MAT 2377C
Final

20 December 2018 **Professor: Mohsen Rezapour**
Time: 180 minutes

Student Number: ________________

Family Name: ________________

First Name: ________________

- This is a closed book exam.
- Your package includes the title page, five pages with questions, a formula sheet and tables.
- Only the calculators TI 30, TI 34, Casio fx-260 and Casio fx-300 are allowed.
- At the end of exam you need to submit the complete exam booklet.
- **Record your answer to each question in the table below.**
- Number of questions: **19**.
- If you see any error in this exam, please report it on your paper.

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Cellular phones, unauthorized electronic devices or course notes
(unless an open-book exam) are not allowed during this exam.
Phones and devices must be turned off and put away in your bag.
Do not keep them in your possession, such as in your pockets.
If caught with such a device or document, the following may occur:
you will be asked to leave immediately the exam and academic fraud
allegations will be filed which may result in you obtaining a
0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you
are complying with the above statement.

X___________________________________
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Page for reporting errors

If you see any error in this exam, please report it on this page.

- **Record your answer to each question in the table below.**
- Shade **ONE** letter for each question. A question with more than one shading answers will not be marked.

Question	a	b	c	d	e
1					
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GOOD LUCK !!!

Q1. An agent believes that a customer will buy a tax-free bonds with a probability of 0.4, will buy a mutual funds with a probability of 0.5, and will buy both tax-free bonds and mutual funds with a probability of 0.15. Find the probability that a customer will buy neither tax-free bonds nor mutual funds.

(a)	0.58	(b)	0.25	(c)	0.7
(d)	0.4	(e)	None of the preceding		

Solution to Q1:

If A and B be the event of investing in tax-free bonds and mutual funds, respectively. Then

$$P((A \cup B)') = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1 - (0.4 + 0.5 - 0.15) = 0.25\,.$$

Answer b). There is no rounding involved here. No other answer accepted.

Q2. In a manufacturing company, three devices, B1, B2, and B3, make 30%, 45%, and 25%, respectively, of the products and 2%, 3%, and 2% of the products made by each device, respectively, are defective. If a randomly selected product is defective, what is the probability that it was made by machine B2?

(a)	0.689	(b)	0.267	(c)	0.551
(d)	0.425	(e)	None of the preceding		

Solution to Q2:

If D be the defective event

$$P(B_2|D) = \frac{P(D|B_2)P(B_2)}{\sum_{i=1}^{3} P(D|B_i)P(B_i)} = \frac{0.03 * 0.45}{0.02 * 0.3 + 0.03 * 0.45 + 0.02 * 0.25}$$

Answer c). There is no rounding involved here. No other answer accepted.

Q3. Consider the discrete random variable X with probability mass function

m	0	1	3
$P(X = m)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

The cumulative distribution function $F(x)$ at 2.5, i.e. $F(2.5)$ equals.

(a)	$\frac{5}{6}$	(b)	$\frac{1}{6}$	(c)	$\frac{1}{2}$
(d)	0	(e)	None of the preceding		

Solution to Q3:

We have $F(2.5) = P(X < 2.5) = P(X = 0) + P(X = 1) = 5/6\,.$
Answer a).

Q4. Consider the continuous random variable X with density function

$$f(x) = \begin{cases} \frac{x^2}{3}\,, & \text{if } -1 < x < 2\,, \\ 0\,, & \text{elsewhere.} \end{cases}$$

Then $P(0 < X \leq 1)$ equals

(a) $\frac{3}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{9}$

(d) 0 (e) None of the preceding

Solution to Q4:

We have $P(0 < X \leq 1) = \int_0^1 f(x)dx = 1/9$.
Answer c).

Q5. A manufacturer knows that on average 70% of its products require repairs within 1 year after they are sold. Find the probability that among the 6 randomly selected products, exactly 2 of them will require repairs.

(a) 0.1 (b) 0.7 (c) 0.8

(d) 0.06 (e) None of the preceding

Solution to Q5:

We have $P(X = 2) = \binom{6}{2}(0.7)^2(0.3)^4 = 0.059535 \sim 0.06$
Answer d).

Q6. The distribution of the number of cars per year that will experience a fault in braking mechanism is a Poisson random variable with mean 6. What is the probability that more than 1 car per year will experience the fault in braking mechanism?

(a) 0.98 (b) 0.85 (c) 0.58

(d) 0.61 (e) None of the preceding

Solution to Q6:

We have $P(X > 1) = 1 - (P(X = 0) + P(X = 1)) = 1 - (e^{-6} + \frac{e^{-6}6}{1!}) = 0.9826487$
Answer a).

Q7. Suppose that $X_1, \ldots, X_5$ is a random sample from a population with unknown mean μ. Consider the following estimators of μ :

$$\hat{\mu}_1 = (X_1 + X_2)/2\,, \quad \hat{\mu}_2 = (X_1 + X_2 + X_3)/2\,, \quad \hat{\mu}_3 = (X_1 + 2X_2)/3\,, \quad \hat{\mu}_4 = (X_1 + X_2 + X_3 + X_4 + X_5)/5\,.$$

Which one of the following statements is <u>not</u> true?

(a) $\hat{\mu}_4$ is the most efficient estimator.

(b) $\hat{\mu}_3$ is an unbiased estimator.

(c) $\hat{\mu}_2$ is a biased estimator.

(d) $\hat{\mu}_3$ is more efficient than $\hat{\mu}_1$.

(e) $\hat{\mu}_3$ is more efficient than $\hat{\mu}_4$.

Solution to Q7:

Both answers d and e are true. If you choose d or e or both of them, or if you mentioned the error in the error page you will get the mark of this question.

Q8. The quality control section of a manufacturer producing cylindrical component parts for the automotive industry claims that they produce parts having a mean diameter of 5.0 millimeters and standard deviation of $\sigma = 0.1$ millimeter. A sample of 100 parts produced by the process is selected randomly and the diameter is measured. What is the probability that the sample mean is bigger than 5.02 millimeters?

(a) 0.9772 (b) 0.0228 (c) 0.3654

(d) 0.106 (e) None of the preceding

Solution to Q8:

We have $P(\bar{X} > 5.2) = P((\bar{X} - \mu)/(\sigma/\sqrt{n}) > 2) = 0.0228$.
Answer b).

Q9. In a certain steel product, two alloys A and B are used. From the past experience we know that the two standard deviations in load capacity are known and equal to 5 tons each. A sample of 50 specimens of each alloy (A and B) are tested. If we assume that $\mu_A = \mu_B$, what is the probability that the absolute value of difference of two sample mean is less than 5 (i.e. $-0.5 < \bar{x}_A - \bar{x}_B < 0.5$)?

(a) 0.772 (b) 0.654 (c) 0.428

(d) 0.383 (e) None of the preceding

Solution to Q9:

We have $P(-0.5 < \dfrac{\bar{x}_A - \bar{x}_B}{5\sqrt{1/50 + 1/50}} < 0.5) = P(Z < 0.5) - p(z < -0.5) = .6915 - .3085 = 0.383$.
Answer d).
There is a typo in the question if your reported it you get the mark.

Q10. In a research study, the water sample was collected by a manufacturer 25 times randomly. From the past experience we know that standard deviation of the population σ is 100 ppm. What is the probability that the sample variance is less than 138.3167? (Assume that the population has normal distribution.)

(a) 0.1 (b) 0.25 (c) 0.75

(d) 0.9 (e) None of the preceding

Solution to Q10:

We have $P(S^2 < 138.3167) = P((n-1)S^2/\sigma^2 < 0.33196) = 1 - P(X^2 > 0.33196)$. According to the chi-squared table $P(X^2 > 0.33196) \sim 1$ therefore, $1 - P(X^2 > 0.33196) \sim 0$, therefore the answer is e. This question want to check if you understand the difference between variance and standard deviation. Although it is not a typo, if you mentioned it as an error you will get the score.

Q11. From past studies we know that the standard deviation of zinc concentration in a river is 0.25 gram per milliliter. A random sample of 25 measurements in different locations of a river is obtained. The average zinc concentration recovered from the sample of measurements is 2.76 grams per milliliter. Find the 99% confidence interval for the mean zinc concentration in the river.

(a) $(2.15, 2.68)$ (b) $(2.63, 2.88)$ (c) $(2.14, 2.96)$
(d) $(2.47, 2.94)$ (e) None of the preceding

Solution to Q11:

The confidence interval is $2.76 - (2.575)0.25/\sqrt{25} < \mu < 2.76 + (2.575)0.25/\sqrt{25}$
Answer b).

Q12. In question 11, if the variance of the population is unknown and the sample variance is 0.49 , then the 95% confidence interval for μ equals.

(a) $(2.07, 2.32)$ (b) $(2.18, 2.78)$ (c) $(2.47, 3.04)$
(d) $(2.23, 2.64)$ (e) None of the preceding

Solution to Q12:

The confidence interval is $2.76 - (2.064)0.7/\sqrt{25} < \mu < 2.76 + (2.064)0.7/\sqrt{25}$
Answer c).

Q13. The following summary of a data represents the length of time, in days, to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication 1	Medication 2
$n = 12$	$m = 16$
$\bar{x} = 16$	$\bar{y} = 20$
$s_1^2 = 1.5$	$s_2^2 = 1.8$

Find a 90% confidence interval for the difference $\mu_2 - \mu_1$ in the mean recovery times for the two medications, assuming normal populations with equal variances. (μ_1 and μ_2 are the mean of length of time to recovery for patients treated with Medication 1 and Medication 2, respectively.)

(a) $(-4.84, -3.15)$ (b) $(-5, -2.99)$ (c) $(3.15, 4.84)$
(d) $(2.70, 5.29)$ (e) None of the preceding

Solution to Q13:

The confidence interval is $20 - 16 - (1.706)s_p\sqrt{1/12 + 1/16} < \mu_2 - \mu_1 < 20 - 16 + (1.706)s_p\sqrt{1/12 + 1/16}$
Answer c).

Q14. A random sample of 31 bags of white cheddar popcorn weighted, on average, 5.23 ounces with a standard deviation of 2.24 ounce. Consider the hypothesis testing $H_0 : \mu = 5.5$ against the alternative hypothesis, $H_1 : \mu < 5.5$ ounces, at the 0.05 level of significance. The P-value of the test falls in the interval

(a) $(0.2, 0.3)$, and we do not have sufficient evidence to reject H_0.

(b) $(0.05, 0.025)$, and we reject H_0.

(c) $(0.2, 0.3)$, and we reject H_0.

(d) $(0.05, 0.025)$, and we do not have sufficient evidence to reject H_0.

(e) None of the preceding

Solution to Q14:

$P - value = P(T < -0.6711145) = P(T > 0.6711145) \in (0.2, 0.3)$
Answer a)

Q15. A random sample of size $n = 15$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\bar{x} = 81$. A second random sample of size $m = 16$, taken from a different normal population with a standard deviation $\sigma_2 = 3.4$, has a mean $\bar{y} = 76$. Which one of the following statements is true about the hypothesis testing $H_0 : \mu_1 = \mu_2$ against the alternative, $H_1 : \mu_1 \neq \mu_2$. Assume that $\alpha = 0.01$.

(a) The test statistics is bigger than $-z_{\alpha/2}$, therefore, we do not have sufficient evidence to reject H_0.

(b) The test statistics is bigger than $-z_\alpha$, therefore, we reject H_0.

(c) The test statistics is bigger than $z_{\alpha/2}$, therefore, we reject H_0.

(d) The test statistics is less than z_α, therefore, we reject H_0.

(e) The test statistics is bigger than z_α, therefore, we do not have sufficient evidence to reject H_0.

Solution to Q15:

Since $\frac{\bar{x} - \bar{y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} = 3.14648$ and $z_{\alpha/2} = 2.57$, therefore part (c) is true.
Answer c).

Q16. In a laboratory effect of a new serum to arrest leukemia was investigated. Survival times, in years, from the time the experiment commenced are as follows:

```
Treatment=c( 2.1, 5.3, 1.4, 4.6, 0.9)
NoTreatment=c(1.9, 0.5, 2.8, 3.1)
```

To investigate whether the serum is effective or no? (hypothesis testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$,) we use the following code in R:

t.test(Treatment,NoTreatment,var.equal=TRUE)

and obtain

```
 Two Sample t-test

data:  Treatment and NoTreatment
t = 0.69899, df = 7, p-value = 0.5071
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
```

```
 -1.870604  3.440604
sample estimates:
mean of x mean of y
    2.860     2.075
```

If we assume the two populations to be normally distributed with equal variances and $\alpha = 0.05$, which of the following statement is true.

(a) We use the correct command. There is no sufficient evidence to justify that the serum is effective;

(b) We use the correct command. There is sufficient evidence to justify that the serum is effective;

(c) We do not use the correct command.

(d) We use the correct command, but based on the output we can not consider the desirable hypothesis testing.

Solution to Q16:

Answer a

For Questions 17 and 18 use the data below:

	1	2	3	4	5	6	7	8	9	10
x	17.3	19.3	19.5	19.7	22.9	23.1	26.4	26.8	27.6	28.1
y	71.7	48.3	88.3	75.0	91.7	100.0	73.3	65.0	75.0	88.3

Hint: $\sum_{i=1}^{n} x_i^2 = 5464.71$, $\sum_{i=1}^{n} y_i^2 = 62324.34$, $\sum_{i=1}^{n} x_i y_i = 18010.23$, $\sum_{i=1}^{n} x_i = 230.7$, and $\sum_{i=1}^{n} y_i = 776.6$,

Q17. A study was conducted at Virginia Tech to determine if certain static arm-strength measures have an influence on the dynamic lift characteristics of an individual.

Ten individuals were subjected to strength tests and then were asked to perform a weightlifting test in which weight was dynamically lifted overhead. Arm-strength, x, as independent variable and the dynamic lift, Y, as a response are measured. The fitted regression $Y = \beta_0 + \beta_1 x + \epsilon$ is:

(a) $\hat{y} = 62.42672 + 0.660307x$

(b) $\hat{y} = 117.1532 - 0.15123x$

(c) $\hat{y} = 5.53349 - 0.003634x$

(d) $\hat{y} = 2.23485 - 0.1521343x$

(e) None of the preceding.

Solution to Q17:

$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 = 5464.71 - (230.7)^2/10 = 142.461$, $S_{xy} = 18010.23 - 230.7 * 776.6/10 = 94.068$, so that $b_1 = S_{xy}/S_{xx} = 94.068/142.461 = 0.660307$, $b_0 = \sum_{i=1}^{n} y_i/n - b_1 \sum_{i=1}^{n} x_i/n = 776.6/10 - 0.660307 * 230.7/10 = 62.42672$.

Answer a)

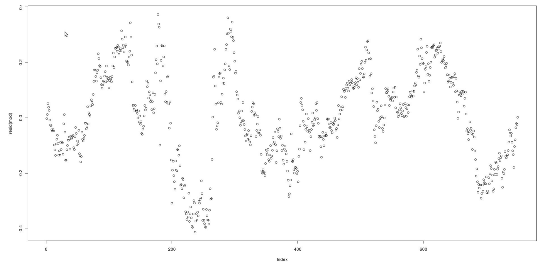

FIGURE 1. Plot of residuals against fitted value

Q18. A 95% prediction interval for single response y_0 given $x = 20$ is
(a) (56.254,85.265)
(b) (47.265,78.245)
(c) (36.739,114.526)
(d) (49.256,81.325)
(e) None of the preceding.

Solution to Q18:

$S^2 = \frac{SSE}{n-2} = \frac{S_{yy} - b_1 S_{xy}}{n-2} = \frac{S_{yy} - b_1 S_{xy}}{n-2} = \frac{62324.34 - 776.6^2/10 - 0.660307 * 94.068}{8} = 243.9338$ The prediction interval is $\hat{y_0} \pm t_{\alpha/2} s \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} = 62.42672 + 0.660307 * 20 \pm 2.306 * \sqrt{243.9338} * \sqrt{1 + 0.1 + (20 - 23.07)^2/142.461}$
Answer c).

Q19. Figure 1 is the plot of the residuals against the fitted values in a regression model. Which one of the following statements about this figure is <u>not</u> true.
(a) This plot confirms that the residuals are not independent.
(b) We can not justify the normality assumption with this plot.
(c) This plot shows that the residuals do not have constant variance.
(d) This plot confirms that the residuals have normal distribution.

Solution to Q19:

Answer d).

This is the last question

MAT 2377
Final Exam Formula Sheet

- Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Total probability rule:

$$\begin{aligned} P(B) &= P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_n) \\ &= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_n)P(A_n) \end{aligned}$$

- Bayes' rule

$$P(A_r|B) = \frac{P(B|A_r)P(A_r)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$$

- Events A and B are independent if $P(A \cap B) = P(A)P(B)$
- Expected value of a discrete random variable X:

$$\mu = E(X) = \sum_x x f(x), \quad \text{where} \quad f(x) = P(X = x)$$

- Expected value of a function of a continuous random variable X:

$$E[h(X)] = \int h(x) f(x) dx, \quad \text{where } f(x) \quad \text{is a density}$$

- If a density f or a distribution F are given, then

$$P(X \le x) = F(x) = \int_{-\infty}^{x} f(u) du.$$

- Binomial random variable: X has Binomial distribution with parameters n and p:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

  Mean: np; Variance $np(1-p)$.
- Geometric random variable: X has Geometric distribution with parameter p:

$$P(X = k) = (1-p)^{k-1} p.$$

  Mean: $1/p$.
- Poisson random variable: X has Poisson distribution with parameter λt:

$$P(X = k) = e^{-(\lambda t)} \frac{(\lambda t)^k}{k!}.$$

  Mean: λ; Variance λ.
- Exponential random variable: X has exponential distribution with parameter $\beta = \frac{1}{\lambda}$:

$$f(x) = \frac{1}{\beta} e^{-x/\beta}, \quad x > 0.$$

  Mean: β.
- Gamma random variable: X has gamma distribution with parameters α and $\beta = \frac{1}{\lambda}$:

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, \quad x > 0.$$

  Mean: $\beta\alpha$.

  $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, for $\alpha > 0$. $\Gamma(1) = 1$, and $\Gamma(n) = (n-1)!$ for positive integer n.

- Standardization: If X is a normal random variable with mean μ and variance σ^2, then

$$Z = \frac{X-\mu}{\sigma} \quad \text{has a standard normal distribution}$$

- Sample mean of the observations $x_1, \ldots, x_n$:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

- Sample variance of the observations $x_1, \ldots, x_n$:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right) = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right].$$

- Statistic used for confidence intervals and tests for a mean μ when σ is known:

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \text{ has a standard normal distribution}$$

- Statistic used for confidence intervals and tests for a mean μ when σ is unknown:

$$T_0 = \frac{\bar{X}-\mu}{S/\sqrt{n}} \text{ has a } T \text{ distribution with } n-1 \text{ d.f.}$$

- Statistic used for confidence intervals and tests for the means μ_1 and μ_2 of two independent populations with variances are known:

$$Z = \frac{\bar{X}-\bar{Y}}{\sqrt{(\sigma_1^2/n + \sigma_2^2/m)}} \text{ has a standard normal distribution}$$

- Statistic used for confidence intervals and tests for the means μ_1 and μ_2 of two independent normal populations with equal variances:

$$T_0 = \frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{S_p\sqrt{(1/n+1/m)}} \text{ has a } T \text{ distribution with } n+m-2 \text{ d.f.}$$

where

$$S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$$

- Statistic used for confidence intervals for the variance σ^2 of a normal population:

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} \text{ has a Chi-squared distribution with } n-1 \text{ d.f.}$$

Formulas for regression analysis:

- Parameter estimation:

$$b_1 = \frac{S_{xy}}{S_{xx}}, \quad b_0 = \bar{y} - b_1\bar{x},$$

$$\begin{aligned} S_{xy} &= \sum_{i=1}^{n} x_i y_i - \frac{1}{n}(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i) \\ S_{xx} &= \sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2 \\ S_{yy} &= \sum_{i=1}^{n} y_i^2 - \frac{1}{n}(\sum_{i=1}^{n} y_i)^2 \end{aligned}$$

- Coefficient of determination $R^2 = 1 - SSE/SST$, where $SST = S_{yy}$ and $SSE = S_{yy} - b1S_{xy}$.

- Statistic used for confidence intervals and tests for the parameter β_0

$$T = \frac{b_0 - \beta_{00}}{S\sqrt{\sum_{i=1}^{n} x_i^2/(nS_{xx})}} \text{ has a } T \text{ distribution with } n-2 \text{ d.f.}$$

where

$$S^2 = \frac{SEE}{n-2} = \frac{S_{yy} - b_1 S_{xy}}{n-2}$$

- Statistic used for confidence intervals and tests for the parameter β_1

$$T = \frac{b_1 - \beta_{10}}{S\sqrt{1/(S_{xx})}} \text{ has a } T \text{ distribution with } n-2 \text{ d.f.}$$

where

$$S^2 = \frac{SEE}{n-2} = \frac{S_{yy} - b_1 S_{xy}}{n-2}$$

- Statistic used for confidence intervals for the mean response $\mu_{Y|x_0}$

$$T = \frac{\hat{y}_0 - \mu_{Y|x_0}}{S\sqrt{\frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}}} \text{ has a } T \text{ distribution with } n-2 \text{ d.f.}$$

- Statistic used for confidence intervals for the single predicted value y_0

$$T = \frac{\hat{y}_0 - y_0}{S\sqrt{1 + \frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}}} \text{ has a } T \text{ distribution with } n-2 \text{ d.f.}$$

- Statistic used for ANOVA

$$f = SSR/S^2 \text{ has a } F\text{-distribution with } \nu_1 = 1 \text{ and } \nu_2 = n-2 \text{ d.f.}$$

where

$$SST = SSR + SSE, SST = S_{yy}\,.$$

Area under the normal curve ($P(Z < z)$, where Z has standard normal distribution).

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Area under the normal curve ($P(Z < z)$, where Z has standard normal distribution).

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical value of t-distribution $P(t > t_\alpha) = \alpha$.

v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical value of Chi-squared distribution $P(X^2 > \chi^2_\alpha) = \alpha$.

ν	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.321
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.698
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.791
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702
40	44.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403
50	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660
60	65.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608

Solutions to multiple choice questions:

Q1 $\longrightarrow$ b

Q2 $\longrightarrow$ c

Q3 $\longrightarrow$ a

Q4 $\longrightarrow$ c

Q5 $\longrightarrow$ d

Q6 $\longrightarrow$ a

Q7 $\longrightarrow$ d

Q7 $\longrightarrow$ e

Q8 $\longrightarrow$ b

Q9 $\longrightarrow$ d

Q10 $\longrightarrow$ e

Q11 $\longrightarrow$ b

Q12 $\longrightarrow$ c

Q13 $\longrightarrow$ c

Q14 $\longrightarrow$ a

Q15 $\longrightarrow$ c

Q16 $\longrightarrow$ a

Q17 $\longrightarrow$ a

Q18 $\longrightarrow$ c

Q19 $\longrightarrow$ d