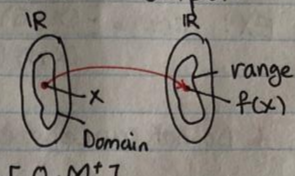
sec 1.3, 2.1

A function f consits of 2 parts.

1. Domain: the set of real numbers allowed as inputs
2. A rule that assigns exactly <u>one</u> output per # in domain

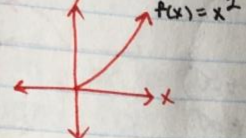
↳ the range of a $f^{CT}$ is the set of all outputs

$\mathbb{R}$ = the set of real #s




EX: consider $f(x) = x^2$ w/ Domain $[0, \infty^+]$.

then:

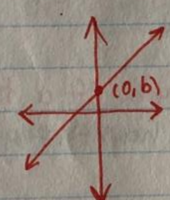



Convention: if f is only given as formula, we assume domain = $\mathbb{R}$ except for parts that don't make sense.

EX. $f(x) = \frac{1}{x}$ ; domain: $(-\infty, 0) \cup (0, +\infty)$

$g(x) = \sqrt{x}$ ; domain $[0, +\infty)$ range: $[0, +\infty)$

## Linear Functions




$f(x) = mx + b \leftarrow f(0)$

↑ slope

## Polynomial Functions

$a < 0$

$a > 0$

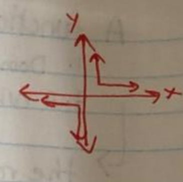
$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

Domain = $(-\infty, +\infty)$

for degree 2 $(n=2) \rightarrow f(x) = ax^2 + bx + c$

## Rational Functions

$r(x) = \frac{p(x)}{q(x)}$ where $p, q$ are polynomials.




Domain: $\mathbb{R}$ except points where $q(x)=0$

EX: $r(x) = \frac{-(x+5)(x-1)}{(x+1)(x-1)}$ ; Domain is $(-\infty,-1) \cup (-1,1) \cup (1,+\infty)$

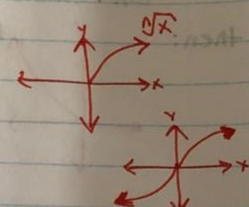
* Do not cancel zero ; $\frac{3 \cdot 0}{2 \cdot 0} \Rightarrow \frac{3}{2} \neq \frac{0}{0}$ *

## Root Functions

$f(x) = \sqrt[n]{x}$ or $x^{\frac{1}{n}}$ ; $n = 1, 2, 3, \ldots$

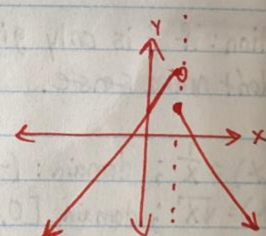
↳ when $n$ is even, domain is $[0, +\infty)$

↳ " " " odd, " " $(-\infty, +\infty)$




## Piecewise-defined Functions

EX: $f(x) = \begin{cases} 2x+1 & \text{if } x<1 \\ -x+2 & \text{if } x \geq 1 \end{cases}$




## The Graph Of a Function pg 16

$f(x)$ is height of graph above n # $x$, or <u>Signed Vertical Distance</u>

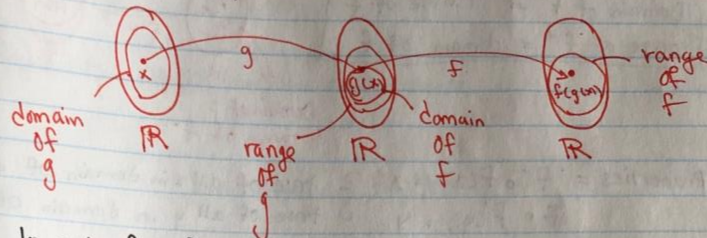
## Vertical Line Test Pg 18

A set of points represents a function if every vertical line crosses the set at most once.

Sec (1.4, 2.2, 2.3)

## Composition of Functions

Let $f$ and $g$ be functions; then the composition $f \circ g$ is the function defined by $f \circ g\,(x) = f(g(x))$




In order for $f(g(x))$ to make sense we need $x$ to be in the domain of $g$ and $g(x)$ to be in the domain of $f$.

Ex. $f(x) = \frac{x-1}{x+1}$, $g(x) = \frac{1}{x}$, find domain of composition $f \circ g$

solution: $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \dfrac{\frac{1}{x} - 1}{\frac{1}{x} + 1} = \dfrac{\frac{1-x}{x}}{\frac{1+x}{x}}$

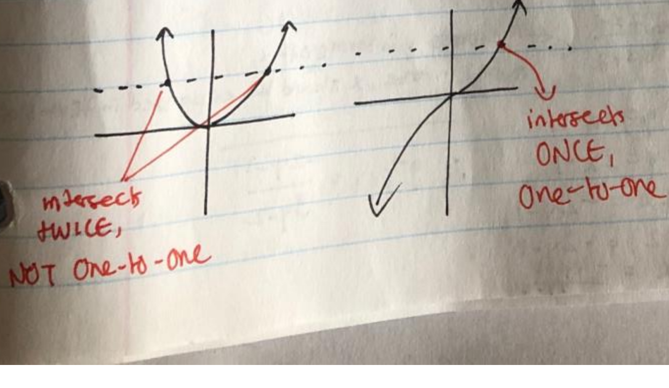
$= \frac{(1-x)}{(1+x)} \frac{x}{x}$ * DO NOT CANCEL $\frac{x}{x}$

Domain is $\{x \in \mathbb{R} \mid x \neq 0, x \neq -1\}$ or $\boxed{(-\infty, -1) \cup (-1, 0) \cup (0, +\infty)}$

## Inverse of Function

a function is one-to-one if no distinct inputs produce the same outp

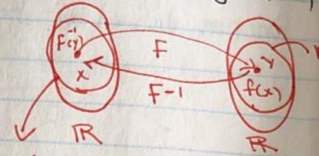
We use 'horizontal line test'



Def. Let $f$ be a one-to-one $f^{CT}$. Then the inverse of that, denoted $f^{-1}$, is defined by the rule:

$$f^{-1}(y) = x \quad , \quad f(x) = y \qquad \text{(y is in the range of x)}$$

| |
|---|
| Domain of $f^{-1}$ = range of $f$ |
| range of $f^{-1}$ = domain of $f$ |




range of $f$ = domain of $f^{-1}$

Domain of $f$ = range of $f^{-1}$

Properties = $f^{-1} \circ f(x) = x$ } true of all $x$ in domain of $f$

$f \circ f^{-1}(y) = y$ } true of all $y$ in domain of $f^{-1}$

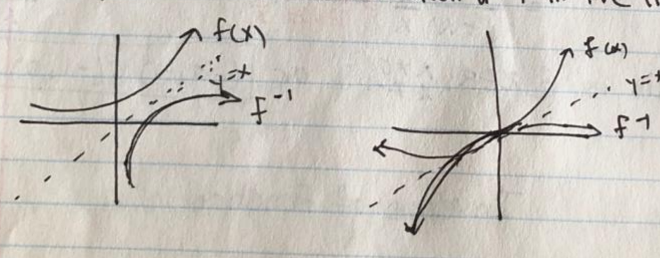
EX. $f(x) = x^3$

solution 1. Is $f^{CT}$ 1 to 1 ? yes.

Then $f^{-1}(y) = y^{\frac{1}{3}}$

$g(y) = \sqrt{y}$ is not the inverse of $f(x) = x^2$

Graph of $f^{-1}$ is the reflection of $f$ in the line $y = x$




EX. Find inverse of $g(x) = \dfrac{2x-1}{3x+2}$

solution: $y = g(x) = \dfrac{2x-1}{3x+2}$ ← express $y$ in terms of $x$

to find inverse, $x$ should be expressed in terms of $y$

$$y = \frac{2x-1}{3x+2} \Rightarrow (3x+2)y = 2x-1$$
$$\Rightarrow 3xy + 2y = 2x - 1$$
$$\Rightarrow 3xy - 2x = -2y - 1$$
$$x(3y-2) = -2y-1$$
$$x = \frac{-2y-1}{3y-2}$$

$$\therefore g^{-1}(y) = \frac{-2y-1}{3y-2}$$