

MAT 1332Midterm #2 v1

1.

a)

$$A = \begin{bmatrix} \boxed{-4} & \boxed{0} & \boxed{0} \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix} \quad \begin{bmatrix} \boxed{+} & \boxed{-} & \boxed{+} \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\begin{aligned} \det A &= -4 \det \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - 0 \det \begin{bmatrix} 0 & 4 \\ 0 & 2 \end{bmatrix} + 0 \det \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} \\ &= -4 \det \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - 0 \det \begin{bmatrix} 0 & 4 \\ 0 & 2 \end{bmatrix} + 0 \det \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} \\ &= -4[(2)(2) - (4)(4)] \\ &= -4(-12) \\ &= 48 \end{aligned}$$

Since  $\det A$  is not equal to 0, the matrix  $A$  is invertible.

b)

$$\left[ \begin{array}{ccc|ccc} -4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 4 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{R_1/-4 \\ R_2/2}} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 0 & -1/4 & 0 & 0 \\ 0 & \underline{1} & 2 & 0 & 1/2 & 0 \\ 0 & 4 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-4R_2+R_3} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 0 & -1/4 & 0 & 0 \\ 0 & \underline{1} & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_3/-6} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 0 & -1/4 & 0 & 0 \\ 0 & \underline{1} & 2 & 0 & 1/2 & 0 \\ 0 & 0 & \underline{1} & 0 & 1/3 & -1/6 \end{array} \right]$$

$$\xrightarrow{-2R_3+R_2} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 0 & -1/4 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 & -1/6 & 1/3 \\ 0 & 0 & \underline{1} & 0 & 1/3 & -1/6 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -1/4 & 0 & 0 \\ 0 & -1/6 & 1/3 \\ 0 & 1/3 & -1/6 \end{bmatrix}$$

c) Two ways to solve. Both are correct. Method #2 is faster.

*Method #1: Solve  $Ax = b$  for  $x$ .*

$$\left[ \begin{array}{ccc|c} -4 & 0 & 0 & 6 \\ 0 & 2 & 4 & 6 \\ 0 & 4 & 2 & 6 \end{array} \right]$$

$$\xrightarrow{\substack{R_1/-4 \\ R_2/2}} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 0 & -3/2 \\ 0 & \underline{1} & 2 & 3 \\ 0 & 4 & 2 & 6 \end{array} \right]$$

$$\xrightarrow{-4R_2+R_3} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 0 & -3/2 \\ 0 & \underline{1} & 2 & 3 \\ 0 & 0 & -6 & -6 \end{array} \right]$$

$$\xrightarrow{R_3/-6} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 0 & -3/2 \\ 0 & \underline{1} & 2 & 3 \\ 0 & 0 & \underline{1} & 1 \end{array} \right]$$

$$\xrightarrow{-2R_3+R_2} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 0 & -3/2 \\ 0 & \underline{1} & 0 & 1 \\ 0 & 0 & \underline{1} & 1 \end{array} \right]$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3/2 \\ 1 \\ 1 \end{bmatrix}$$

Method #2: Solve  $x = A^{-1}b$ .

$$\begin{aligned}\vec{x} &= A^{-1}\vec{b} \\ &= \begin{bmatrix} -1/4 & 0 & 0 \\ 0 & -1/6 & 1/3 \\ 0 & 1/3 & -1/6 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} (-1/4)(6) + (0)(6) + (0)(6) \\ (0)(6) + (-1/6)(6) + (1/3)(6) \\ (0)(6) + (1/3)(6) + (-1/6)(6) \end{bmatrix} \\ &= \begin{bmatrix} -3/2 \\ 1 \\ 1 \end{bmatrix}\end{aligned}$$

d)

$$\lambda I - A = \begin{bmatrix} \boxed{\lambda+4} & \boxed{0} & \boxed{0} \\ 0 & \lambda-2 & -4 \\ 0 & -4 & \lambda-2 \end{bmatrix} \quad \begin{bmatrix} \boxed{+} & \boxed{-} & \boxed{+} \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\det(\lambda I - A) = (\lambda+4) \det \begin{bmatrix} \lambda-2 & -4 \\ -4 & \lambda-2 \end{bmatrix} - 0 \det \begin{bmatrix} 0 & -4 \\ 0 & \lambda-2 \end{bmatrix} + 0 \det \begin{bmatrix} 0 & \lambda-2 \\ 0 & -4 \end{bmatrix}$$

$$= (\lambda+4) \det \begin{bmatrix} \lambda-2 & -4 \\ -4 & \lambda-2 \end{bmatrix} - 0 \det \begin{bmatrix} 0 & -4 \\ 0 & \lambda-2 \end{bmatrix} + 0 \det \begin{bmatrix} 0 & \lambda-2 \\ 0 & -4 \end{bmatrix}$$

$$= (\lambda+4) [(\lambda-2)(\lambda-2) - (-4)(-4)]$$

$$= (\lambda+4)(\lambda^2 - 4\lambda + 4 - 16)$$

$$= (\lambda+4)(\lambda^2 - 4\lambda - 12)$$

$$= (\lambda+4)(\lambda-6)(\lambda+2)$$

$$\det(-4I - A) = (-4+4)(-4-6)(-4+2)$$

$$= 0$$

$\therefore \lambda = -4$  is an eigenvalue

$\lambda = -4, -2, 6$  are the three eigenvalues of  $A$

e)

$$(-4I - A) = \left[ \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -6 & -4 & 0 \\ 0 & -4 & -6 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 \text{ swap } R_3} \left[ \begin{array}{ccc|c} 0 & -4 & -6 & 0 \\ 0 & -6 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 / -4} \left[ \begin{array}{ccc|c} 0 & \underline{1} & 3/2 & 0 \\ 0 & -6 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{6R_1 + R_2} \left[ \begin{array}{ccc|c} 0 & \underline{1} & 3/2 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_2 / 5} \left[ \begin{array}{ccc|c} 0 & \underline{1} & 3/2 & 0 \\ 0 & 0 & \underline{1} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{-3/2R_2 + R_1} \left[ \begin{array}{ccc|c} \boxed{0} & \underline{1} & 0 & 0 \\ \boxed{0} & 0 & \underline{1} & 0 \\ \boxed{0} & 0 & 0 & 0 \end{array} \right]$$

$$x = t$$

$$y = 0$$

$$z = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ are the eigenvectors of } \lambda = -4$$

2.

$$\left[ \begin{array}{cc|c} 1 & a & 1 \\ b & 5 & 2 \end{array} \right]$$

Usually, you would need to use lots of complex matrix row reduction to solve this system. Instead, we will use logic! Ahhh... logic...

We will answer the question backwards, as it's easier to determine what conditions give us *no solutions* and *infinite solutions* first.

**To get infinite solutions**, we need to get a parameter/free variable. If  $a = 5/2$ , and  $b = 2$ , the two rows are multiples, and they cancel out each other. Then, after row reduction, we would have

$$\left[ \begin{array}{cc|c} 1 & 5/2 & 1 \\ 2 & 5 & 2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 5/2 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

which shows a parameter/free variable in the  $y$  column. BOOM! Infinite solutions.

**To get NO solutions**, we need to get a zombie row. If  $a = 1$  and  $b = 5$ , we get the following after row reduction...

$$\left[ \begin{array}{cc|c} 1 & 1 & 1 \\ 5 & 5 & 2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & -3 \end{array} \right]$$

which is inconsistent (a zombie row), and therefore has no solutions.

**To get a unique solution**, we just need to avoid the other two situations. Therefore, if  $(a \neq 1, b \neq 5)$  or  $(a \neq 5/2, b \neq 2)$  we will get a unique solution.

3.

$$\left[ \begin{array}{ccc|c} 2 & 3 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 3 & 5 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 \text{ swap } R_2} \left[ \begin{array}{ccc|c} \underline{1} & 2 & -3 & -2 \\ 2 & 3 & 4 & 3 \\ 3 & 5 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_1+R_2 \\ -3R_1+R_3}} \left[ \begin{array}{ccc|c} \underline{1} & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & -1 & 10 & 7 \end{array} \right]$$

$$\xrightarrow{-R_2} \left[ \begin{array}{ccc|c} \underline{1} & 2 & -3 & -2 \\ 0 & \underline{1} & -10 & -7 \\ 0 & -1 & 10 & 7 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_2+R_1 \\ R_2+R_3}} \left[ \begin{array}{ccc|c} \underline{1} & 0 & \boxed{17} & 12 \\ 0 & \underline{1} & \boxed{-10} & -7 \\ 0 & 0 & \boxed{0} & 0 \end{array} \right]$$

$$x_1 + 17x_3 = 12$$

$$x_2 - 10x_3 = -7$$

$$x_3 = t$$

$$x_1 = 12 - 17t$$

$$x_2 = -7 + 10t$$

$$x_3 = t$$

$$4. a) \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 1 & 5 \\ 1 & -3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & -5 \\ -1 & \lambda + 3 \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= (\lambda - 1)(\lambda + 3) - (-5)(-1) \\ &= \lambda^2 + 2\lambda - 3 - 5 \\ &= \lambda^2 + 2\lambda - 8 \\ &= (\lambda + 4)(\lambda - 2) \end{aligned}$$

$\lambda = -4, 2$  are the eigenvalues

$$\begin{aligned} -4I - A &= \begin{bmatrix} -5 & -5 & | & 0 \\ -1 & -1 & | & 0 \end{bmatrix} \\ &\xrightarrow{R_1 / -5} \begin{bmatrix} 1 & 1 & | & 0 \\ -1 & -1 & | & 0 \end{bmatrix} \\ &\xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \end{aligned}$$

$$x + y = 0$$

$$y = t$$

$$x = -t$$

$$y = t$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ are the eigenvectors for } \lambda = -4$$

$$2I - A = \left[ \begin{array}{cc|c} 1 & -5 & 0 \\ -1 & 5 & 0 \end{array} \right]$$

$$\xrightarrow{R_1+R_2} \left[ \begin{array}{cc|c} \boxed{1} & \boxed{-5} & 0 \\ 0 & \boxed{0} & 0 \end{array} \right]$$

$$x - 5y = 0$$

$$y = t$$

$$x = 5t$$

$$y = t$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 5 \\ 1 \end{bmatrix} \text{ are the eigenvectors for } \lambda = 2$$

c) The general solution is  $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-4t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ .

d) We are given initial conditions  $x(0) = 11, y(0) = 1$ . We can therefore solve for the particular solution of the system.

$$\begin{bmatrix} 11 \\ 1 \end{bmatrix} = C_1 e^{-4(0)} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2 e^{2(0)} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$11 = -C_1 + 5C_2$$

$$1 = C_1 + C_2$$

I will use substitution to solve this system of equations.

$$C_2 = 1 - C_1$$

$$11 = -C_1 + 5(1 - C_1)$$

$$= -C_1 + 5 - 5C_1$$

$$6C_1 = -6$$

$$C_1 = -1$$

$$C_2 = 1 - (-1) = 2$$

The particular solution is  $\begin{bmatrix} x \\ y \end{bmatrix} = -e^{-4t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + 2e^{2t} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ .

e) The eigenvalue  $\lambda = -4$  will converge to zero because it is a negative eigenvalue. The eigenvalue  $\lambda = 2$  is greater than zero, and will therefore cause the solution to increase to infinity as  $t \rightarrow \infty$ . Therefore, **no**, not all solutions will converge to zero.

5.

a)

$$A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 5 & 2 \\ -1 & \lambda - 3 \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= (\lambda - 5)(\lambda - 3) - (2)(-1) \\ &= \lambda^2 - 8\lambda + 15 + 2 \\ &= \lambda^2 - 8\lambda + 17 \end{aligned}$$

$$\begin{aligned} \det[(4+i)I - A] &= (4+i)^2 - 8(4+i) + 17 \\ &= (16 + 8i + i^2) - (32 + 8i) + 17 \\ &= 16 + 8i + (-1) - 32 - 8i + 17 \\ &= 0 \\ \therefore \lambda = 4 + i &\text{ is an eigenvalue of } A \end{aligned}$$

$$\begin{aligned} \det[(4-i)I - A] &= (4-i)^2 - 8(4-i) + 17 \\ &= (16 - 8i + i^2) - (32 - 8i) + 17 \\ &= 16 - 8i + (-1) - 32 + 8i + 17 \\ &= 0 \\ \therefore \lambda = 4 - i &\text{ is an eigenvalue of } A \end{aligned}$$

b)

$$\begin{aligned}
 (4+i)I - A &= \left[ \begin{array}{cc|c} -1+i & 2 & 0 \\ -1 & 1+i & 0 \end{array} \right] \\
 &\xrightarrow{R_1 \text{ swap } R_2} \left[ \begin{array}{cc|c} -1 & 1+i & 0 \\ -1+i & 2 & 0 \end{array} \right] \\
 &\xrightarrow{-R_1} \left[ \begin{array}{cc|c} 1 & -1-i & 0 \\ -1+i & 2 & 0 \end{array} \right] \\
 &\xrightarrow{(1-i)(R_1)+R_2} \left[ \begin{array}{cc|c} 1 & \boxed{-1-i} & 0 \\ 0 & \boxed{0} & 0 \end{array} \right]
 \end{aligned}$$

$$x + (-1-i)y = 0$$

$$y = t$$

$$x = (1+i)t$$

$$y = t$$

$$\begin{aligned}
 \begin{bmatrix} x \\ y \end{bmatrix} &= t \begin{bmatrix} 1+i \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + i \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
 &= \cos(\beta t) \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} - \sin(\beta t) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\
 &= \cos t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \sin t \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
 &= \begin{bmatrix} \cos t - \sin t \\ \cos t \end{bmatrix}
 \end{aligned}$$

$$6. f(x, y) = \frac{1}{\sqrt{x-y-4}}$$

a) The domain is all  $(x, y)$  where the function is possible. Since there is a square root AND a denominator, the function inside the square root must be greater than zero.

$$x - y - 4 > 0$$

$$x - y > 4$$

$$-y > 4 - x$$

$$y < x - 4$$

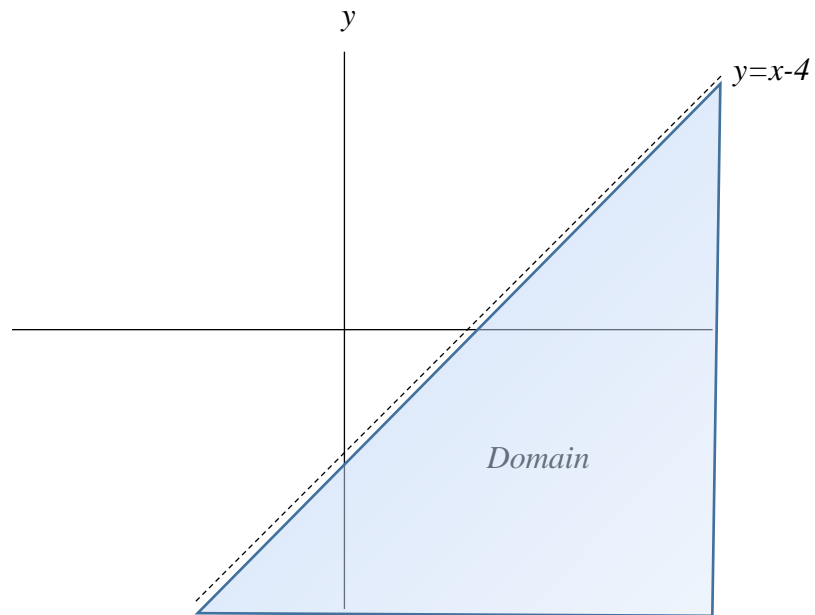
Therefore, the domain of the function is

$$D: \{(x, y) \in \mathbb{R}^2 \mid y < x - 4\}$$

b) The range of the function is “what are all possible  $z$  values of this function”. Since there is a square root in this function, *no negative values of  $z$  are possible*. Furthermore, zero is not possible because there are no variables in the numerator to allow us to “plug in”  $x$  or  $y = 0$ . Therefore, the range is

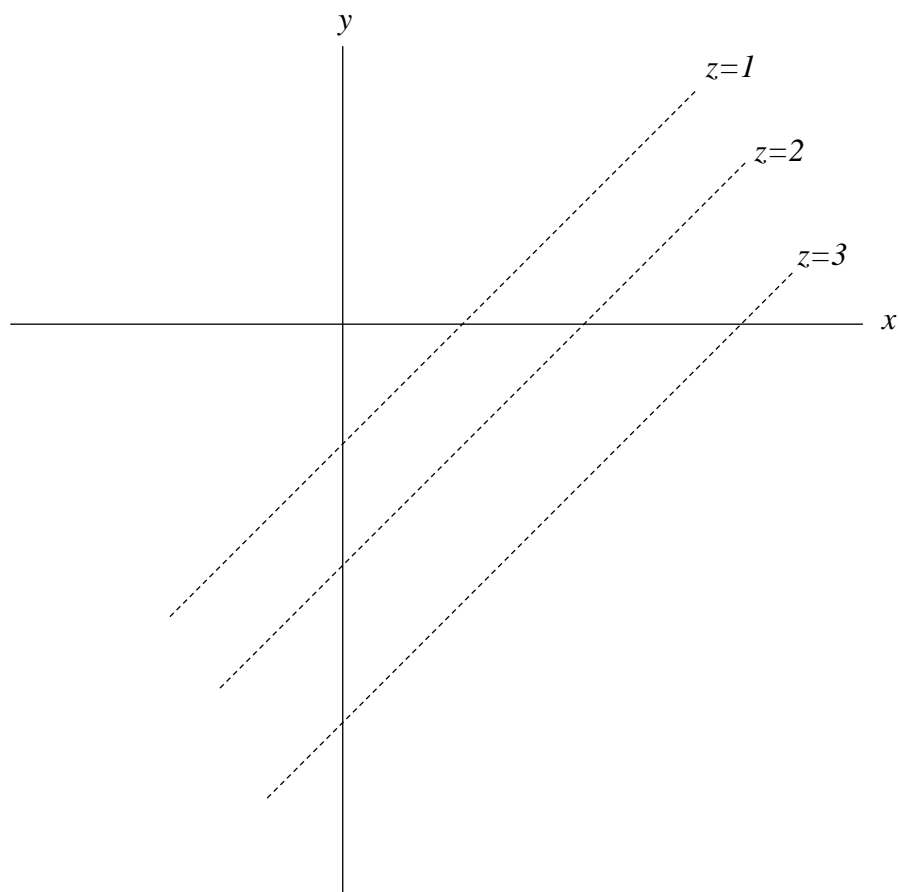
$$\text{range} = \{z \in \mathbb{R} \mid z > 0\}$$

c) The domain is bordered by a straight line.



d) Draw 3 level curves.

Level	Resulting Formula	Level Curve (ie. formula solved for y)
$z = 1$	$\sqrt{x - y - 4} = 1$	$y = x - 5$
$z = 2$	$\sqrt{x - y - 4} = 2$	$y = x - 8$
$z = 3$	$\sqrt{x - y - 4} = 3$	$y = x - 13$



**MAT 1332****Midterm #2 v2**

$$1. \frac{dx}{dt} = 25x - x^3$$

a)

$$25x - x^3 = 0$$

$$x(25 - x^2) = 0$$

$$x^* = -5, 0, 5$$

b)

$$\left(\frac{dx}{dt}\right)' = 25 - 3x^2$$

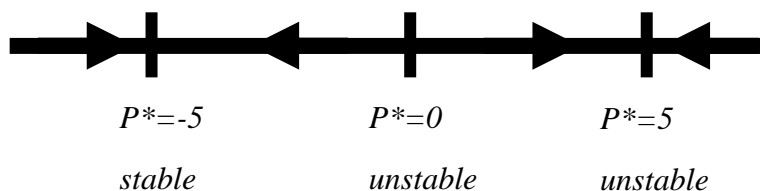
$$\left(\frac{dx}{dt}\right)' \Big|_{x^*=-5} = 25 - 3(-5)^2 = -50 \rightarrow \text{stable}$$

$$\left(\frac{dx}{dt}\right)' \Big|_{x^*=0} = 25 - 3(0)^2 = 25 \rightarrow \text{unstable}$$

$$\left(\frac{dx}{dt}\right)' \Big|_{x^*=5} = 25 - 3(5)^2 = -50 \rightarrow \text{stable}$$

c)

	$x < -5$	$-5 < x < 0$	$0 < x < 5$	$x > 5$
$dx/dt$	+	-	+	-
Phase diagram	Arrow to right	Arrow to left	Arrow to right	Arrow to left



$$2. x^3 - 6x^2 + 13x - 10 = 0$$

a) Plug  $x_1 = 2$  into the cubic equation. If the equation = 0, then  $x_1 = 2$  is a solution.

$$\begin{aligned} (2)^3 - 6(2)^2 + 13(2) - 10 &= 8 - 24 + 26 - 10 \\ &= -16 + 16 \\ &= 0 \end{aligned}$$

b) Long division.

$$\begin{array}{r} x^2 - 4x + 5 \\ x - 2 \overline{) x^3 - 6x^2 + 13x - 10} \\ \underline{-(x^3 - 2x^2)} \phantom{-10} \\ -4x^2 + 13x - 10 \\ \underline{-(-4x^2 + 8x)} \phantom{-10} \\ 5x - 10 \\ \underline{-(5x - 10)} \\ 0 \end{array}$$

$$x^2 - 4x + 5 = 0$$

$$\boxed{x_2 = 2 - i}$$

$$\begin{aligned} (2 - i)^2 - 4(2 - i) + 5 &= 4 - 4i + i^2 - 8 + 4i + 5 \\ &= 4 - \cancel{4i} + (-1) - 8 + \cancel{4i} + 5 \\ &= 0 \end{aligned}$$

$$\boxed{x_3 = 2 + i}$$

$$\begin{aligned} (2 + i)^2 - 4(2 + i) + 5 &= 4 + 4i + i^2 - 8 - 4i + 5 \\ &= 4 + \cancel{4i} + (-1) - 8 - \cancel{4i} + 5 \\ &= 0 \end{aligned}$$

c)

$$\begin{aligned}x_2 x_3 &= (2-i)(2+i) \\ &= 4 - 2i + 2i - i^2 \\ &= 4 - \cancel{2i} + \cancel{2i} - (-1) \\ &= 5\end{aligned}$$

d)

$$\begin{aligned}\frac{x_2}{x_3} &= \frac{2-i}{2+i} \\ &= \left(\frac{2-i}{2+i}\right)\left(\frac{2-i}{2-i}\right) \\ &= \frac{4-2i-2i+i^2}{4+2i-2i+i^2} \\ &= \frac{4-2i-2i+(-1)}{4+\cancel{2i}-\cancel{2i}+(-1)} \\ &= \frac{3-4i}{5} \\ &= \frac{3}{5} - \frac{4}{5}i\end{aligned}$$

We want to get rid of the  $i$  terms in the bottom because it's not "pretty" enough. To do that, we can multiply the top and bottom by the complex conjugate of the bottom term, and then simplify everything.

e)

$$\begin{aligned}
 (3+i)^2 &= (3+i)(3+i) \\
 &= 9+3i+3i+i^2 \\
 &= 9+3i+3i+(-1) \\
 &= 8+6i
 \end{aligned}$$

f)  $re^{i\phi\pi}$  is the general form of a complex number in exponential form. Since we are given  $e^i$ , this implies that  $r = 1$  and  $\phi = 1$ .

We also know that  $r^2 = a^2 + b^2$  and  $\phi = \arctan\left(\frac{b}{a}\right)$ . We know  $r$  and  $\phi$  and need to solve for  $a$  and  $b$ .

$$\begin{aligned}
 \tan 1 &= \frac{b}{a} \\
 b &= (\tan 1)a
 \end{aligned}$$

$$\begin{aligned}
 a^2 + b^2 &= 1 \\
 a^2 + [(\tan 1)a]^2 &= 1 \\
 a^2(1 + \tan^2 1) &= 1
 \end{aligned}$$

$$\begin{aligned}
 a^2 &= \frac{1}{1 + \tan^2 1} \cong 0.292 \\
 a &\cong 0.540 \\
 b &= (\tan 1)(0.540) \cong 0.841
 \end{aligned}$$

$$\therefore a + bi = 0.540 + 0.841i$$

We only take the positive  $a$  root because of the location of the complex number. The angle of 1 radian tells us that the complex number is located in the first quadrant where  $x$  and  $y$  are positive.

3.

a)

$$B = \begin{bmatrix} \boxed{1} & \boxed{0} & \boxed{c} \\ 2 & 4 & 5 \\ 3 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} \boxed{+} & \boxed{-} & \boxed{+} \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\begin{aligned} \det B &= 1 \det \begin{bmatrix} 4 & 5 \\ 1 & 0 \end{bmatrix} - 0 \det \begin{bmatrix} 2 & 5 \\ 3 & 0 \end{bmatrix} + c \det \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \\ &= 1[(4)(0) - (5)(1)] + c[(2)(1) - (4)(3)] \\ &= 1(-5) + c(2 - 12) \\ &= -5 - 10c \end{aligned}$$

$$\det B \neq 0$$

$$-5 - 10c \neq 0$$

$$-10c \neq 5$$

$$c \neq -\frac{1}{2}$$

b)

$$\left[ \begin{array}{ccc|c} 5 & 3 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 3 & -1 & a & b \end{array} \right]$$

$$\xrightarrow{R_1 \text{ swap } R_2} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 5 & 3 & 1 & 1 \\ 3 & -1 & a & b \end{array} \right]$$

$$\xrightarrow{\substack{-5R_1+R_2 \\ -3R_1+R_3}} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -7 & -4 & -9 \\ 0 & -7 & a-3 & b-6 \end{array} \right]$$

$$\xrightarrow{R_2 / -7} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 4/7 & 9/7 \\ 0 & -7 & a-3 & b-6 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_2+R_1 \\ 7R_2+R_3}} \left[ \begin{array}{ccc|c} 1 & 0 & -1/7 & -4/7 \\ 0 & 1 & 4/7 & 9/7 \\ 0 & 0 & a+1 & b-3 \end{array} \right]$$

**To get a unique solution**, we need to have three leading 1's. If  $a \neq -1$ , this will happen (note that  $b$  can be any real number).

**To get infinite solutions**, we need to get a parameter/free variable. If  $a = -1$ , and  $b = 3$ , we have a row of zeroes, and the third column is a free variable.

**To get NO solutions**, we need to get a zombie row. If  $a = -1$  and  $b \neq 3$ , we get a zombie row and therefore we have no solutions.

4.

a)

$$A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 5 & 2 \\ -1 & \lambda - 3 \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= (\lambda - 5)(\lambda - 3) - (2)(-1) \\ &= \lambda^2 - 8\lambda + 15 + 2 \\ &= \lambda^2 - 8\lambda + 17 \end{aligned}$$

$$\begin{aligned} \det[(4+i)I - A] &= (4+i)^2 - 8(4+i) + 17 \\ &= (16 + 8i + i^2) - (32 + 8i) + 17 \\ &= 16 + 8i + (-1) - 32 - 8i + 17 \\ &= 0 \\ \therefore \lambda = 4 + i &\text{ is an eigenvalue of } A \end{aligned}$$

$$\begin{aligned} \det[(4-i)I - A] &= (4-i)^2 - 8(4-i) + 17 \\ &= (16 - 8i + i^2) - (32 - 8i) + 17 \\ &= 16 - 8i + (-1) - 32 + 8i + 17 \\ &= 0 \\ \therefore \lambda = 4 - i &\text{ is an eigenvalue of } A \end{aligned}$$

b)

$$\begin{aligned}
 (4+i)I - A &= \left[ \begin{array}{cc|c} -1+i & 2 & 0 \\ -1 & 1+i & 0 \end{array} \right] \\
 &\xrightarrow{R_1 \text{ swap } R_2} \left[ \begin{array}{cc|c} -1 & 1+i & 0 \\ -1+i & 2 & 0 \end{array} \right] \\
 &\xrightarrow{-R_1} \left[ \begin{array}{cc|c} 1 & -1-i & 0 \\ -1+i & 2 & 0 \end{array} \right] \\
 &\xrightarrow{(1-i)(R_1)+R_2} \left[ \begin{array}{cc|c} 1 & \boxed{-1-i} & 0 \\ 0 & \boxed{0} & 0 \end{array} \right]
 \end{aligned}$$

$$x + (-1-i)y = 0$$

$$y = t$$

$$x = (1+i)t$$

$$y = t$$

$$\begin{aligned}
 \begin{bmatrix} x \\ y \end{bmatrix} &= t \begin{bmatrix} 1+i \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + i \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
 &= \cos(\beta t) \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} - \sin(\beta t) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\
 &= \cos t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \sin t \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
 &= \begin{bmatrix} \cos t - \sin t \\ \cos t \end{bmatrix}
 \end{aligned}$$

5.

$$\left[ \begin{array}{ccc|c} 2 & 3 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 3 & 5 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 \text{ swap } R_2} \left[ \begin{array}{ccc|c} \underline{1} & 2 & -3 & -2 \\ 2 & 3 & 4 & 3 \\ 3 & 5 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_1+R_2 \\ -3R_1+R_3}} \left[ \begin{array}{ccc|c} \underline{1} & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & -1 & 10 & 7 \end{array} \right]$$

$$\xrightarrow{-R_2} \left[ \begin{array}{ccc|c} \underline{1} & 2 & -3 & -2 \\ 0 & \underline{1} & -10 & -7 \\ 0 & -1 & 10 & 7 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_2+R_1 \\ R_2+R_3}} \left[ \begin{array}{ccc|c} \underline{1} & 0 & \boxed{17} & 12 \\ 0 & \underline{1} & \boxed{-10} & -7 \\ 0 & 0 & \boxed{0} & 0 \end{array} \right]$$

$$x_1 + 17x_3 = 12$$

$$x_2 - 10x_3 = -7$$

$$x_3 = t$$

$$x_1 = 12 - 17t$$

$$x_2 = -7 + 10t$$

$$x_3 = t$$

6.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -5 & 0 \\ 4 & 0 & 5 \end{bmatrix}$$

a)

$$\left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 2 & 1 & 0 & 0 \\ 0 & -5 & 0 & 0 & 1 & 0 \\ 4 & 0 & 5 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-4R_1+R_3} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 2 & 1 & 0 & 0 \\ 0 & -5 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & -4 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{R_2/-5 \\ R_3/-3}} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 2 & 1 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 & -1/5 & 0 \\ 0 & 0 & \underline{1} & 4/3 & 0 & -1/3 \end{array} \right]$$

$$\xrightarrow{-2R_3+R_1} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 0 & -5/3 & 0 & 2/3 \\ 0 & \underline{1} & 0 & 0 & -1/5 & 0 \\ 0 & 0 & \underline{1} & 4/3 & 0 & -1/3 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -5/3 & 0 & 2/3 \\ 0 & -1/5 & 0 \\ 4/3 & 0 & -1/3 \end{bmatrix}$$

b) Two ways to solve. Both are correct. Method #2 is faster.

*Method #1: Solve  $Ax = b$  for  $x$ .*

$$\left[ \begin{array}{ccc|c} \underline{1} & 0 & 2 & 3 \\ 0 & -5 & 0 & 5 \\ 4 & 0 & 5 & 3 \end{array} \right]$$

$$\xrightarrow{-4R_1+R_3} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 2 & 3 \\ 0 & -5 & 0 & 5 \\ 0 & 0 & -3 & -9 \end{array} \right]$$

$$\xrightarrow{\substack{R_2/-5 \\ R_3/-3}} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 2 & 3 \\ 0 & \underline{1} & 0 & -1 \\ 0 & 0 & \underline{1} & 3 \end{array} \right]$$

$$\xrightarrow{-2R_3+R_1} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 0 & -3 \\ 0 & \underline{1} & 0 & -1 \\ 0 & 0 & \underline{1} & 3 \end{array} \right]$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

*Method #2: Solve  $x = A^{-1}b$ .*

$$\vec{x} = A^{-1}\vec{b}$$

$$= \begin{bmatrix} -5/3 & 0 & 2/3 \\ 0 & -1/5 & 0 \\ 4/3 & 0 & -1/3 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-5/3)(3) + (0)(5) + (2/3)(3) \\ (0)(3) + (-1/5)(5) + (0)(3) \\ (4/3)(3) + (0)(5) + (-1/3)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

c)

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & \boxed{0} & -2 \\ 0 & \boxed{\lambda + 5} & 0 \\ -4 & \boxed{0} & \lambda - 5 \end{bmatrix} \quad \begin{bmatrix} + & \boxed{-} & + \\ - & \boxed{+} & - \\ + & \boxed{-} & + \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= -0 \det \begin{bmatrix} 0 & 0 \\ -4 & \lambda - 5 \end{bmatrix} + (\lambda + 5) \det \begin{bmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 5 \end{bmatrix} - 0 \det \begin{bmatrix} \lambda - 1 & -2 \\ 0 & 0 \end{bmatrix} \\ &= \cancel{-0 \det \begin{bmatrix} 0 & 0 \\ -4 & \lambda - 5 \end{bmatrix}} + (\lambda + 5) \det \begin{bmatrix} \lambda - 1 & -2 \\ -4 & \lambda - 5 \end{bmatrix} - \cancel{0 \det \begin{bmatrix} \lambda - 1 & -2 \\ 0 & 0 \end{bmatrix}} \\ &= (\lambda + 5) [(\lambda - 1)(\lambda - 5) - (-2)(-4)] \\ &= (\lambda + 5)(\lambda^2 - 6\lambda + 5 - 8) \\ &= (\lambda + 5)(\lambda^2 - 6\lambda - 3) \end{aligned}$$

$$\begin{aligned} \det(-5I - A) &= (-5 + 5) [(-5)^2 - 6(-5) - 3] \\ &= 0 \\ \therefore \lambda = -5 &\text{ is an eigenvalue} \end{aligned}$$

$$\lambda^2 - 6\lambda - 3 = 0$$

Can't factor. Use quadratic formula.

$$\begin{aligned} \lambda &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{6 \pm \sqrt{48}}{2} \\ &= \frac{6 \pm \sqrt{16}\sqrt{3}}{2} \\ &= \frac{6 \pm 4\sqrt{3}}{2} \\ &= 3 \pm 2\sqrt{3} \end{aligned}$$

$\lambda = -5, 3 - 2\sqrt{3}, 3 + 2\sqrt{3}$  are the three eigenvalues of  $A$

d)

$$-5I - A = \left[ \begin{array}{ccc|c} -6 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & -10 & 0 \end{array} \right]$$

$$\xrightarrow{R_1/-6} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & -10 & 0 \end{array} \right]$$

$$\xrightarrow{4R_1+R_2} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -26/3 & 0 \end{array} \right]$$

$$\xrightarrow{R_2/(-26/3)} \left[ \begin{array}{ccc|c} \underline{1} & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 \end{array} \right]$$

$$\xrightarrow{-(1/3)R_2+R_1} \left[ \begin{array}{ccc|c} \underline{1} & \boxed{0} & 0 & 0 \\ 0 & \boxed{0} & 0 & 0 \\ 0 & \boxed{0} & \underline{1} & 0 \end{array} \right]$$

$$x = 0$$

$$y = t$$

$$z = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ is an eigenvector for } \lambda = -5$$

**MAT 1332****Midterm #2 v3**

1.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

a)

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & -2 \\ 0 & \lambda - 4 \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= (\lambda - 1)(\lambda - 4) - (-2)(0) \\ &= (\lambda - 1)(\lambda - 4) \end{aligned}$$

$\lambda = 1, 4$  are the eigenvalues of  $A$

b)

$$\begin{aligned} \boxed{\lambda = 1} \\ I - A &= \begin{bmatrix} 0 & -2 & | & 0 \\ 0 & -3 & | & 0 \end{bmatrix} \\ \xrightarrow{R_1 / -2} & \begin{bmatrix} 0 & 1 & | & 0 \\ 0 & -3 & | & 0 \end{bmatrix} \\ \xrightarrow{3R_1 + R_2} & \begin{bmatrix} 0 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \end{aligned}$$

$$x = t$$

$$y = 0$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  is an eigenvector for  $\lambda = 1$

$$\begin{aligned} \boxed{\lambda = 4} \\ 4I - A &= \begin{bmatrix} 3 & -2 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \\ \xrightarrow{R_1 / 3} & \begin{bmatrix} 1 & -2/3 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \end{aligned}$$

$$x - 2/3y = 0$$

$$y = t$$

$$x = 2/3t$$

$$y = t$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$  is an eigenvector for  $\lambda = 4$

$$2. y' = 12y + 3y^2$$

a)

$$12y + 3y^2 = 0$$

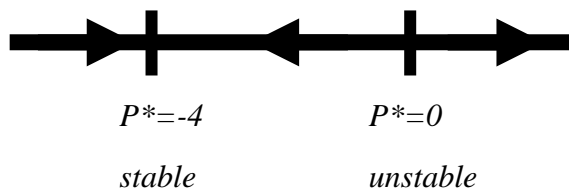
$$3y(4 + y) = 0$$

$y^* = -4, 0$  are the equilibrium points

b)

	$x < -4$	$-4 < x < 0$	$x > 0$
$y'$	+	-	+
Phase diagram	Arrow to right	Arrow to left	Arrow to right

c)



$$3. \begin{bmatrix} 1 & 0 & -1 \\ 2 & -2 & -a \\ 3 & a & -2 \end{bmatrix}$$

a) For a matrix to be invertible, its determinant must NOT be 0. Therefore, we will determine what values of  $a$  make the determinant zero, and then those values of  $a$  will NOT be allowed.

$$\det \begin{bmatrix} \boxed{1} & \boxed{0} & \boxed{-1} \\ 2 & -2 & -a \\ 3 & a & -2 \end{bmatrix} = 0 \quad \begin{bmatrix} \boxed{+} & \boxed{-} & \boxed{+} \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$0 = 1 \det \begin{bmatrix} -2 & -a \\ a & -2 \end{bmatrix} - 0 \det \begin{bmatrix} 2 & -a \\ 3 & -2 \end{bmatrix} + (-1) \det \begin{bmatrix} 2 & -2 \\ 3 & a \end{bmatrix}$$

$$= 1 \det \begin{bmatrix} -2 & -a \\ a & -2 \end{bmatrix} - 0 \det \begin{bmatrix} 2 & -a \\ 3 & -2 \end{bmatrix} + (-1) \det \begin{bmatrix} 2 & -2 \\ 3 & a \end{bmatrix}$$

$$= 1 \det \begin{bmatrix} -2 & -a \\ a & -2 \end{bmatrix} + (-1) \det \begin{bmatrix} 2 & -2 \\ 3 & a \end{bmatrix}$$

$$= 1[(-2)(-2) - (-a)(a)] - 1[(2)(a) - (-2)(3)]$$

$$= (4 + a^2) - (2a + 6)$$

$$= a^2 - 2a - 2$$

Can't factor. Use quadratic formula.

$$a = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{12}}{2}$$

$$= \frac{2 \pm \sqrt{4}\sqrt{3}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2}$$

$$= 1 \pm \sqrt{3}$$

Therefore, if  $a \neq 1 \pm \sqrt{3}$  the matrix is invertible.

b)

$$\left[ \begin{array}{ccc|ccc} \underline{1} & 0 & -1 & 1 & 0 & 0 \\ 2 & -2 & 0 & 0 & 1 & 0 \\ 3 & 0 & -2 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_1+R_2 \\ -3R_1+R_3}} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & -1 & 1 & 0 & 0 \\ 0 & -2 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2/-2} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & -1 & 1 & 0 & 0 \\ 0 & \underline{1} & -1 & 1 & -1/2 & 0 \\ 0 & 0 & \underline{1} & -3 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{R_3+R_1 \\ R_3+R_2}} \left[ \begin{array}{ccc|ccc} \underline{1} & 0 & 0 & -2 & 0 & 1 \\ 0 & \underline{1} & 0 & -2 & -1/2 & 1 \\ 0 & 0 & \underline{1} & -3 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ -2 & -1/2 & 1 \\ -3 & 0 & 1 \end{bmatrix}$$

c)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & \boxed{0} & 1 \\ -2 & \boxed{\lambda + 2} & 0 \\ -3 & \boxed{0} & \lambda + 2 \end{bmatrix} \quad \begin{bmatrix} + & \boxed{-} & + \\ - & \boxed{+} & - \\ + & \boxed{-} & + \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= -0 \det \begin{bmatrix} -2 & 0 \\ -3 & \lambda + 2 \end{bmatrix} + (\lambda + 2) \det \begin{bmatrix} \lambda - 1 & 1 \\ -3 & \lambda + 2 \end{bmatrix} - 0 \det \begin{bmatrix} \lambda - 1 & 1 \\ -2 & 0 \end{bmatrix} \\ &= -0 \det \begin{bmatrix} -2 & 0 \\ -3 & \lambda + 2 \end{bmatrix} + (\lambda + 2) \det \begin{bmatrix} \lambda - 1 & 1 \\ -3 & \lambda + 2 \end{bmatrix} - 0 \det \begin{bmatrix} \lambda - 1 & 1 \\ -2 & 0 \end{bmatrix} \\ &= (\lambda + 2) \det \begin{bmatrix} \lambda - 1 & 1 \\ -3 & \lambda + 2 \end{bmatrix} \\ &= (\lambda + 2) [(\lambda - 1)(\lambda + 2) - (1)(-3)] \\ &= (\lambda + 2)(\lambda^2 + \lambda - 2 + 3) \\ &= (\lambda + 2)(\lambda^2 + \lambda + 1) \end{aligned}$$

$$\det(-2I - A) = (-2 + 2) [(-2)^2 + (-2) + 1] = 0$$

$\therefore \lambda = -2$  is an eigenvalue of  $A$

d)

$$-2I - A = \left[ \begin{array}{ccc|c} -3 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 \text{ swap } R_2} \left[ \begin{array}{ccc|c} -2 & 0 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 / -2} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{\substack{3R_1 + R_2 \\ 3R_1 + R_3}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x = 0$$

$$y = t$$

$$z = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ is an eigenvector of } \lambda = -2$$

4.

$$\left[ \begin{array}{ccc|c} 3 & 2 & 4 & 1 \\ 2 & 1 & 2 & -1 \\ 1 & 2 & 4 & 7 \end{array} \right]$$

$$\xrightarrow{R_1 \text{ swap } R_3} \left[ \begin{array}{ccc|c} \underline{1} & 2 & 4 & 7 \\ 2 & 1 & 2 & -1 \\ 3 & 2 & 4 & 1 \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}} \left[ \begin{array}{ccc|c} \underline{1} & 2 & 4 & 7 \\ 0 & -3 & -6 & -15 \\ 0 & -4 & -8 & -20 \end{array} \right]$$

$$\xrightarrow{R_2 / -3} \left[ \begin{array}{ccc|c} \underline{1} & 2 & 4 & 7 \\ 0 & \underline{1} & 2 & 5 \\ 0 & -4 & -8 & -20 \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} -2R_2 + R_1 \\ 4R_2 + R_3 \end{array}} \left[ \begin{array}{ccc|c} \underline{1} & 0 & \boxed{0} & -3 \\ 0 & \underline{1} & \boxed{2} & 5 \\ 0 & 0 & \boxed{0} & 0 \end{array} \right]$$

$$x = -3$$

$$y + 2z = 5$$

$$z = t$$

$$x = -3$$

$$y = 5 - 2t$$

$$z = t$$

$$5. f(x, y) = \cos(\sqrt{x+y})$$

a) The domain is all  $(x, y)$  where the function is possible. Since there is a square root, the function inside the square root must be greater than or equal to zero.

$$x + y \geq 0$$

$$y \geq -x$$

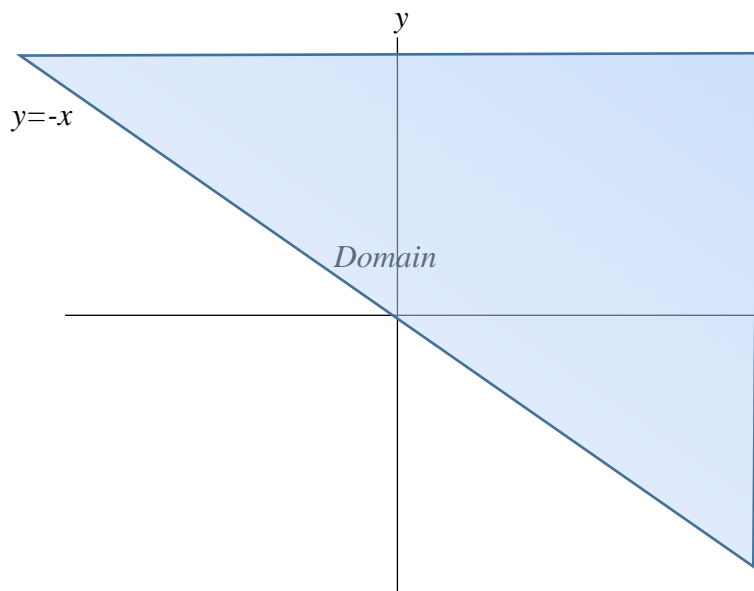
Therefore, the domain of the function is

$$D: \{(x, y) \in \mathbb{R}^2 \mid y \geq -x\}$$

b) The range of the function is “what are all possible  $z$  values of this function”. Since there is a square root in this function, *no negative values of  $z$  are possible*. Therefore, the range is

$$\text{range} = \{z \in \mathbb{R} \mid z \geq 0\}$$

c) The domain is bordered by a straight line.



d) Draw 3 level curves.

Level	Resulting Formula	Level Curve (ie. formula solved for y)
$z = 0$	$\cos(\sqrt{x+y}) = 0$	$y = (\arccos 0)^2 - x$ $= \frac{\pi^2}{4} - x$
$z = \sqrt{2}/2$	$\cos(\sqrt{x+y}) = \sqrt{2}/2$	$y = \left(\arccos \frac{\sqrt{2}}{2}\right)^2 - x$ $= \frac{\pi^2}{16} - x$
$z = 1$	$\cos(\sqrt{x+y}) = 1$	$y = (\arccos 1)^2 - x$ $= -x$

