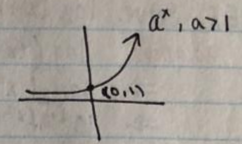
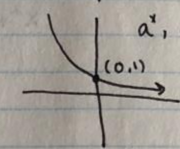
Exponential Functions

$f(x) = a^x$ where $a > 0$, a is called base.



Domain is $(-\infty, +\infty)$
Range is $(0, +\infty)$

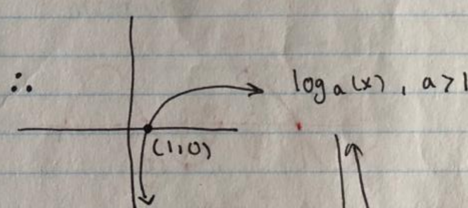


Properties: pg 73 for laws of exponants.

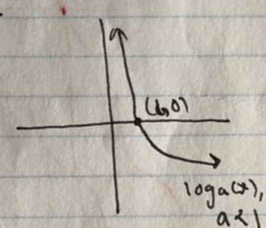
Logrithmic Function

for $a > 0$ AND $a \neq 1$, the log F^{CT} $\log_a(x)$ is defined as the inverse of the function a^x.

$\log_a(x) = y \Leftrightarrow a^y = x$, $\therefore$



Domain is $(0, +\infty)$
Range is $(-\infty, +\infty)$



$\left.\begin{array}{l}\log_a(a^x) = x \\ a^{\log_a(x)} = x\end{array}\right\}$ inverse function undo eachother

$\ln(x) = \log_e(x)$

Properties: pg 76 for laws of logrithms

EX. Solve $2^{x+3} = 16^{2x-1}$ for x.

solution: $2^{x+3} = (2^4)^{2x-1} \Rightarrow 2^{x+3} = 2^{8x-4}$

$\Rightarrow \log_2(2^{x+3}) = \log_2(2^{8x-4}) \Rightarrow x+3 = 8x-4$

$7x = 7$

$\boxed{x = 1}$

EX. Solve $\log(x+1) + \log(x+4) = 1$ for x

solution: $\log\left((x+1)(x+4)\right) = 1$

$\Rightarrow \log_{10}(x^2+5x+4) = 1$

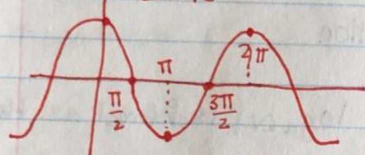
$\Rightarrow 10^{\log_{10}(x^2+5x+4)} = 10^1$

$x^2+5x+4 = 10 \Rightarrow x^2+5x-6=0 \Rightarrow (x+6)(x-1)=0$ $\boxed{\cancel{x=-6} \quad x=1}$ Cant have (−) logs

* $x=-6$ is NOT in the domain of the F^{ct}

Trigonometric functions

Cosine

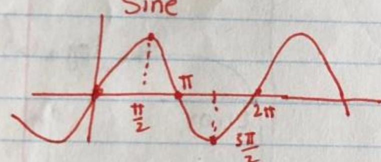


Domain: $(-\infty, +\infty)$
range: $[-1, +1]$
Period: 2π

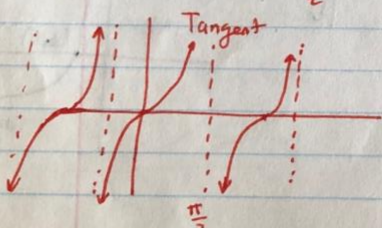
⇓

//

Sine



Tangent

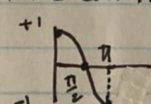


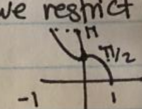
Domain: $\{x \in \mathbb{R} \mid x \neq \frac{n\pi}{2} \ n = \pm1, \pm3, \ldots\}$
range: $(-\infty, +\infty)$
Period: π

Identities: $(\sin x)^2 + (\cos x)^2 = 1$
: $\sin(x+y) = \sin x \cos y + \cos x \sin y$
: $\cos(x+y) = \cos x \cos y - \sin x \sin y$

Inverse Trig F^{ct}: in general, trig functions are not one-to-one, so don't have inverses in general. BUT we can if we restrict domain.

EX. $\cos\theta$ on $[0, \pi]$





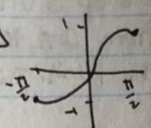
arccos(x)

$x = \frac{1}{3y-2}$

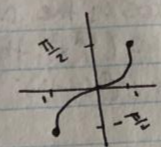
	Domain	Range
arcsin(x)	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
arccos(x)	$[-1, 1]$	$[0, \pi]$
arctan(x)	$(-\infty, +\infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

CONT.

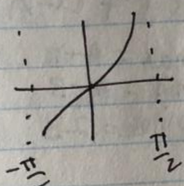
Sin θ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$



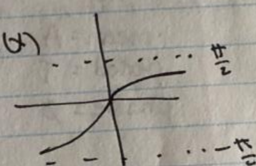
arcsin(x)



tan ϕ on $(-\frac{\pi}{2}, \frac{\pi}{2})$



arctan(x)



$\arcsin(x) = \phi \iff \sin\theta = x$

$\arccos(x) = \phi \iff \cos\phi = x$

$\arctan(x) = \phi \iff \tan\phi = x$

EX. Find the values of $\arcsin(\frac{1}{2})$ and $\arccos(-\frac{1}{\sqrt{2}})$

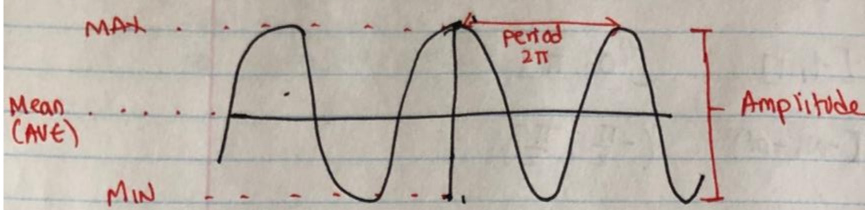
SOLUTION: $\arcsin(\frac{1}{2}) = \phi \;(=)\; \sin\phi = \frac{1}{2}$

We need $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$, so $\boxed{\phi = \frac{\pi}{6}}$

arccose $(-\frac{1}{\sqrt{2}}) \;(=)\; \cos\phi = -\frac{1}{\sqrt{2}}$

We need $0 \le \phi \le \pi$, so $\phi: \frac{\pi}{2} + \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

Transformation of Cosine



EX → $2\cos x$: amplitude doubles

→ $3+2\cos x$: the ave + whole graph moves up 3 units

→ $3+2\cos\left(\frac{2\pi}{4}x\right)$: the period changes from 2π to 4 ← $\frac{2\pi}{4}x=2\pi$ ∴ $x=4$

→ $3+2\cos\left(\frac{2\pi}{4}(x-1)\right)$: graph shifts to right by 1 unit. This # is called phase.

IN GENERAL $A+B\cos\left(\frac{2\pi}{T}(x-\phi)\right)$

· mean $=A$
period $=T$
phase $=\phi$

max $=A+B$
min $=A-B$
Amplitude $=2B$