Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 1320C **FINAL EXAMINATION**

Prof : Weixuan Li **Duration: 3 hours**

Your seat number _____

Last name _____ First name _____ Student number _____

Instructions:

1. The exam is closed book. Only non-programmable and non-graphing calculators (TI-30 or equivalent) are allowed.
2. The exam has three parts in 14 pages (including this cover page) with a total of 50 marks. Part 1 has eight multiple choice questions (2 marks each), Part 2 has eight short answer questions (2 marks each), and Part 3 has three long-answer questions (18 marks). In Part 2 and part 3, give enough details to show how the final result is obtained.
3. Write your solution in the space provided for each question. If you need extra space to write your solution, use the back of the pages. Please clearly indicate where your answer is whenever you do this.
4. Check your answer carefully to avoid arithmetic calculation mistakes.
5. Do not separate the pages.

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Part 1. Multiple-choice Questions

Write your answers to questions in this part in the following boxes:

1.	2.	3.	4.	5.	6.	7.	8.

1. Evaluate $\int_0^1 \frac{\arctan x}{1+x^2}dx$.

(A) $\pi/12$; (B) $\pi^2/30$; (C) $\pi/\sqrt{2}$; (D) $\pi^2/24$; (E) $\pi/10$; (F) $\pi^2/32$.

2. Consider the function $f(x) = x^3 - 2x^2 + x$. Which of the following statements is true?

(A) $f(x)$ attains a local (i.e., relative) maximum at $x = 1$.
(B) $f(x)$ attains a local (i.e., relative) minimum at $x = 1$.
(C) $(1, 0)$ is a critical point of $f(x)$, but it is not a local maximum or a local minimum.
(D) $(1, 0)$ is not a critical point, but is an inflection point.
(E) $(1, 0)$ is not a critical point, nor an inflection point.

3. If $f(x) = \ln(x + \sqrt{x^2+9})$, then $f'(4) =$

(A) 2 / 3; (B) 2; (C) 2 / 5; (D) 1 / 2; (E) 1 / 3; (F) 1 / 5.

4. Which one of the following values is closest to $\int_0^2 3^x dx$?

(A) 8; (B) 9; (C) 7.28; (D) 9.89; (E) 8.19; (F) 8.79.

5. On what interval(s) is the function $f(x) = (x^2 - 4x + 5)e^x$ concave down?

(A) It is never concave down; (B) $x > 1$ only; (C) $x < -1$ and $x > 1$ only;
(D) $-\infty < x < \infty$; (E) $-1 < x < 1$ only; (F) $x < -1$ only.

6. Newton's Method is being used to find a root of $f(x) = x^4 + 6x - 4$ with $x_1 = 1$. What is the value of x_2?

(A) 0.70; (B) 3.00; (C) 10.00; (D) 0.64; (E) 0.50; (F) 1.30.

7. What is the slope of the tangent line to the curve $xe^y + x^2y^2 + y = 2$ at the point $(x, y) = (0, 2)$?

(A) undefined; (B) 1; (C) 1.2; (D) $-1/2$; (E) -1; (F) 0.

8. At what value(s) of x does the graph of $y = (1 + x^2)^3$ have a horizontal tangent line?

(A) 1, and -1 only; (B) never; (C) 0, 1, and -1 only;
(D) 0 only; (E) 2 and -2 only; (F) 0, 2, and -2 only.

Section B: Short Answer Questions (2 marks each)

9. If $g(x) = \int_2^{x^2} \cos(t^2)dt$, what is $\frac{d}{dx}g(x)$?

Answer.

10. If $f(x)$ is a continuous function such that $\int_1^7 f(x)dx = 9$ and $\int_4^7 f(x)dx = 15$, what is $\int_1^4 (f(x)+2)dx$?

Answer.

11. Where are the critical points of the function $f(x) = x^{2/3}(x-5)$?

Answer.

12. Using the left sum and right sum with the velocity data in the table below, make both an overestimate and underestimate of the distance the car travels in the 5 seconds.

t (seconds)	0	1	2	3	4	5
v (m / sec)	30	26	22	18	13	8

13. Find the linear approximation to $f(x) = \sqrt{x+2}$ near $x = 7$, and use it to approximate $\sqrt{10}$ to three decimal places.

14. An ice cube (shaped like a perfect cube) with sides of length 5 cm is dropped in the Rideau Canal. As the cube melts, its sides decrease in length at a rate of 0.2 cm / minute. How fast is the volume decreasing when the cube has volume 8 cm^3?

15. Evaluate $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{2x}$.

16. What is the derivative of $y = \sin(x^2 + \ln(x + e^x))$?

Section C. Long Answer Questions

17. (4 marks) If 4800 cm^2 material is available to make a box with a square base and an open top, find the largest possible volume of the box.

18. (8 marks) Find the following integrals:

(a) $\int \frac{1}{(2x+1)(x-2)}\,dx$.

(b) $\int t^2 e^{3t}\,dt$.

(Continue on next page)

(c) $\int_1^4 \frac{1}{1+\sqrt{x}}\,dx$.

(d) $\int x(2x^2-6)^{5/2}\,dx$.

19. (6 marks) (a) (4 marks) Approximate the integral $\int_1^3 \ln x dx$ numerically to 4 decimal places with $n = 6$ subdivisions with the Trapezoidal Rule and Simpson's Rule (i.e. calculate $T(6)$ and $S(6)$).

(b) (2 marks) Evaluate this definite integral to find its true value.