TELFER

VOTRE LIEN AVEC CE QUI COMPTE — CONNECTS YOU TO WHAT MATTERS

Mid-term Exam #2

ADM 3351 A
Fixed Income Investments
Date: November 14, 2017

Professor: C. Guo

Duration: 80 Minutes

INSTRUCTIONS
1. Books and notes **are not** permitted.
2. Calculators **are** permitted.
3. Answer in the designated spaces.

NAME: [redacted]
STUDENT # [redacted]

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I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

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Evaluation

Question	Credit	Mark
1	9	9
2	10	10
3	6	5
Total	25	24

Question 1 (9 points)

You have purchased a freehold house (i.e., no condo fee) for $300,000 with 20% down payment and the rest borrowed from your local bank as a 30-year mortgage loan at 6% (APR with monthly compounding). The mortgage can be paid off any time without penalty, i.e., it allows prepayment.

(a) (1 point) What is your loan to value (LTV) ratio?
(b) (2 points) What is your monthly payment?
(c) (1 point) If your gross annual income is $72,000, and the property tax on the house is $6,000 per year, what is your payment to income (PTI) ratio?
(d) (1 point) Why does your prepayment option affect the profitability of your bank when the interest rate falls?
(e) (1 point) Why is your prepayment option equivalent to your right for refinancing?
(f) (3 points) Assume that the mortgage rate falls to 5.4% (APR with monthly compounding) in three years. Will you refinance your loan despite that your bank charges you $2,000 refinancing fee? Why or why not? Be numerically specific.

(Extra spaces on the next two pages)

a) LTV = 1 − 0.2 = 80% ✓

b) N = 30(12) = 360
I/Y = 0.06/12 = 0.5%
PV = 300,000 − 60,000 = 240,000
PMT = ?
FV = 0

DP = 300,000 × 0.2 = 60,000

N	I/Y	PV	PMT	FV
360	0.5	240,000	?	0

PMT = −1,438.921260 ✓

c) $PTI = \frac{(1{,}438.921260 \times 12) + 6000}{72{,}000} = 0.3231534433$ or 32.315% ✓

d) The value the bank receives as prepayment is lower the what it would monthly receive as a regular payment. Also, the bank will hold that value while the interest rate is lower, which makes it less optimal.

e) Because any bank will provide a prepayment option. If a bank refuses prepayment, then the customer can find another bank to do it. ✓

(Extra space for Question 1)

f) $N = 360 - (3 \cdot 12) = 324$

$I/Y = 0.5\%$

$PV = ?$

$PMT = \$1,438.921260$

$FV = 0$

N	I/Y	PV	PMT	FV
324	0.5	?	1438.921260	0

$PV = \$230,601.7682$ ✓

$N = 324$

$I/Y = \frac{5.4\%}{12} = 0.45\%$

$PV = 230,601.7682 + 2,000$

$= \$232,601.7682$

$PMT = ?$

$FV = 0$

N	I/Y	PV	PMT	FV
324	0.45	232,601.7682	?	0

$PMT = \$1,365.503176.$ ✓

Yes, I would still refinance the loan with the $2000 because the monthly payment is lower than the previous monthly payment. ✓

(Extra space for Question 1)

Question 2 (10 points)

A mortgage pass-through security of 30-year confirming mortgages is issued with initial balance of $250 million and initial WAM of 354 months. The WAC is 8%, and coupon rate (to the investors) is 7%. The rates are quoted as monthly simple interest rates (i.e., APR with monthly compounding).

To ensure sufficient accuracy, you are required to keep at least 6 decimals, e.g., x.yyyy%, on the monthly mortgage rate. The accuracy on the dollar amount can be rounded to dollar.

(a) (2 points) What mortgage payment is to be received by the pass-through in the first month?
(b) (1 point) For simplicity, the single monthly mortality (SMM) for the first month is given as 0.001767. What is the **total** amount of principal to be received by the pass-through in this month?
(c) (1 point) What is the **cash flow** to the investors of the pass-through in the first month?
(d) (1 point) What mortgage payment is to be received by the pass-through in the second month?
(e) (2 points) If the prepayment speed on the second month is calculated based on the PSA 150, what is the **cash flow** to the investors of the pass-through in the second month?
(f) (2 points) The pass-through is divided into 1 million shares and sold to the investors at 95% of par (i.e., the original balance). An investor who bought one share of the pass-through wants to know his **cash flow yield**. Write an explicit mathematical expression to explain to the investor on the cash flow yield and indicate how it can be found and how it will be reported on the basis of Bond Equivalent Yield (BEY) (i.e., APR based on semiannual compounding assumption). You are required to show specific values if they are already known, and to indicate the unknowns with appropriate variables.
(g) (1 point) Explain to the investor on why the cash flow yield in part (f) depends on the assumption of the prepayment speed.

(Extra space on the next two pages)

a) $N = 354$

$I/Y = \frac{8\%}{12} = 0.6666667$

$PV = 250\,000\,000$

$PMT = ?$

$FV = 0$

N	I/Y	PV	PMT	FV
354	0.6666667	250,000,000	?	0

$PMT = \$1,841,951.596$

b) $SPP: 1,841,951.596 - \left(\frac{8\%}{12} \cdot 250,000,000\right) = \$175,284.9293$

$PP: 0.001767 \cdot (250,000,000 - 175,284.9293) = 441,460.2715$

$TPP: 175,284.9293 + 441,460.2715 = \$616,725.2008$

c) $\left(\frac{0.07}{12} \cdot 250,000,000\right) + \$616,725.2008 = \$2,075,058.534 = CF_1$

(Extra space for Question 2)

d) $MB_2 = 250,000,000 - 616,725.2008 = \$249,383,274.80$

$N = 354 - 2 + 1 = 353$

$I/Y = \frac{8\%}{12} = 0.6666667$

$PV = \$249,383,274.80$

$PMT = ?$

$FV = 0$

N	I/Y	PV	PMT	FV
353	0.6666667	249,383,274.80	?	0

$PMT = \$1,838,696.868.$

c) $CPR = 6\% \cdot \left(\frac{360-353+1}{30}\right) \cdot 1.5 = 0.024$

$SMM = 1 - (1-0.024)^{1/12} = 0.002022\,34335$

$SPP = 1,838,696.868 - \left(\frac{0.08}{12} \cdot 249,353,274.80\right) = \$176,141.7027$

$PP = 0.002022\,34335 \times (249,353,274.80 - 176,141.7027)$

$= \$503,982.3884$

$CF_2 = \left(\frac{0.07}{12} \cdot \$249,383,274.80\right) + \$176,141.7027 + \$503,982.3884$

$= \$2,134,859.861$

f) $\frac{250\text{ m}}{1\text{ m}} = 250 \cdot 95\% = \$237.50$

$CF_1 = \frac{2,075,058.534}{1,000,000} = 2.07$

$CF_2 = \frac{2,134,859.861}{1,000,000} = 2.13.$

$237.50 = \frac{2.07}{(1+r_m)} + \frac{2.13}{(1+r_m)^2} + \cdots + \frac{CF_{354}}{(1+r_m)^{354}}$

BEY: $\left[(1+r_m)^6 - 1\right] \times 2$

g) Because the higher the prepay rate the speed the lower the cash flows yield

(Extra space for Question 2)

Question 3 (6 points)

3-1 (2 points)

A 5% fixed-rate coupon tranche of $75 million is split into a floater of $50 million with coupon payment equal to LIBOR + 0.5%, and an inverse floater of $25 million with coupon payment determined by the formula: K-L×LIBOR. Specify the parameters of the formula for the inverse floater.

$$\frac{L}{1+L} = \frac{50}{75} = \frac{2}{3}, \quad L = 2.$$

$$\frac{50}{75}(\text{LIBOR} + 0.5\%) + \frac{25}{75}(K - L \times \text{LIBOR}) = 5\%$$

$$\frac{2}{3}(2 + 0.5\%) + \frac{1}{3}(K - 2 \times 2) = 5\%$$

$$\frac{401}{300} + \frac{1}{3}K - \frac{4}{3} = 5\%$$

$$K = 0.14$$

3-2 (2 points)

The following is a Sequential-Pay CMO:

Tranche	Par Amount	Coupon Rate (%)
A	$194,500,000	7.5
B	36,000,000	7.5
C	96,500,000	7.5
D	73,000,000	7.5
Total	$400,000,000	

The principal payment is to be received first by Tranche A, then B, then C, then D.

(a) (1 point) Before Tranche A is paid off, what is the total cash flow to the Tranche B investors per month?
(b) (1 point) Which tranche is the most sensitive to prepayment risk? Why?

a) $36,000,000 \times \frac{7.5\%}{12} = \$225,000$ ✓

b) Tranche D, because it is the last tranche in the CMO, so it will have the lowest interest value out of the other tranches. ✓ −1

3-3 (2 points)

A CMO contains one Planned Amortization Class (PAC) bond, and one supporting bond S. The PAC bond is created between PSA 90 and PSA 300. Part of the principal payments are calculated and shown in the following table. Assume that the supporting bond S will be exhausted by Month 210. Given the actual principal payments in the 4th column, fill out the respective principal payments to this PAC bondholders in the last column, then, explain your answers.

Month	At 90% PSA	At 300% PSA	Actual Principal Payment	Principal Payment to the PAC Bondholder
1	508,169	1,075,931	600,000	$ 508,169
2	569,843	1,279,412	800,000	$ 569,843
…				
(The supporting bond S has gone on Month 210)				
211	949,282	213,309	200,000	$ 200,000
212	946,033	209,409	300,000	$ 300,000
…				
349	613,875	12,314		
350	*612,292*	*12,008*		

Before month 210, there is a supported bond available so we choose the lowest principle payment available, which are $508,169 and $569,843. After Month 210, the supported bond is exhausted, so we must go back to the unsupported bond for $200,000 and $300,000.