 101325 \times 2 = 202650Pa$, $V_f = 25.0L = 25 \times 10^{-3}m^3$. We want V_i and T.

For isothermic process:

$$W = nRT \ln \frac{V_f}{V_i}$$

But we don't have T or V_i. Let's use the ideal gas equation:

$$P_f V_f = nRT$$

Hence,

$$W = P_f V_f \ln \frac{V_f}{V_i} \Rightarrow \ln \frac{V_f}{V_i} = \frac{W}{P_f V_f}$$

$$\frac{V_f}{V_i} = e^{\frac{W}{P_f V_f}}$$

$$V_i = \frac{V_f}{e^{\frac{W}{P_f V_f}}}$$

$$V_i = \frac{25}{e^{\frac{5000}{202650\times 25\times 10^{-3}}}} = 9.32L$$

Now, calculating the temperature T:

$$P_f V_f = nRT \Rightarrow T = \frac{P_f V_f}{nR}$$

$$T = \frac{2\times 25}{1\times 0.082} = 609.76K$$

(b)

We have: $n = 1mol$, $W = -3500J$, $T_i = 500K$, $P_i = 3.60atm = 101325 \times 3.60 = 364770Pa$. Also, since it's a monoatomic gas: $\gamma = \frac{5}{3} = 1.7$. We want T_f and P_f.

For adiabatic process:

$$W = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

But,

$$P_i V_i = nRT_i$$

And,

$$P_f V_f = nRT_f$$

Hence,

$$W = \frac{nRT_f - nRT_i}{\gamma - 1}$$

Therefore,

$$T_f = T_i + \frac{W(\gamma - 1)}{nR}$$

$$T_f = 500 + \frac{-3500(1.7-1)}{1\times 8.31} = 205K$$

To calculate the final pressure, we have to use the property that adiabatic processes have:

$$PV^{\gamma} = Const$$

Hence,

$$P_f V_f^{\gamma} = P_i V_i^{\gamma}$$

But

$$V_i = \frac{nRT_i}{P_i}; V_f = \frac{nRT_f}{P_f}$$

Therefore,

$$P_f \left(\frac{nRT_f}{P_f}\right)^{\gamma} = P_i \left(\frac{nRT_i}{P_i}\right)^{\gamma}$$

$$P_f \left(\frac{T_f}{P_f}\right)^{\gamma} = P_i \left(\frac{T_i}{P_i}\right)^{\gamma}$$

$$P_f^{1-\gamma} T_f^{\gamma} = P_i \left(\frac{T_i}{P_i}\right)^{\gamma}$$

$$P_f^{1-\gamma} = P_i \left(\frac{T_i}{T_f P_i}\right)^{\gamma}$$

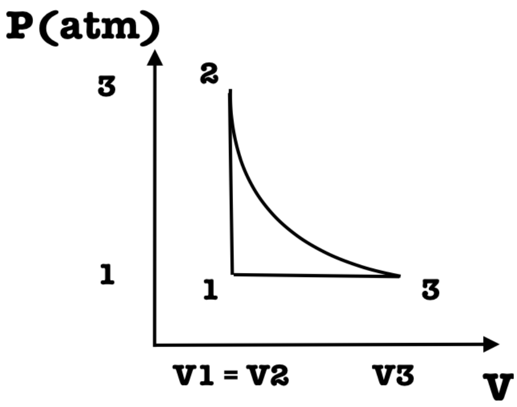
$$P_f = \left[P_i \left(\frac{T_i}{T_f P_i}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}}$$

$$P_f = \left[3.6 \left(\frac{500}{205 \times 3.6}\right)^{1.7}\right]^{\frac{1}{1-1.7}} = 0.41 atm$$

Question 5

We know that $\gamma = 1.4$, $V_1 = 4L$, $P_1 = 1atm$, $T_1 = 300K$, $P_2 = 3atm$

(a)



(b)

We want to know V_3. Because process 1-3 is adiabatic, we can use:

$$P_3 V_3^\gamma = P_2 V_2^\gamma$$

$$V_3^\gamma = \frac{P_2}{P_3} V_2^\gamma$$

$$V_3 = V_2 \left(\frac{P_2}{P_3}\right)^{\frac{1}{\gamma}}$$

$$V_3 = 4 \left(\frac{3}{1}\right)^{\frac{1}{1.4}} = 8.77L$$

(c)

We want to know T_2. From 1-2, we can use:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$T_2 = \frac{P_2 T_1}{P_1} = \frac{3 \times 300}{1} = 900K$$

(d)

We want to know T_1 (end of the cycle). But that information was given. Therefore,

$$T_1 = 300K$$

PS: There is a possibility that the professor wanted to ask for T_3, so just for precaution I'm going to show you how to calculate it:

From 3-1:

$$\frac{V_3}{T_3} = \frac{V_1}{T_1}$$

$$T_3 = \frac{V_3 T_1}{V_1} = \frac{8.77 \times 300}{4} = 657.7K$$

(e)

$$W_{net} = W_{12} + W_{23} + W_{31}$$

$$W_{net} = 0 + \frac{P_3V_3 - P_2V_2}{\gamma - 1} - P_3(V_1 - V_3)$$

$$W_{net} = \frac{101325 \times 8.77 \times 10^{-3} - 3 \times 101325 \times 4 \times 10^{-3}}{1.4 - 1} - 101325(4 - 8.77) \times 10^{-3} = -818 + 483$$

$$W_{net} = -335J$$

Question 6

From the first law of thermodynamics:

$$dE = dW + dQ$$

Since we are talking about adiabatic process:

$$dQ = 0$$

Moreover, we have that:

$$dE = nC_v dT$$

and

$$dW = -PdV$$

Therefore,

$$dE = dW + dQ = dW$$

$$nC_v dT = -PdV \tag{1}$$

Since we have 3 variables on the equation above, we cannot integrate both sides. Let's find a another relation between P, V and T (the ideal gas equation):

$$PV = nRT$$

Deriving both sides:

$$d(PV) = d(nRT)$$

From the chain rule , we have that

$$d(PV) = PdV + VdP$$

Therefore,

$$PdV + VdP = nRdT$$

So,

$$ndT = \frac{PdV + VdP}{R}$$

Now we can substitute this result in Equation (1).

$$C_v \frac{PdV + VdP}{R} = -PdV$$

Now we have one equation with 2 variables. Let's put everything that has P to one side and everything that has V to the other.

$$PdV + VdP = -\frac{R}{C_v} PdV$$

$$PdV \left(1 + \frac{R}{C_v}\right) = -VdP$$

$$\left(1 + \frac{R}{C_v}\right) \frac{dV}{V} = -\frac{dP}{P}$$

Just a wuick math: $1 + \frac{R}{C_v} = \frac{C_v + R}{C_v} = \frac{C_p}{C_v}$. Therefore,

$$\left(\frac{C_p}{C_v}\right) \frac{dV}{V} = -\frac{dP}{P}$$

Now, let's integrate both sides:

$$\left(\frac{C_p}{C_v}\right) \int \frac{dV}{V} = -\int \frac{dP}{P}$$

$$\left(\frac{C_p}{C_v}\right) \ln V = -\ln P + C$$

$$\ln V^{\left(\frac{C_p}{C_v}\right)} + \ln P = C$$

$$\ln PV^{\left(\frac{C_p}{C_v}\right)} = C$$

$$PV^{\left(\frac{C_p}{C_v}\right)} = e^C$$

Calling $\frac{C_p}{C_v} = \gamma$ and $e^C = Const$, we have that:

$$PV^\gamma = Const$$