Tutorial 1: Getting started w/ Limits

If f is continuous at b and $\lim_{x\to a} g(x) = b$ then $\lim_{x\to a} f(g(x)) = f(b)$

In other words: $\lim_{x\to a} f(g(x)) = f\lim_{x\to a} g(x)$

p. 8

1. $\lim_{x\to 3} f(x) = 14 \quad \lim_{x\to 3} g(x) = -2$

$$\lim_{x\to 3}\left(3(f(x)g(x) + \frac{g(x)}{(f(x))^2} - \sqrt{14 f(x)} + \ln(3+g(x))\right).$$

$$= 3\lim_{x\to 3} f(x)g(x) + \lim_{x\to 3}\frac{g(x)}{(f(x))^2} - \sqrt{14\lim_{x\to 3} f(x)} + \ln(3+\lim_{x\to 3} g(x))$$

$$= 3(14)(-2) + \frac{-2}{(14)^2} - \sqrt{14(14)} + \ln(3+-2)$$

$$= -84 - \frac{1}{98} - 14 + 0 = -98$$

2. $g(x) = \sin(\frac{\pi}{x})$

a) evaluate $g(\frac{1}{10}) = 0 \quad g(\frac{1}{100}) = 0 \quad g(\frac{1}{1000}) = 0$ guess it is zero

b) $g(\frac{3}{10}) = -\sqrt{3}/2 \quad g(\frac{3}{100}) = -\frac{\sqrt{3}}{2} \quad g(\frac{3}{1000}) = -0.866 \quad \frac{-\sqrt{3}}{2}$

c) doesn't exist b/c there is no specific point → as shown in a vs b

* d) sandwich Therom $\lim_{x\to 0} x^2 g(x)$

$$-|x^2| \le g(x) \le |x^2|$$

$$\lim_{x\to 0} -|x^2| = 0 \qquad \lim_{x\to 0} |x^2| = 0$$

3. find function $f(x)$ and $g(x)$ so neither $\lim_{x\to 2} f(x)$ nor $\lim_{x\to 2} g(x)$ exists but $\lim_{x\to 2} \frac{f(x)}{g(x)}$ does.

$f(x) = \frac{1}{x-2}$ $\quad \lim_{x\to 2} \frac{\frac{1}{x-2}}{g(x)} = \frac{1}{x-2}\cdot\frac{x-2}{1}$ $\quad g(x) = \frac{-1}{x-2}$ $\quad = -1$

4. $x\to -2^- = \quad x\to -2^+ = 0$ → b/c not same

$0^- - 2 \qquad 0^+ = -2$ →

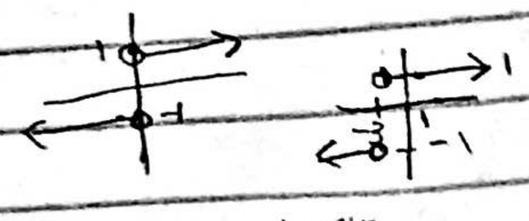
$0^- f(fx) = 0 \qquad = 0$ → Sarah

Tutorial #2: Limits + Continuity

( ) excluding (no real endpoints) [ ] including

p. 16

1. $y = \frac{|x|}{x}$ $y = \frac{|x+3|}{x+3}$



no b/c not approaching same limit as $x \to 0$ and $x \to -3$

2. $g(x) = (-\infty, \infty)$ domain $\lim_{x \to 4^-} g(x) = 4$ $\lim_{x \to 4^+} g(x) = -1$

$\lim_{x \to 4} g(x) = DNE$ $\lim_{x \to 2} g(x^2) = DNE$

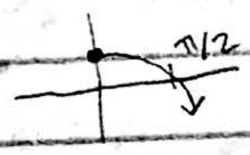
$\lim_{x \to 4^+} (20 - x^2)$

$\lim_{x \to 4^+} 20 - \lim_{x \to 4^+} x^2$

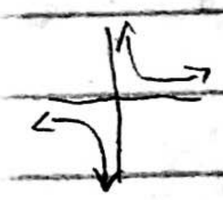
$20 - (4)^2 = 4$

3. W/out finding it, explain how we know there must be a # c such that $\cos(c) = \sqrt{c}$

b/c the IVT applied to $f(x) = \cos(x) - \sqrt{x}$ where we observe $f(0) > 0$ and $f(\pi/2) < 0$ $(0, \infty)$ $\checkmark$ continuous



* 4. $\lim_{x \to 0} \frac{1}{x^n}$



a) n? does $\lim_{x \to 0} \frac{1}{x^n} = \infty$ → positive even integer

b) $= -\infty$ none

c) = doesn't exist → positive odd integer

d) $= 0$ negative integer

5. look @ book

6. $x \leq 1$ jump

$x = 2$ removable