CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 1004 A, B, C, D, E, F
December 2014

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics,
MATH 1004 A, B, C, D, E, F.
Course Instructor(s): Dr. A.B. Mingarelli (Sect. A), Dr. B. Fodden (Sect. B), Mr. M. Blenkinsop (Sect. C, D), Dr. Z. Montazeri (Sect. E, F).

AUTHORIZED MEMORANDA
STUDIO 56 SCIENTIFIC CALCULATOR ONLY AS PER COURSE OUTLINE.

This exam may be released to the Library and may be taken away by the student. **In addition to the examination paper students will require an EXAMINATION BOOKLET, and a SCANTRON SHEET.**

1. Please verify that you are in possession of a Scantron FORM
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D, E, F), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination consists of two sheets of legal size paper.** It is out of a total of 100 and consists of 25 multiple choice questions each worth 4 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.,* you get 0 marks for that question).

Return only the Scantron form not the examination nor your work.

1 Find the equation of the tangent line to the graph of $y = \sqrt{4x^2+5}$ at the point $(1, 3)$.
(a) $y = 4x - 2$ **(b)** $y = 4x - 6$ **(c)** $3y = 4x + 5$ **(d)** None of these

2 Let $f(x) = \dfrac{\cos 3x - 1}{2x}$, for $x \neq 0$, and $f(x) = L$, for $x = 0$. What value of L will make f continuous at $x = 0$?
(a) $L = 1$ **(b)** $L = 0$ **(c)** $L = -1$ **(d)** $L = 1/3$.

3 Evaluate $L = \lim\limits_{x\to\infty} (x^3 + 2x - 1)\, e^{-2x}$.
(a) $L = 1$ **(b)** $L = \frac{1}{2}$ **(c)** $L = \frac{4}{3}$ **(d)** $L = 0$.

4 Let $f(x) = x \sin(1/x)$. Evaluate $L = \lim\limits_{x\to-\infty} f(x)$.
(a) $L = 1$ **(b)** $L = 0$ **(c)** $L = -1$ **(d)** This limit does not exist.

5 Two functions f, g are defined by $f(x) = 5\cos(x^2)$ and $g(x) = \sqrt{2x}$.
What is the value of their composition $f(g(0))$?
(a) 0 **(b)** 10 **(c)** 5 **(d)** $5\sqrt{2}$

6 Find the derivative of the function $y = 2^x \operatorname{Arctan} e^x$ at $x = 0$.
(a) $\dfrac{\pi \ln 2}{4} + \dfrac{1}{2}$ **(b)** $\dfrac{\pi}{4} + \dfrac{1}{2}$ **(c)** $\dfrac{\ln 2}{3} + \dfrac{1}{2}$ **(d)** $\dfrac{\pi \ln 4}{3} + \dfrac{1}{2}$

7. Find the derivative of the function $y = 2x^3e^{-2x}$.
   (a) $2x^2e^{-2x}(2x-3)$ **(b)** $x^2e^{-2x}(2-3x)$ **(c)** $2x^2e^{-2x}(3-2x)$ **(d)** $-12x^2e^{-2x}$

8. Find the derivative of the function $y = \ln(e^{\sqrt{x}} - 1)$.
   (a) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}(e^{\sqrt{x}}-1)}$ **(b)** $\dfrac{1}{2\sqrt{x}e^{\sqrt{x}}}$ **(c)** $\dfrac{1}{2\sqrt{x}(e^{\sqrt{x}}-1)}$ **(d)** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}\ln(e^{\sqrt{x}}-1)}$

9. Find all local maximum or minimum points of the function $y = 2x^3 + 9x^2 + 1$.
   (a) Maximum at $(-3, 28)$ **(b)** Minimum at $(-3, 28)$, maximum at $(0, 1)$ **(c)** Minimum at $\left(-\frac{3}{2}, \frac{29}{2}\right)$
   (d) Minimum at $(0, 1)$, maximum at $(-3, 28)$

10. Which of the following statements is true?
    (a) $f(x) = x^3 + 7$ is concave up for all x, and has no points of inflection.
    (b) $f(x) = x^2 - 1$ is concave down for $x < 0$, concave up for $x > 0$, and has a point of inflection at $(0, -1)$.
    (c) $f(x) = -3e^x$ is concave up for all x, and has no points of inflection.
    (d) $f(x) = (x-3)^5$ is concave down for $x < 3$, concave up for $x > 3$, and has a point of inflection at $(3, 0)$.

11. Evaluate $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx$.
    (a) $-2\cos(\sqrt{x}) + C$ **(b)** $2\sin(\sqrt{x}) + C$ **(c)** $-\dfrac{1}{2}\cos(\sqrt{x}) + C$ **(d)** $2\cos(\sqrt{x}) + C$

12. Evaluate the definite integral $\displaystyle\int_0^{2\pi} \sin^2\left(\frac{x}{4}\right)\cos^2\left(\frac{x}{4}\right)dx$.
    (a) 0 **(b)** $\dfrac{\pi}{2}$ **(c)** $\dfrac{\pi}{4}$ **(d)** $\dfrac{\pi}{8}$

13 Evaluate
$$I = \int_0^{\frac{\pi}{2}} \sin 2x \sin 5x \; dx$$
    (a) $-\dfrac{2}{21}$ **(b)** $-\dfrac{2}{29}$ **(c)** $\dfrac{2}{21}$ **(d)** $\dfrac{5}{21}$

14 Evaluate the integral
$$\int e^x(x^2 + 4x) \; dx$$
    (a) $e^x(x^2-6x+6)+C$ **(b)** $e^x(x^2+2x-2)+C$ **(c)** $e^x(x^2-6x+2)+C$ **(d)** $e^x(x^2-2x-6)+C$

15 Evaluate the integral
$$I = \int \sin(\ln x^2) \; dx$$
    (a) $\dfrac{x}{5}(\sin(\ln x^2) - 2\cos(\ln x^2)) + C$ **(b)** $\dfrac{x}{5}(\sin(\ln x^2) + 2\cos(\ln x^2)) + C$ **(c)** $\dfrac{x}{3}(2\cos(\ln x^2) - \sin(\ln x^2)) + C$ **(d)** $\dfrac{x}{3}(-\sin(\ln x^2) - 2\cos(\ln x^2)) + C$

16 Evaluate the integral
$$\int \frac{3x^2 - 4x + 1}{x^3 - 2x^2 + x - 2} \; dx$$
    using partial fractions. (Hint: $x - 2$ is a factor of the denominator.)
    (a) $\dfrac{3}{2}\ln(x^2+1) - \tan^{-1}(x) + \ln|x-2| + C$ **(b)** $\ln|x-2| + C$ **(c)** $\ln|x-2| - \tan^{-1}(x) + C$
    (d) $\ln(x^2+1) + \ln|x-2| + C$

17 Let $f(x) = \dfrac{1 + \ln x}{e^{2x-1}}$. Evaluate $f'(1)$.
    (a) $-\dfrac{1}{e}$ **(b)** $\dfrac{1}{e}$ **(c)** $\dfrac{2}{e}$ **(d)** $\dfrac{3}{e}$

18 Evaluate the limit

$$\lim_{x\to 0^+} x\cot 3x$$

(a) 3 **(b)** -3 **(c)** $\frac{1}{3}$ **(d)** 1

19 Evaluate the following derivative: $\dfrac{d}{dx}\displaystyle\int_1^{x^2}\sqrt{1+t^3}\,dt.$

(a) $2x\sqrt{1+x^6}$ **(b)** $\sqrt{1+x^6}$ **(c)** $2x\sqrt{1+x^3}$ **(d)** None of these

20 Given that f is such that its inverse F exists, $f'(1)=1/2,\ F(0)=1,\ F(2)=0$. Find the value of the derivative of the inverse function F at $x=0$.

(a) 4 **(b)** 2 **(c)** 5 **(d)** 1/5

21 Evaluate $\displaystyle\int_1^2 \frac{3^{2\ln x}}{x}\,dx.$

(a) $3^{\ln 4}-1$ **(b)** $3^{\ln 4}$ **(c)** 3 **(d)** $\dfrac{3^{\ln 4}-1}{2\ln 3}$

22 Let $f(x)=x|x|$. Calculate $L=\displaystyle\lim_{h\to 0}\frac{f(-5+h)-f(-5)}{h}$.

(a) $L=10$ **(b)** $L=1$ **(c)** $L=5$ **(d)** This limit does not exist.

23 Let $f(x)=e^{-x^2}$. Evaluate $f''(0)$. In other words, find the second derivative of f at $x=0$.

(a) $f''(0)=4$ **(b)** $f''(0)=-2$ **(c)** $f''(0)=1/2$ **(d)** $f''(0)$ does not exist

24 Find an expression for the volume V of the solid of revolution obtained by rotating the region bounded by the graph of $y=x^3$, $y=x^2+1$, $x=0$ and $x=1$ about the $y-$axis.

(a) $\displaystyle\int_0^1 \pi x(x^2-x^3)\,dx$ **(b)** $\displaystyle\int_0^1 2\pi x(1+x^2-x^3)\,dx$ **(c)** $\displaystyle\int_0^1 2\pi x(1-x^2-x^3)\,dx$

(d) $\displaystyle\int_0^{1/2} 2\pi x^2(1+x^2-x^3)\,dx$

25 Evaluate the improper integral $\displaystyle\int_0^1 x\,\ln x\,dx.$

(a) 0 **(b)** -1 **(c)** $-1/4$ **(d)** 1/2

[Total: 100 marks]

END OF THE EXAMINATION.