Solutions

MATH 151 [A01]
Midterm 1 (Version A)
October 15, 2018

Instructor: Dr. Jill Simmons

Last Name, First Name: ____________________

Student Number: V00____________

TO BE ANSWERED ON THE EXAM
DURATION: 50 minutes

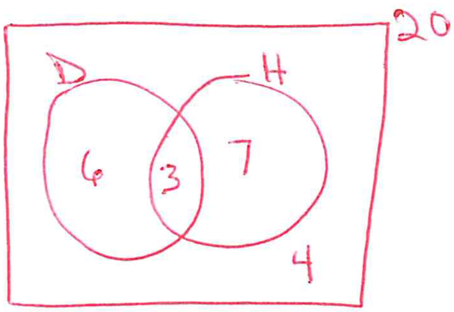
SCORE: /24

THIS EXAM HAS 5 PAGES, PLUS COVER.

Instructions:

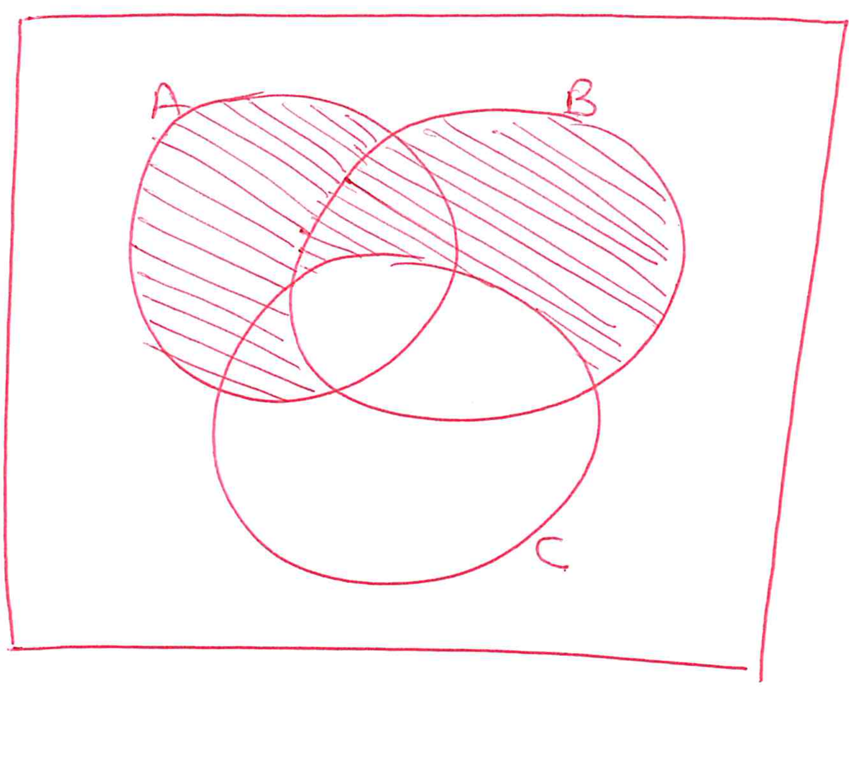
- The only calculators permitted are Sharp EL-510R series. No other electronic devices are permitted.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- If extra space is required, you may use the backs of the exam pages, but be sure to indicate where you have done so. No outside paper of any kind is allowed.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- For numeric answers that are not integer-valued (such as probabilities), you may state your answer as a fraction or as a decimal to at least 3 decimal places unless otherwise requested.

1. (2 marks) Victor knows how to cook 20 different meals. Four of the meals are neither healthy nor delicious, 7 are healthy but not delicious, and 6 are delicious but not healthy! How many of the meals that Victor knows how to make are delicious?



$n(D) = 6 + 3 = \underline{9}$

2. (2 marks) Draw a Venn diagram depicting 3 sets A, B, and C, and shade the region that represents $(A \cap B') \cup (B \cap C')$.



3. (2 marks) A daycare has 4 pieces of playground equipment that need painting: a swing, a slide, monkey bars, and a teeter-totter. Each piece of equipment can be painted with any of 6 available colours.

   (a) How many ways can the daycare paint the equipment if there are no restrictions?

   $6^4 = \underline{\underline{1296}}$

   (b) How many ways can the daycare paint the equipment if it wants each of the 4 pieces of equipment get painted a different colour?

   $6 \cdot 5 \cdot 4 \cdot 3 = 6P4 = \underline{\underline{360}}$

4. (2 marks) If E and F are independent events with $Pr(E) = 0.25$ and $Pr(F|E) = 0.6$, find $Pr(E \cup F)$.

   $Pr(F) = .6$

   $Pr(E \cap F) = (.25)(.6) = .15$

   $Pr(E \cup F) = .25 + .6 - .15 = \underline{\underline{.7}}$

5. (4 marks) Consider the linear arrangements of the 13 letters in the word TYRANNOSAURUS.

(a) How many different linear arrangements are there, if there are no restrictions?

T, Y, $\begin{smallmatrix}R\\R\end{smallmatrix}$, $\begin{smallmatrix}A\\A\end{smallmatrix}$, $\begin{smallmatrix}N\\N\end{smallmatrix}$, O, $\begin{smallmatrix}S\\S\end{smallmatrix}$, $\begin{smallmatrix}U\\U\end{smallmatrix}$

$$\frac{13!}{(2!)^5} = \underline{194{,}594{,}400}$$

(b) How many of the linear arrangements have all pairs of duplicate letters adjacent? i.e. The two Rs must be adjacent, the two As must be adjacent, etc.

Arrange T, Y, O, RR, AA, NN, SS, UU ← 8 objects

$$8! = \underline{40320}$$

6. (3 marks) Three cards are dealt at random from a well-shuffled standard deck of 52 cards. What is the probability the 3 dealt cards will each be of a different suit? Recall, the deck contains 13 cards in each of the suits ♡, ♢, ♣, ♠. An example of such a collection of 3 cards is 9♡, 4♢, 9♠.

$$n(S) = \binom{52}{3}$$

$$\frac{\binom{4}{3}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{3}} = \frac{169}{425} = \underline{.398}$$

7. (6 marks) A container holds 5 red, 10 white, and 8 blue balls. Two balls are to be drawn from the container. Let E be the event that at least one red ball is drawn and let F be the event that the two balls have the same colour.

(a) Find $Pr(F)$.

$$Pr(F) = \frac{\binom{5}{2}+\binom{10}{2}+\binom{8}{2}}{\binom{23}{2}} = \frac{83}{253} = .328$$

(b) Find $Pr(F|E)$.

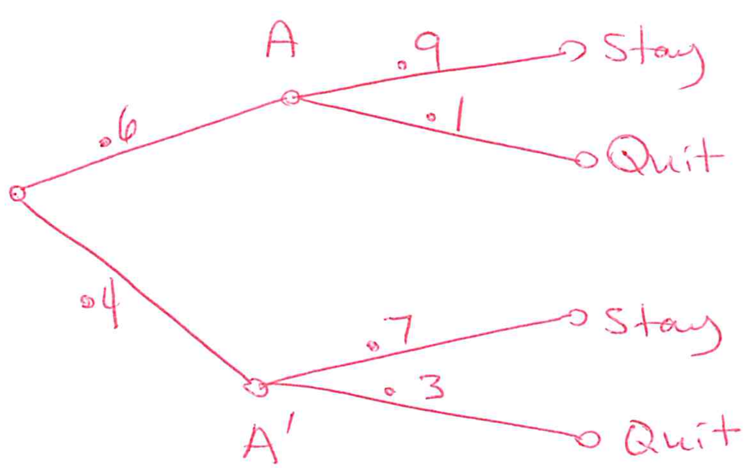
$$Pr(F|E) = \frac{Pr(E \cap F)}{Pr(E)} = \frac{\binom{5}{2}/\binom{23}{2}}{1-\binom{18}{2}/\binom{23}{2}}$$

$$= \frac{10}{100} = \frac{1}{10} = .1$$

(c) Are E and F independent events? Your answer must include either an explanation or a calculation to justify your decision.

No. If they were independent then $Pr(F)$ would equal $Pr(F|E)$.

8. (3 marks) In an attempt to improve employee retention, a company invites its new employees to a series of team building events. Over the last several years, 60% of new employees attended all of the events. Of those who attended all the events, 90% stayed with the company for at least a year, whereas 70% of those who did not attend all the events stayed for at least a year. If an employee recently quit prior to having been with the company for a year, what is the probability they attended all of the team building events?



$$Pr(A \mid Quit) = \frac{Pr(A \cap Quit)}{Pr(Quit)}$$

$$= \frac{(.6)(.1)}{(.6)(.1)+(.4)(.3)}$$

$$= \frac{.06}{.18} = \frac{1}{3} = \underline{\underline{.333}}$$

- END -

Solutions

MATH 151 [A01]
Midterm 1 (Version B)
October 15, 2018

Instructor: Dr. Jill Simmons

Last Name, First Name: ______________________________

Student Number: V00____________

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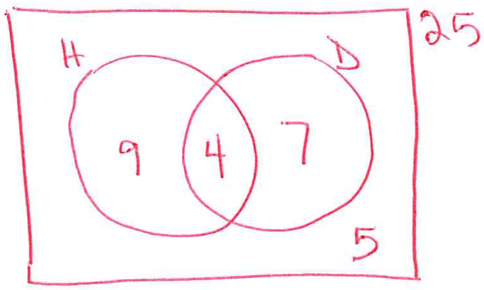
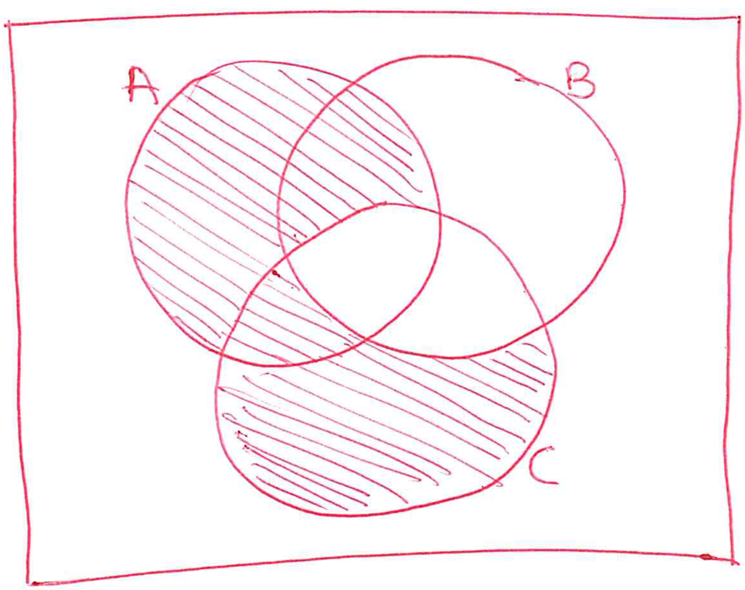
- For numeric answers that are not integer-valued (such as probabilities), you may state your answer as a fraction or as a decimal to at least 3 decimal places unless otherwise requested.

1. (2 marks) Victor knows how to cook 25 different meals. Five of the meals are neither healthy nor delicious, 9 are healthy but not delicious, and 7 are delicious but not healthy! How many of the meals that Victor knows how to make are healthy?



$n(H) = 9 + 4 = \underline{13}$

2. (2 marks) Draw a Venn diagram depicting 3 sets A, B, and C, and shade the region that represents $(A \cap C') \cup (B' \cap C)$.



3. (2 marks) A daycare has 4 pieces of playground equipment that need painting: a swing, a slide, monkey bars, and a teeter-totter. Each piece of equipment can be painted with any of 7 available colours.

(a) How many ways can the daycare paint the equipment if there are no restrictions?

$7^4 = \underline{2401}$

(b) How many ways can the daycare paint the equipment if it wants each of the 4 pieces of equipment get painted a different colour?

$7 \cdot 6 \cdot 5 \cdot 4 = 7P4 = \underline{840}$

4. (2 marks) If E and F are independent events with $Pr(E) = 0.35$ and $Pr(F|E) = 0.6$, find $Pr(E \cup F)$.

$Pr(F) = .6$

$Pr(E \cap F) = (.35)(.6) = .21$

$Pr(E \cup F) = .35 + .6 - .21 = \underline{.74}$

5. (4 marks) Consider the linear arrangements of the 13 letters in the word TYRANNOSAURUS.

(a) How many different linear arrangements are there, if there are no restrictions?

T, Y, R, A, N, O, S, U
R A N S U

$$\frac{13!}{2!\,2!\,2!\,2!\,2!} = \underline{\underline{194,594,400}}$$

(b) How many of the linear arrangements have all pairs of duplicate letters adjacent? i.e. The two Rs must be adjacent, the two As must be adjacent, etc.

Arrange T, Y, O, RR, AA, NN, SS, UU ← 8 objects

$$8! = \underline{\underline{40320}}$$

6. (3 marks) Three cards are dealt at random from a well-shuffled standard deck of 52 cards. What is the probability the 3 dealt cards will each be of a different suit? Recall, the deck contains 13 cards in each of the suits $\heartsuit, \diamondsuit, \clubsuit, \spadesuit$. An example of such a collection of 3 cards is $9\heartsuit, 4\diamondsuit, 9\spadesuit$.

$n(S) = \binom{52}{3}$.

$$\frac{\binom{4}{3}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{3}} = \underline{\underline{.398}} \quad \text{or} \quad \frac{169}{425}$$

7. (6 marks) A container holds 8 red, 12 white, and 6 blue balls. Two balls are to be drawn from the container. Let E be the event that at least one red ball is drawn and let F be the event that the two balls have the same colour.

(a) Find $Pr(F)$.

$$Pr(F) = \frac{\binom{8}{2}+\binom{12}{2}+\binom{6}{2}}{\binom{26}{2}} = \frac{109}{325} = .335$$

(b) Find $Pr(F|E)$.

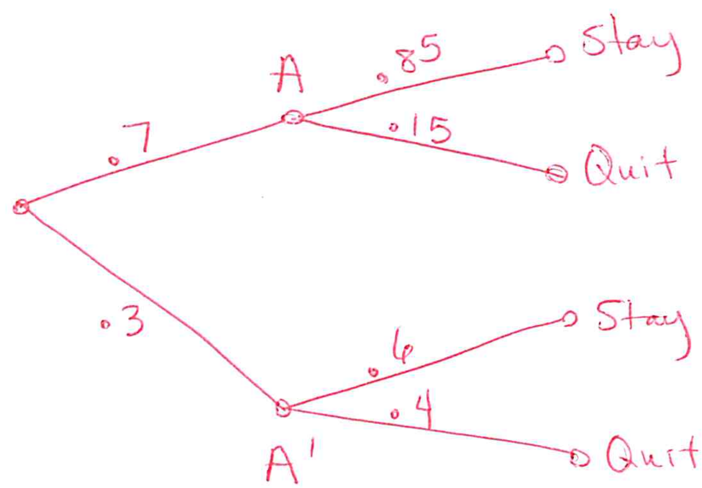
$$Pr(F|E) = \frac{Pr(E \cap F)}{Pr(E)} = \frac{\binom{8}{2}/\binom{26}{2}}{1-\binom{18}{2}/\binom{26}{2}}$$

$$= \frac{28}{172} = \frac{7}{43} = .163$$

(c) Are E and F independent events? Your answer must include either an explanation or a calculation to justify your decision.

No. If they were, then $Pr(F) = Pr(F|E)$.

8. (3 marks) In an attempt to improve employee retention, a company invites its new employees to a series of team building events. Over the last several years, 70% of new employees attended all of the events. Of those who attended all the events, 85% stayed with the company for at least a year, whereas 60% of those who did not attend all the events stayed for at least a year. If an employee recently quit prior to having been with the company for a year, what is the probability they attended all of the team building events?



$$\Pr(A \mid Quit) = \frac{\Pr(A \cap Quit)}{\Pr(Quit)}$$

$$= \frac{(.7)(.15)}{(.7)(.15) + (.3)(.4)}$$

$$= \underline{.467}$$