Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 1320D: Calculus I

Instructor : Cheikh Ndongo

First Midterm Exam - V.B

February 6, 2019

Family Name ______________________ First Name ______________________

Student # ______________________

Instructions:

(1) You have 80 minutes to complete this exam. **No calculators are permitted**.
(2) Write your student number at the top of each page in the space provided.
(3) There are 3 multiple choice questions worth 1 mark each and 4 longer answer questions worth 17 marks for a total of 20.
(4) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(5) No notes, books, scrap paper, calculators or other electronic devices are allowed.
(6) You are strongly recommended to write in **pen**, not pencil.
(7) You may use the last page of the exam as scrap paper.
(8) Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. **Phones and devices must be turned off and put away in your bag**. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature ______________________

Good luck!!!

Question	1	2	3	4	5	6	7	Total
Maximum	1	1	1	4	3	4	6	20
Answer				X	X	X	X	X
Grade								

Multiple Choice Questions

The questions 1 to 3 are multiple choice and worth 1 mark each. **You may write** *your answers to the multiple choice questions in the second page. You don't need to justify your answers for multiple choice questions.*

Question 1. [**1 mark**] If $f(x) = \frac{3x+1}{2x-3}$, then $f^{-1}(1) =$

(A) -1 **(B)** -2 **(C)** -4 (D) $\frac{-1}{2}$ **(E)** $\frac{-1}{4}$ **(F)** does not exist.

Solution: (C) We have $f^{-1}(x) = \frac{3x+1}{2x-3}$ and $f^{-1}(1) = -4$.

Question 2. [**1 mark**] Solve for x the equation $\ln(x-1) + \ln(x-2) = \ln(6)$.

(A) 1 and -4 **(B)** -4 **(C)** 1 (D) -1 and 4 **(E)** -1 **(F)** 4

Solution: (F) The equation is equivalent to $\ln((x-1)(x-2)) = \ln(6)$. Then we compose with e^x to obtain $(x-1)(x-2) = 6 \Leftrightarrow x^2 - 3x - 4 = 0$ and$x = -1$ or $x = 4$. But the domain of the equation is $x > 2$, so $x = 4$.

Question 3. [**1 mark**] Find an equation of the tangent line to the curve $f(x) = 4x^2 - 7x + 4$ at $x = 1$?

(A) $y = x + 1$ **(B)** $y = -x + 1$ **(C)** $y = x$ (D) $y = 1$ **(E)** $y = -x$ **(F)** $y = -1$

Solution: (C). Since $f'(x) = 8x - 7 \Rightarrow f'(1) = 1$ and $f(1) = 1$. Thus an equation of the tangent line at $x = 1$ is $y = 1(x-1) + 1$ or $y = x$.

Long Answer.

For questions 4 to 7. You must show all relevant steps in your solutions in order to obtain full marks, unless indicated otherwise.

Question 4. **[4 marks]** Evaluate each of the following limits. (You can't use Hospital's rule).

(a) $\lim_{x\to 1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)$.

Solution: Put the same denominator then simplify to get
$\lim_{x\to 1}\left(\frac{2}{x^2-1}-\frac{1}{x+1}\right)=\lim_{x\to 1}\left(\frac{-x-1+2}{x^2-1}\right)=\lim_{x\to 1}\left(\frac{-1}{x+1}\right)=-\frac{1}{2}$.

(b) $\lim_{x\to\infty}\left(\sqrt{x^2-2x-1}-x\right)$.

Solution: We multiply by the conjugate then we simplify to obtain

$$\lim_{x\to\infty}\left(\sqrt{x^2-2x-1}-x\right)=\lim_{x\to\infty}\frac{(\sqrt{x^2-2x-1}-x)(x+\sqrt{x^2-2x-1})}{(x+\sqrt{x^2-2x-1})}$$

$$=\lim_{x\to\infty}\frac{-2x-1}{(x+\sqrt{x^2-2x-1})}=\lim_{x\to\infty}\frac{-x(2+(1/x))}{x((\sqrt{1-(2/x)-(1/x^2)}+1))}=\frac{-2}{2}=-1.$$

Question 5. [**3 points**] Find the values of a and b that make f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} 2x+1 & \text{si } x < -1 \\ ax^2 + b & \text{si } -1 \leq x \leq 2 \\ x^2 - 2x - 1 & \text{si } x > 2 \end{cases}$$

Solution: f is continuous on $(-\infty, \infty)$ if and only if

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$$

and

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2).$$

Thus,

$$\lim_{x \to -1^-} f(x) = -1 = \lim_{x \to -1^+} f(x) = a + b = f(-1)$$

and

$$\lim_{x \to 2^-} f(x) = 4a + b = f(2) = \lim_{x \to 2^+} f(x) = -1.$$

Then, we obtain $a = 0$ and $b = -1$.

Question 6. **[4 points]**

a) Give the definition of the derivative of a function f at a point $x = a$.

Solution: $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$.

b) Using the limit definition of the derivative (not the rules), find the derivative of

$$f(x) = \frac{x}{x+2}$$

at $x = 1$. Make sure you show all steps starting from the limit definition.

Solution: We have

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\frac{1+h}{1+h+2} - \frac{1}{1+2}}{h} = \lim_{h\to 0} \frac{3(1+h) - (h+3)}{3h(h+3)} = \lim_{h\to 0} \frac{2}{3(h+3)} = \frac{2}{9}.$$

Question 7. [**6 points**] Find the derivative of each of the following functions using appropriate rules. You need to simplify your answer if it's possible.

(a) $f(x) = \dfrac{x^3 + x^{2/5}}{x}$

Solution: Simplify

$$f(x) = \frac{x^3 + \sqrt[5]{x^2}}{x} = x^2 + x^{-3/5}$$
$$f'(x) = 2x - \frac{3}{5}x^{-8/5}\,.$$

(b) $g(x) = \dfrac{x}{\tan(x)}$

Solution: Using the Quotient Rule, we have

$$g'(x) = \frac{(x)'\tan(x) - \tan'(x)x}{(\tan(x))^2} = \frac{\tan(x) - \sec^2(x)x}{\tan^2(x)} =_{ou\ bien} \frac{\sin(x)\cos^2(x) - x}{\sin^2(x)}.$$

(c) $h(x) = \sin\left(2x^3 - 3x^2 + 6\right)$

Solution: Using the Chain Rule we have

$$h'(x) = \left(2x^3 - 3x^2 + 6\right)'\sin'\left(2x^3 - 3x^2 + 6\right) = \left(6x^2 - 6x\right)\cos\left(2x^3 - 3x^2 + 6\right).$$

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