University of Ottawa

Department of Mathematics and Statistics

MAT 1330X-Summer 2019-June 6th - Midterm 1 Version B

Professor: Xinhou Hua

Family Name ____________________ First Name ____________________

Student # ____________________ DGD (Thu. or Fri.) ____________________

- You have 75 minutes to complete this exam.
- Closed book exam. No notes. You can only use Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*).
- Questions 1-5 are multiple choice, worth 1 point each. **Record your answers into the boxes on the top of the second page.**
- Questions 6-9 are long answer questions. **You must show your work!**

• Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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Question	1 - 5	6	7	8	9	Total
Max Mark	5	2	2	6	5	20
Answer		X	X	X	X	X
Your Mark						

Question	1	2	3	4	5
Answer					

Part I: Multiple Choice Questions, no partial marks. Put your answers into the boxes above.

1. [1 point] How many solutions to the equation $\tan(2x) = 1$, $0 \le x \le 2\pi$?

(A) 1 **(B)** 2 **(C)** 3 (D) 4 **(E)** 5 **(F)** 6

Solution: (D). $0 \le 2x \le 4\pi$. Thus $2x = \frac{\pi}{4}, \frac{5\pi}{4}, 2\pi + \frac{\pi}{4}, 2\pi + \frac{5\pi}{4}$.

2. [1 point] Simplify $\log_c(c^a c^b)$.

(A) $a - b$ **(B)** $a + b$ **(C)** ab (D) abc **(E)** $\frac{a}{b}$ **(F)** $c + a + b$.

Solution: (B). $\log_c(c^a c^b) = \log_c(c^a) + \log_c(c^b) = a + b$.

3. [1 point] Find the domain of the function $f(x) = \ln(4x - 8)$.

(A) $x < 8$ **(B)** $x < 4$ **(C)** $x > 2$ (D) $x > 3$ **(E)** $x > 4$ **(F)** $x > 8$

Solution: (C). $4x - 8 > 0$, $x > 2$.

4. [1 point] If $f(x)$ is the updating function of the DTDS: $x_{t+1} = 0.8x_t + 5.4$, then $f^{-1}(8) =$

(A) 3/7 **(B)** 2/5 **(C)** 2 (D) -3 **(E)** 4/7 **(F)** 13/4

Solution: (F). $f(x) = 0.8x + 5.4$, $0.8x + 5.4 = 8$, $x = 13/4$.

5. [1 point] $\lim_{x\to 1} \dfrac{x^4 - 3x + 2}{x^2 - 1} =$

(A) 0.6 **(B)** 0.5 **(C)** 0.8 (D) −0.8 **(E)** 0.2 **(F)** 0.1

Solution: (B).

$$\lim_{x\to 1} \frac{x^4 - 3x + 2}{x^2 - 1} = \lim_{x\to 1} \frac{(x-1)(x^3 + x^2 + x - 2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x^3 + x^2 + x - 2}{x+1} = \frac{1}{2}.$$

Part II: Long Answer Questions, you have to show your work clearly.

6. [2 points] Find the values of a and b such that the function $f(x)$ is continues in $\mathbb{R}$, where

$$f(x) = \begin{cases} 2x + b & \text{if } x < 1 \\ ax^2 + b & \text{if } x > 1 \\ 4a - 2b & \text{if } x = 1 \end{cases}$$

Solution: To have continuity, we need:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$
$$2 + b = a + b = 4a - 2b$$

Thus $a = 2$ and $b = 2$.

Grading Scheme: 1 for work, 1 for conclusion.

7. [2 points] Calculate the following limit:

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x+5}-x\right).$$

Solution:

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x+5}-x\right)=\lim_{x\to\infty}\frac{(\sqrt{x^2+4x+5}-x)(\sqrt{x^2+4x+5}+x)}{(\sqrt{x^2+4x+5}+x)}$$

$$=\lim_{x\to\infty}\frac{4x+5}{(\sqrt{x^2+4x+5}+x)}=\lim_{x\to\infty}\frac{4+(5/x)}{(\sqrt{1+(4/x)+(5/x^2)}+1)}=\frac{4}{2}=2.$$

Grading Scheme: (1 pt for multiplying conjugate, 0.5 pt for simplification, 0.5 pt for conclusion.)

8. [6 points] Solve the following two equations for the variable x:

(a) [3 points] $3(2^{2x}) - 8(2^x) + 5 = 0.$

Solution: Let $z = 2^x$. Then

$$3z^2 - 8z + 5 = 0, \quad z = \frac{5}{3}, 1.$$

If $z = \frac{5}{3}$, $2^x = \frac{5}{3}$, $x = \ln(5/3)/\ln 2$.
If $z = 1$, $2^x = 1$, $x = 0$.

The solutions are $x = \ln(5/3)/\ln 2,\ 0$.

(b) [3 points] $\log_3(x+3) + \log_3(x-5) = 2.$

Solution:

$$\begin{gathered} \log_3[(x+3)(x-5)] = 2 \\ (x+3)(x-5) = 3^2, \\ x^2 - 2x - 24 = 0, \\ x = -4, 6. \end{gathered}$$

Note that $x = -4$ is not in the domain. Thus the solution is $x = 6$.

9. [5 points] The DTDS $x_{t+1} = \dfrac{4.6x_t}{8 - x_t}$ models a certain population.

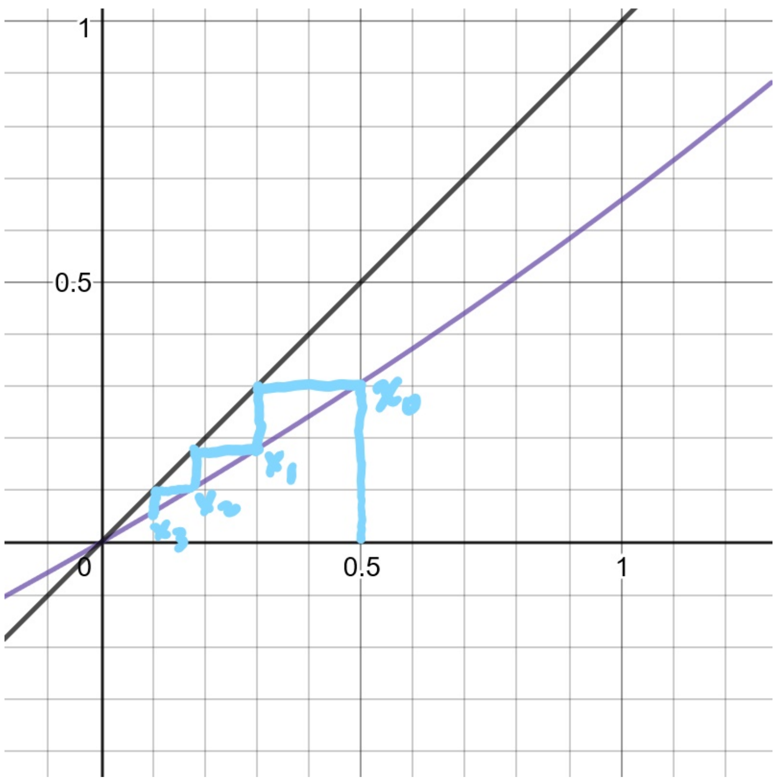
(a) (2 points) Solve for all fixed points of the DTDS exactly.

Solution: $x^* = \dfrac{4.6x^*}{8 - x^*}, \quad x^* = 0, 3.4.$

Grading Scheme: (1 for equation; 1 for the answer.)

Your answer:

(b) (2 points) The graph of the updating function of this DTDS is given below. Suppose the initial value is $x_0 = 0.5$. Draw a cobweb diagram on the graph below for this DTDS with 4 **steps**. Label the points $x_0 \cdots x_4$.



(c) (1 point) Indicate "stable" or "unstable" of the fixed point(s) from (a).

Solution: $x^* = 0$ is stable, $\quad x^* = 3.4$ is unstable.