

**CONCORDIA UNIVERSITY**  
**Department of Mathematics & Statistics**

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Course	Number	Section(s)
Mathematics	209	All

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Examination	Date	Pages
Final	April 22 2017	3

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Instructors	Course Examiner
ALL	R. Raphael

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**Special Instructions**

- ▷ Ruled booklets to be used.
  - ▷ Approved calculators allowed.
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**MARKS**

[6] 1. (a) Find the following limits

(i)  $\lim_{x \rightarrow 1} \frac{2x^5 + 7x - 1}{x^2 + 5x + 3}$

(ii)  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 3x - 2}$

(b) Prove or disprove by giving an example: there exists a function  $f$  from the real numbers to the real numbers that is discontinuous at exactly three points.

[4] 2. Find the derivative  $f'(x)$  of the functions  $f(x)$ : (Do not simplify)

(a)  $f(x) = 4x^5 - 9x^2 + x - 22$

(b)  $f(x) = \frac{x^{-7}}{8} + \frac{1}{\sqrt[2]{x}}$

- [10] 3. Find  $\frac{dy}{dx}$  (do not simplify):
- (a)  $y = \frac{7 - x^3}{e^{3x}}$
  - (b)  $y = \ln(3x^4 + 7)$
  - (c)  $y = (4x - 5)^3(3x^2 + 4)$
  - (d)  $y = (5 + x^3 \ln x)^3$
- [8] 4. Let  $f(x) = 4x^4 - x^2 - 7$
- (a) Find the slope of the tangent line to the curve when  $x = 1$
  - (b) Find the equation of the tangent line to the curve when  $x = 1$
- [13] 5. Let  $f(x) = x^4 - 2x^3$   
Find
- (a) the critical and inflection points of  $f(x)$
  - (b) the intervals where  $f(x)$  is increasing and where it is decreasing
  - (c) the intervals on which  $f(x)$  is concave up and on which it is concave down
  - (d) use the above to sketch the graph
- [9] 6. If the cost of a seminar is \$400 per person 1000 people attend. For every \$5 dollar reduction in cost 20 more people will attend the seminar.  
How much should be charged for the seminar to maximize revenue?
- [6] 7. Find the absolute extrema of the function  $f(x) = x^3 - 12x$  on the interval  $[-5, 5]$ .
- [4] 8. A country has Lorenz curve  $f(x) = x$ . Find its Gini index. What can you conclude from the Gini index?
- [10] 9. Find the equation(s) of the tangent line(s) to the graph of  $y - xy^2 + x^2 + 1 = 0$  at the point(s) with  $x = 1$ .

[10] 10. Compute these antiderivatives:

(a)  $\int (5x^7 - 4x^3 - 9) dx$

(b)  $\int \frac{e^{-2x}}{4 + e^{-2x}} dx$

(c)  $\int \frac{x^2}{\sqrt{x-5}} dx$

[10] 11. Evaluate the integrals:

(a)  $\int_0^1 (x^4 - 5) dx$

(b)  $\int_6^{10} \frac{2}{x-4} dx$

(c)  $\int_4^7 \sqrt{x-2} dx$

[10] 12. Find the area bounded by the graphs of  $f(x) = 5 - x^2$  and  $g(x) = 2 - 2x$ .

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