MATH 1300 E-MIDTERM # 2-2012

Last Name: ________________ **First Name:** ________________

ID# Solutions

Instructions:

- This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.
- **Place your answers to the multiple choice questions in the boxes below.**
- **For long answer questions, YOU MUST SHOW YOUR WORK**
- You may use the backs of pages. If you need additional scrap paper, it will be provided by the proctors.

NO CALCULATORS. NO BOOKS. NO NOTES.

TURN OFF YOUR CELL PHONES AND PUT THEM AWAY.

ANSWERS:

E	B	B	B
#1.	#2	#3	#4

Multiple Choice Section Questions (1-4)

Question 1. *The number of a certain type of bacteria increases exponentially. There are 1000 present at an initial time and 2000 present 4 hours later. After how many hours will there be 3500?*

A) $\frac{6\ln(\frac{7}{4})}{\ln(3)}$ B) $\frac{\ln(7)}{\ln(2)}$ C) $\frac{\ln(\frac{7}{4})}{\ln(6)}$ D) $\frac{\ln(\frac{2}{7})}{4\ln(2)}$ ✓ E) $\frac{4\ln(\frac{7}{2})}{\ln(2)}$

$t=0$ $P_0 = 10^3$

$t=4$ $P(4) = 2\times10^3$

$P(4) = P_0 e^{tK}$

$2\times10^3 = 10^3 e^{4k} \Rightarrow \ln 2 = 4k$

$K = \frac{\ln 2}{4}$

$P(t) = 35\times10^2 = 10^3 e^{\frac{\ln 2}{4}t}$

$\frac{7}{5} = e^{\frac{\ln 2}{4}t} \Rightarrow t = \frac{4\ln(7/2)}{\ln 2}.$

Question 2. *Consider the function* $g(x) = x^3 - 9x^2 + 24x - 3$. *On what interval or intervals is the function concave up?*

A) $(2,\infty)$ ✓ B) $(3,\infty)$ C) $(-2,4)$ D) $(2,4)$ E) It is never concave-up.

$g'(x) = 3x^2 - 18x + 24$

$g''(x) = 6x - 18$

$= 0 \Rightarrow x = 3$

		$x=3$	
g	$\frown$		$\smile$
g''	$-$		$+$

$(3,\infty)$

Question 3. *Suppose that the demand function for a product is given by $p = x^2 - 3x + 5$. What is the elasticity of demand when $x = 3$? Is demand elastic or inelastic?*

A) $\eta = \frac{5}{9}$, elastic B) $\eta = \frac{5}{9}$, inelastic C) $\eta = \frac{9}{5}$, elastic
D) $\eta = \frac{9}{5}$, inelastic E) $\eta = -1$, unit elastic

$$\eta = \frac{\frac{P(3)}{3}}{P'(3)} = \frac{\frac{9-9+5}{3}}{2(3)-3} = \frac{\frac{5}{3}}{3} = \frac{5}{9} < 1$$

inelastic

Question 4. *Suppose that x and y are related by the equation*

$$\ln(x^2 y) + 6x = 12.$$

Use implicit differentiation to find $\frac{dy}{dx}$ at $(2, \frac{1}{4})$.

A) $\frac{13}{7}$ B) $\frac{-7}{4}$ C) $\frac{17}{5}$ D) $\frac{-8}{3}$ E) $\frac{\ln(2)}{3}$

$$\frac{[2xy + x^2 y']}{x^2 y} + 6 = 0$$

$$\frac{2}{x} + \frac{y'}{y} = -6 \Big|_{(2, 1/4)}$$

$$1 + \frac{y'}{\frac{1}{4}} = -6$$

$$4y' = -7$$

$$y' = -7/4$$

Long Answer Section Questions (5-7)

Question 5. (14 points) *For the following function find the appropriate information (listed next page) to sketch the graph of the following function.*

$$f(x) = \frac{x}{x-4}$$

1 point $D_f = \mathbb{R} - \{4\}$

1 point x intercept $\Rightarrow f(0) = 0$ $(0,0)$

y intercept $x = 0 \longrightarrow y = 0$

2 points (1 for each Asy) $\lim_{x \to \infty} f(x) = 1$ $y = 1$ H. A

$x = 4$ V. A

1 ← $f'(x) = \dfrac{x-4-x}{(x-4)^2} = \dfrac{-4}{(x-4)^2} < 0$

1 ← always → f decreases everywhere

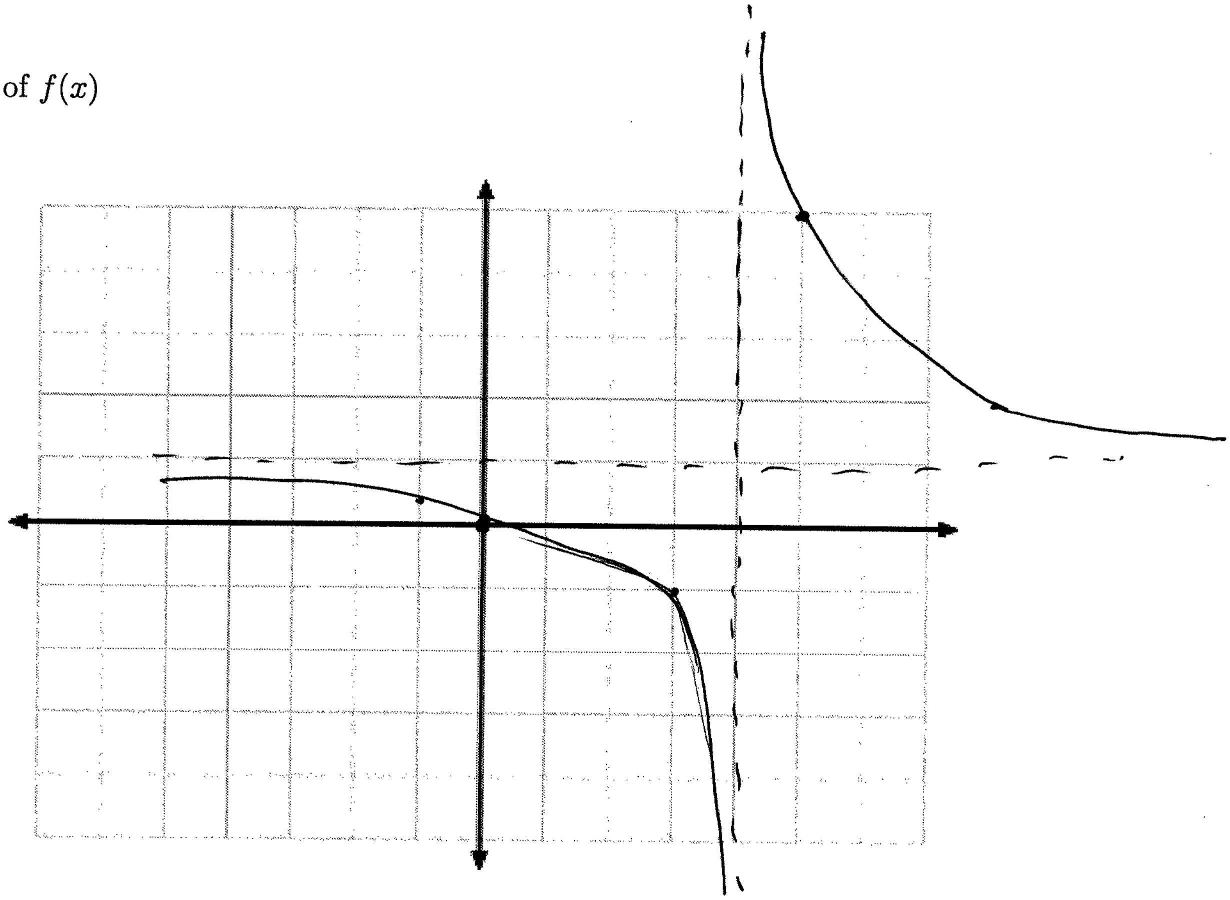
1 ← No local extremas.

1 ← $f''(x) = \dfrac{8}{(x-4)^3}$ $\neq 0$ → NO P.O.I → 1

1 ←

		$x=4$	
f	$\frown$		$\cup$
f''	$-$		$+$

Graph of $f(x)$



1. Find the domain of the function
2. Find the y-intercept and plot it
3. Find the x-intercepts and plot them
4. Find the horizontal asymptotes and plot them
5. Find the vertical asymptotes and plot them
6. Find the critical numbers
7. Find the intervals of increase and decrease
8. Identify the relative extrema and plot them
9. Find the intervals of concave up and concave down
10. Identify the points of inflection and plot them
11. Fill in the rest of the graph using (7) and (9)

4 Points for the graph

(All or Nothing)

Question 6. (12 points)

(a) (4 points) Suppose $f'(x) = 8x^3 + 6x^2 - 3$, and that $f(1) = 2$. Find $f(x)$.

2 points

$$f(x) = \int (8x^3 + 6x^2 - 3)\,dx$$

$$= \frac{8}{4}x^4 + \frac{6}{3}x^3 - 3x + C$$

$$= 2x^4 + 2x^3 - 3x + C$$

$f(1) = 2 + 2 - 3 + C = 2$

1 point $\leftarrow$ $\underline{C = 1}$ $\rightarrow$ $f(x) = 2x^4 + 2x^3 - 3x + 1$

1 point

(b) (8 points) Calculate

$$\int (x^3 + \frac{x}{3} + e^{3x} + 3^x + \pi)dx =$$

$$\underbrace{\frac{x^4}{4}}_{1} + \underbrace{\frac{1}{2}\frac{x}{3}}_{1} + \underbrace{\frac{1}{3}e^{3x}}_{2} + \underbrace{\frac{1}{\ln 3}3^x}_{2} + \underbrace{\pi x}_{1} + \underbrace{C}_{1}$$

Question 7. (14 points) *We wish to design an open box (no top) with a square base. The bottom costs 5 dollars per square centimetre and the sides cost 1 dollar per square centimetre. The total cost is limited at \$240. Find the dimensions which maximize the volume of the box.* **Be sure to explain why your answer is an absolute maximum, and not just a local maximum.**

2 points $\rightarrow$ $C(x,h) = 5x^2 + (1)\,4xh = 240 \Rightarrow h = \dfrac{240 - 5x^2}{4x}$

2 points

$V(x,h) = x^2 h$

$V(x) = x^2\left[\dfrac{240 - 5x^2}{4x}\right] = \dfrac{1}{4}\left[240x - 5x^3\right]$

4 points $\rightarrow$ $V'(x) = \dfrac{1}{4}\left[240 - 15x^2\right] = 0$

1 point

$x^2 = \dfrac{240}{15} = 16$

$x = 4$

2 points $\rightarrow$ $h = \dfrac{240 - 5(4)^2}{16} = \dfrac{160}{16} = 10.$

2 points

$V''(x) = \dfrac{1}{4}\left[-30x\right] < 0$

$V'' < 0 \rightarrow$ max.

Space for additional work

.MATH 1300 E-MIDTERM # 2-2012.

LAST Name: ________________ First Name: ________________
ID# Solutions

Instructions:

- This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.
- **Place your answers to the multiple choice questions in the boxes below.**
- **For long answer questions, YOU MUST SHOW YOUR WORK**
- You may use the backs of pages. If you need additional scrap paper, it will be provided by the proctors.

NO CALCULATORS. NO BOOKS. NO NOTES.

TURN OFF YOUR CELL PHONES AND

PUT THEM AWAY.

If you need additional scrap paper, it will be provided by the proctors.

ANSWERS:

#1	#2.	#3	#4
A	D	E	C

Question 1. *Suppose that x and y are related by the equation*

$$\ln(xy^2) + 4x = 1.$$

Use implicit differentiation to find $\frac{dy}{dx}$ at $(\frac{1}{4}, 2)$.

A) -8 B) $\frac{-11}{4}$ C) -4 D) $\frac{-6}{5}$ E) $\frac{ln(2)}{3}$

$$\frac{y^2 + x2yy'}{xy^2} + 4 = 0$$

$$\frac{1}{x} + \frac{2y'}{y} + 4 = 0 \Big|_{1/4, 2}$$

$$4 + \frac{2y'}{2} + 4 = 0$$

$$\boxed{y' = -8}$$

Question 2. *Suppose that the demand function for a product is given by $p = -x^2 - 6x + 11$. What is the elasticity of demand when $x = 2$? Is demand elastic or inelastic?*

A) $\eta = 4$, elastic B) $\eta = 4$, inelastic C) $\eta = \frac{1}{4}$, elastic
D) $\eta = \frac{1}{4}$, inelastic E) $\eta = -1$, unit elastic

$$\eta = \frac{\frac{P(2)}{2}}{p'(2)} = \frac{\frac{-4 - 12 + 11}{2}}{-2(2) - 6} = \frac{\frac{-5}{2}}{-10} = \frac{-5}{-20} = \frac{1}{4}$$

$$\eta = \frac{1}{4} < 1$$

inelastic

Question 3. *The number of a certain type of bacteria increases at an exponential growth. There are 80 present at an initial time and 160 present 4 hours later. After how many hours will there be 300?*

A) $\dfrac{6\ln(\frac{15}{4})}{\ln(3)}$ B) $\dfrac{\ln(5)}{\ln(2)}$ C) $\dfrac{\ln(\frac{5}{4})}{\ln(6)}$ D) $\dfrac{\ln(\frac{15}{2})}{4\ln(2)}$ E) $\dfrac{4\ln(\frac{15}{4})}{\ln(2)}$

$t = 0 \quad P_0 = 80$

$t = 4 \quad P_4 = 160$

$P(4) = 80 \times e^{K4} = 160$

$e^{4K} = 2 \Rightarrow 4K = \ln 2$

$K = \dfrac{\ln 2}{4}$

$P(t) = 300 = 80 \times e^{t \cdot \frac{\ln 2}{4}}$

$\dfrac{30}{8} = e^{t\frac{\ln 2}{4}}$

$\ln\dfrac{15}{4} = t\dfrac{\ln 2}{4}$

$t = \dfrac{4\ln 15/4}{\ln 2}$

Question 4. *Consider the function* $g(x) = x^3 + 6x^2 + 9x - 17$. *On what interval or intervals is the function decreasing?*

A) $(-\infty, 0)$ B) $(-3, \infty)$ C) $(-3, -1)$ D) $(-\infty, -3)$ E) It is never decreasing.

$g'(x) = 3x^2 + 12x + 9$

$= 3[x^2 + 4x + 3]$

$= 3[x+1)(x+3) = 0$

$x = -1 \quad x = -3$

g	$\nearrow$	$x=-3$ $\searrow$	$x=-1$	$\nearrow$
g'	$+$	$-$		$+$

Question 6. (14 points) *For the following function find the appropriate information (listed next page) to sketch the graph of the following function.*

$$f(x) = \frac{x}{x+4}$$

$D = \mathbb{R} - \{-4\}$

x intercept $\quad y=0 \rightarrow x=0 \quad (0,0)$

y - intercept $\quad x=0 \rightarrow y=0$

$\lim_{x \to \infty} f(x) = 1 \qquad y = 1$ H. A

$x = -4$ V. A

$$f'(x) = \frac{x+4-x}{(x+4)^2} = \frac{4}{(x+4)^2} > 0$$

always positive

Always increases.

No local extremas.

$$f''(x) = \frac{-8}{(x+4)^3} \neq 0$$

No P.O.I

		$x=-4$	
f''	$\cup$		$\cap$
f	$+$		$-$

Long Answer Section Questions (5-7)

Question 5. (14 points) *We wish to design a storage box (with top and bottom) with square base and top. The bottom and top costs 4 dollars per square centimetre and the sides cost 1 dollar per square centimetre. The total cost is limited at \$600. Find the dimensions which maximize the volume of the box.* **Be sure to explain why your answer is an absolute maximum, and not just a local maximum.**

$$C(x,h) = 2x^2(4) + (1)4xh = 600$$

$$8x^2 + 4xh = 600$$

$$\Rightarrow h = \frac{600 - 8x^2}{4x}$$

$$V(x,h) = x^2 h$$

$$\Rightarrow V(x) = x^2\left(\frac{600 - 8x^2}{4x}\right)$$

$$= \frac{1}{4}(600x - 8x^3)$$

$$V'(x) = \frac{1}{4}(600 - 24x^2)$$

$$= 0 \quad \Rightarrow \quad x^2 = \frac{600}{24} = 25$$

$$x = 5$$ (x only can be positiv

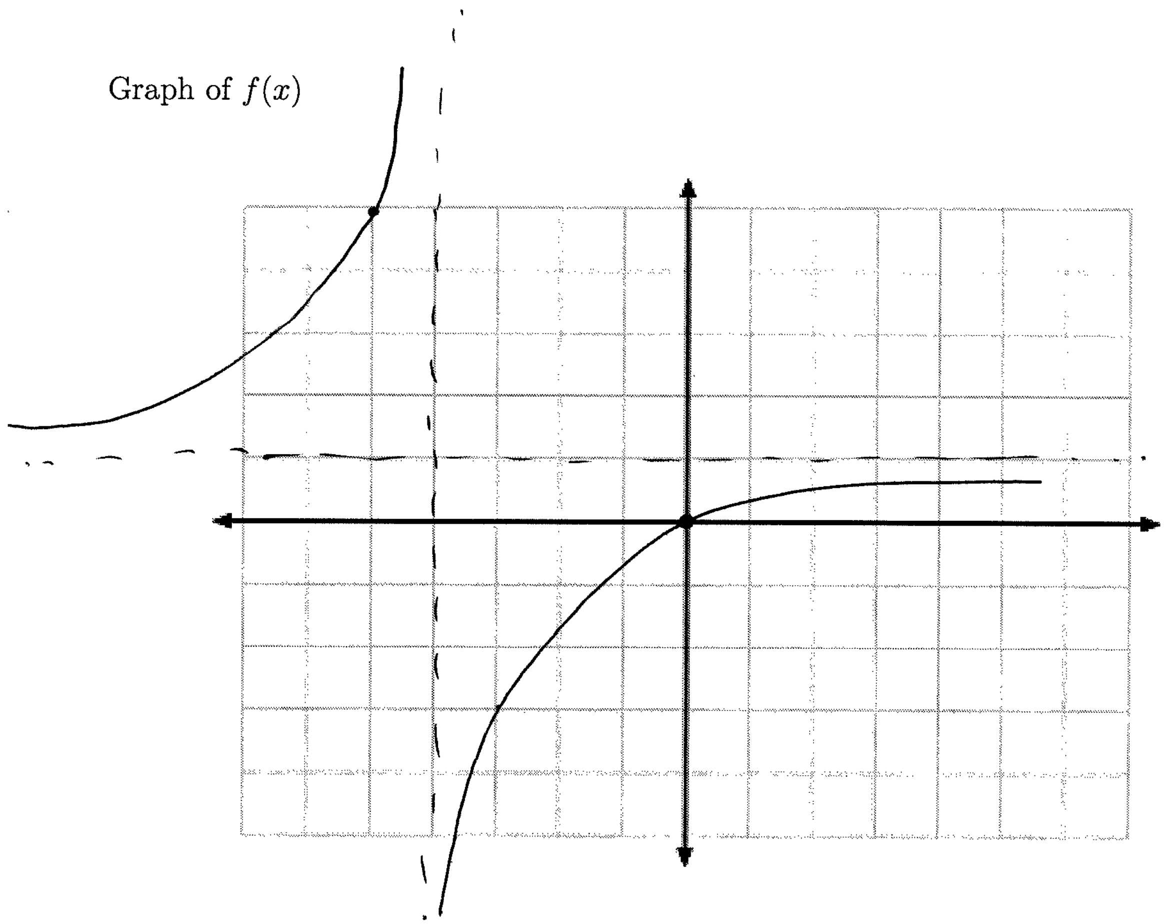
$$h = \frac{600 - 8(25)}{20} =$$

$$\frac{600 - 200}{20} = 20.$$

$$V''(x) = \frac{1}{4}(-48x) < 0$$

$\mapsto$ max.

Graph of $f(x)$



1. Find the domain of the function
2. Find the y-intercept and plot it
3. Find the x-intercepts and plot them
4. Find the horizontal asymptotes and plot them
5. Find the vertical asymptotes and plot them
6. Find the critical numbers
7. Find the intervals of increase and decrease
8. Identify the relative extrema and plot them
9. Find the intervals of concave up and concave down
10. Identify the points of inflection and plot them
11. Fill in the rest of the graph using (7) and (9)

Question 7. (12 points)

(a) (4 points) Suppose $f'(x) = 9x^2 + 2x - 3$, *and that* $f(1) = 2$. *Find* $f(x)$.

$$f(x) = \int (9x^2 + 2x - 3)\,dx$$

$$= 3x^3 + x^2 - 3x + C$$

$$f(1) = 3 + 1 - 3 + C = 2$$

$$\underline{C = 1}$$

$$f(x) = 3x^3 + x^2 - 3x + 1$$

(b) (8 points) Calculate

$$\int (x^5 + \frac{x}{5} + e^{5x} + 5^x + e^5)dx$$

$$= \frac{x^6}{6} + \frac{x^2}{10} + \frac{1}{5}e^{5x} + \frac{5^x}{\ln 5} + e^5 x + C$$

Space for additional work