MAT 1341E Assignment 1, 2011

Due: 11:30 am, 6-October, 2011.
Instructor: Sanghoon Baek

Family Name:_____

First Name:_____

Student number:_____

(For the marker's use only →)

1	
2	
3	
4	
5	
6	
7	
[Bonus] 8	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 6 to 8, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 5 are worth 1 point each, and no part marks will be given. However, you must show some work to obtain the point. Simply writing the correct answer will earn you 0.
3. Questions 6 and 7 and are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.** Question 8 is a bonus question and is worth 4 points. Earning points here will be much more difficult than in questions 1-7.
4. Homework is collected at the beginning of the lecture. **No late assignments will be accepted**.

1. (1 mark) Which <u>two</u> of the following subsets of $\mathbb{R}^4$ are closed under the usual operation of multiplication by scalars?

A. $\{(a,\ b,\ c,\ d) |\ ab = 0\}$

B. $\{(a,\ b,\ c,\ d) |\ a = 1, b = 0 \text{ and } c + d = 1\}$

C. $\{(a,\ b,\ c,\ d) |\ a > 0 \text{ and } b < 0\}$

D. $\{(a,\ b,\ c,\ d) |\ a > 0 \text{ and } b > 0\}$

E. $\{(a,\ b,\ c,\ d) |\ a + b + c + d = 0\}$

ANSWER

2. (1 mark) Which <u>two</u> of the following are subspaces of $F[0,1] = \{f \mid f : [0,1] \to \mathbb{R}\}$?

$U = \{f \in F[0,1] \mid f(0)f(1) = 0\}$

$V = \{f \in F[0,1] \mid f(0) + f(1) = 0\}$

$S = \{f \in F[0,1] \mid f(x) = -2f(x),\ \forall x \in [0,1]\}$

$T = \{f \in F[0,1] \mid f(1) \leq 0\}$

ANSWER

3. (1 mark) Which <u>three</u> of the following statements are true?

I. The span of two distinct vectors u and v in $\mathbb{R}^3$ is a plane through the origin.

II. The span of a single nonzero vector u in $\mathbb{R}^2$ is a line.

III. A set of vectors $\{u, v, w\} \subseteq X$ spans a vector space X if every $x \in X$ is a linear combination of v and w.

IV. Any spanning set for $\mathbb{R}^2$ contains at least two elements.

V. $\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}\}$ spans $M_{2,2}(\mathbb{R})$.

ANSWER

4. (1 mark) Which <u>three</u> of the following sets are linearly independent?

A. $\{(1,\ -5,\ 2)\}$

B. $\{(5,\ 1,\ 0),\ (0,\ 0,\ 0),\ (-2,\ 0,\ 1)\}$

C. $\{(5,\ 1,\ 0),\ (2,\ 2,\ 2)\}$

D. $\{(0,\ 0,\ 0)\}$

E. $\{(5,\ 1,\ 0),\ (-2,\ 0,\ 1),\ (3,\ 2,\ 1)\}$

ANSWER

5. (1 mark) If we give $W = \mathbb{R}^2$ the *non-standard* operations

$$(x,\ y) \oplus (x',\ y') = (x + x' + 9,\ y + y' - 5) \quad (\text{ vector addition})$$

and

$$k \otimes (x,\ y) = (kx + 9k - 9,\ ky - 5k + 5) \quad (\text{multiplication by scalars}),$$

then W *is* a vector space over $\mathbb{R}$.

- (0.5 mark) What is the zero vector of W?
- (0.5 mark) If $\mathbf{v} = (x,\ y)$ is in W then what is $-\mathbf{v}$?

ANSWER

$0 =$

$-\mathbf{v} =$

6. (6 marks) Let $u = (3, -2, 1)$, and $U = \{w \in \mathbb{R}^3 \mid w \cdot u = 0\}$

(a) (2 marks) Is U a subspace of $\mathbb{R}^3$?

(b) (2 marks) Find a spanning set for U.

(c) (2 marks) Give a complete geometric description of U.

(*You must justify your answers.*)

Space for problem 6

7. (6 marks) Consider the vector space $F(\mathbb{R}) = \{f \mid f{:}\mathbb{R} \to \mathbb{R}\}$, with the standard operations. Recall that the zero of $F(\mathbb{R})$ is the function that has the value 0 for all $x \in \mathbb{R}$.

Let $W = \{f \in F(\mathbb{R}) \mid f(0) = f(\pi)\}$ be the subspace of functions which have the same value at $x = 0$ and $x = \pi$.

(a) (2 marks) Show that $\cos 2x$ belongs to W and show that $\cos x$ does not belong to W.

(b) (1 mark) Show that $\sin^2 x \in \text{span}\{1, \cos 2x\}$.

(c) (1 mark) Show that $\sin^2 x \notin \text{span}\{1, \sin 2x\}$.

(d) (2 marks) By any means, show that $\{1, \sin^2 x, \sin 2x\}$ is linearly independent.

(*You must justify your answers.*)

Space for problem 7

8. [Bonus](4 marks) Let $E = \{\text{“}ax + by + cz = d\text{”} \mid a, b, c, d \in \mathbb{R}\}$ be the set of linear equations with real coefficients in the variables x, y and z. Equip E with the usual operations on equations that you learned in high school: addition of equations, denoted here by “$\oplus$” and multiplication by scalars, denoted here by “$\circledast$”, as follows:

$$\text{“}ax + by + cz = d\text{”} \oplus \text{“}ex + fy + gz = h\text{”} := \text{“}(a + e)x + (b + f)y + (c + g)z = d + h\text{”}$$

and

$$\forall k \in \mathbb{R}, \quad k \circledast \text{“}ax + by + cz = d\text{”} := \text{“}ka\,x + kb\,y + kc\,z = k\,d\text{”}.$$

You may assume without proof that E is a vector space.

(a) (2 marks) Find the zero vector of E.
(b) (2 marks) Find a spanning set for E.

(*You must justify your answers.*)

Space for problem 8