Mid-term Examination 2019 (Winter)
ECON 464/564

Time: 75 minutes
Full Grade: 50

You have 75 minutes to complete this exam.

Write down the relevant steps or calculations and/or explain your answers in as much detail as necessary. In particular, do not skip important steps, but omit details that are not feasible to write down. Use your judgment regarding what is important and what is feasible.

1. (20 points) A professional baseball team determines a new player's contract by submitting a salary offer to an arbitrator. Simultaneously, the new player also submits a salary request to the arbitrator, without knowing the team's offer. Assume that both the salary offer and the request will be either \$20 million or \$10 million. If the offer is at least as much as the request, then the arbitrator chooses the average of the two as the salary, which determines the payoffs. For example, if both the offer and request are \$20 million then the payoffs are (-20, 20) to (T, P), where T denotes the team and P denotes the new player. If the offer is less than the request then the arbitrator simply chooses the amount of the offer as the salary which, if not appealed, yields payoffs of (-10, 10). However, the player has the right to appeal in this case. Suppose that if there is an appeal then the new player is awarded the requested \$20 million salary, but only after a costly arbitration process which costs both parties \$1 million and thus results in net final payoffs of (-21, 19).

   (a) Provide an extensive form representation of this 2-player game. Choose the representation where T decides at the initial node. (6 points)

   (b) Write the game in normal form. (Let T be the row player and P be the column player.) (6 points)

   (c) What strategies are rationalizable strategies ? (8 points)

2. (15 points) Two people divide up \$10 between themselves. They use the following procedure. Each person names an integer between 0 and 10. If the sum of the two numbers is at most 10 then each gets the amount of money she named, and the rest of the money will be destroyed. If the sum of the two numbers exceeds 10 then the person announcing the smaller number gets that amount, and the other person receives the remaining money. If the sum of the two numbers exceeds 10 and the two people announce the same number then each person receives \$5.

   (a) For each player, determine the best responses to each of the other player's pure strategies, i.e., determine the best response function. (10 points)

(b) Find all the Nash Equilibria of the game. (5 points)

3. (15 points) A public facility needs to be located on a street , which I denote by the interval $[0, 1]$. In the city there are n voters. Each voter i has an ideal location p_i, $0 \leq p_i \leq 1$, where she wants the facility to be located. If the facility is located at l, $0 \leq l \leq 1$, then the utility of voter i is

$$-(p_i - l)^2$$

The following voting game is played to decide on the location. Every citizen/ voter i votes for a location x_1, where $0 \leq x_1 \leq 1$. Given the voted location profile, $x = (x_1, \cdots, x_n)$ the facility is chosen at location $W(x)$ where $W(x)$ is a rule that picks a location for every voted location profile x. We call $W(x)$ a *voting rule*. The specific $W(x)$ that the city uses, is the following:

$$W(x) = W(x_1, \cdots, x_n) = \min\{x_1, \cdots, x_n\}$$

(a) Define the game in normal form. (3 points)

(b) (7 points) Is it the case that, it is a weakly dominant strategy for each agent to vote for her ideal location, given the voting rule described above. Explain why, or why not.

(c) (5 points) If instead the city used the voting rule,

$$V(x) = V(x_1, \cdots, x_n) = \max\{x_1, \cdots, x_n\}$$

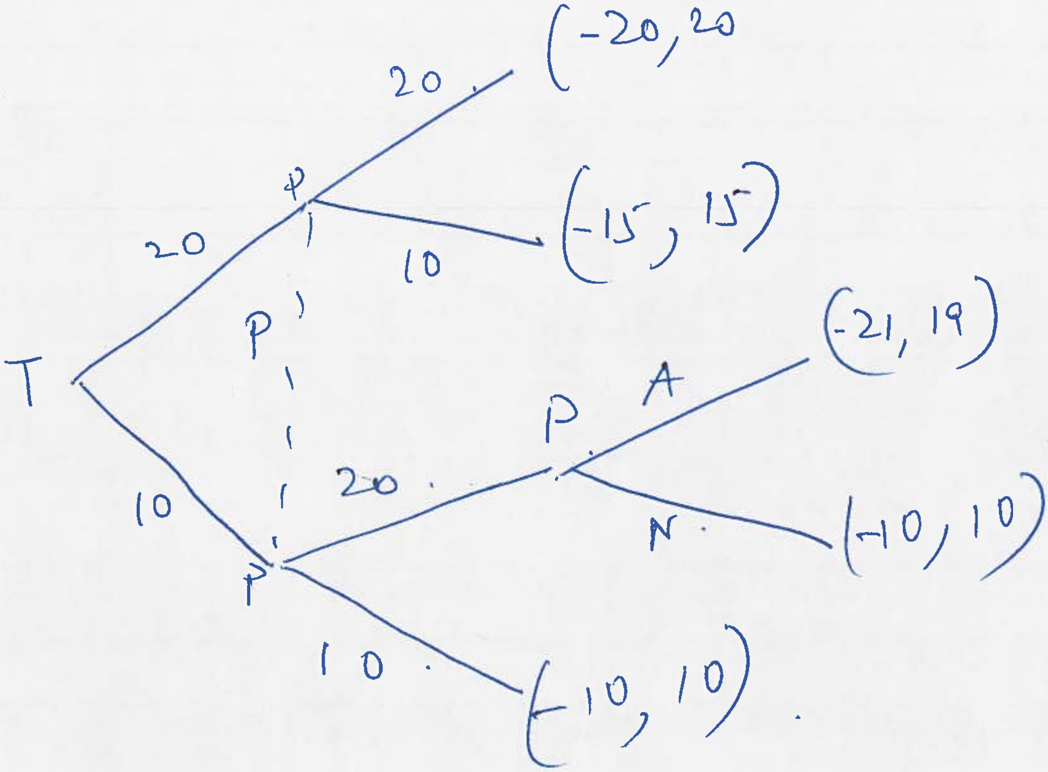
would you answer to part (b) change ? Why, or why not ?

Midterm Answer Key. 464.564

(2019) Winter

• Answer to question 1:

1 (a).



A → appeal

N →. Don't Appeal.

(b)

T		20A	20N	10A	10N
	20	−20, 20	−20, 20	−15, 15	−15, 15
	10	−21, 19	−10, 10	−10, 10	−10, 10

(c) Rationalizable Strategies for T : $R_T = \{20, 10\}$.

Rationalizable Strategies for P : $R_P = \{20A, 20N\}$.

Answer to Question 2:-

(a). For $i = 1, 2$, I write below person i's best Response for person j. (This is a SYMMETRIC game).

Person j (fixed $i \neq j$)	Person i's BRs	Person i's Payoff.
0	10	10
1	9, 10	9
2	8, 9, 10	8.
3	7, 8, 9, 10	7.
4	6, 7, 8, 9, 10	6
5	5, 6, 7, 8, 9, 10	5
6	5, 6	5
7	6	6
8	7	7
9	8	8
10	9	9

(b) There are 4 N.E. (b)

(5,5) (5,6), (6,5) & (6,6).

Answer to Question 3 (Midterm).

(a) Game in normal form.

Players – voters $\{1, \ldots, n\}$.

Strategy set for player i = $S_i = [0,1]$.

• Payoff function : as given in the question.

$$u_i(x_1 \ldots x_n) = -\left(p_i - \ell(x_1, \ldots x_n)\right)^2.$$

(b) Yes there is : I will show that voting for $x_i = p_i$ one's ideal location ; ie voting $x_i = p_i$ is a dominant strategy for each player i. The analysis is similar to that for 2nd price Auction.

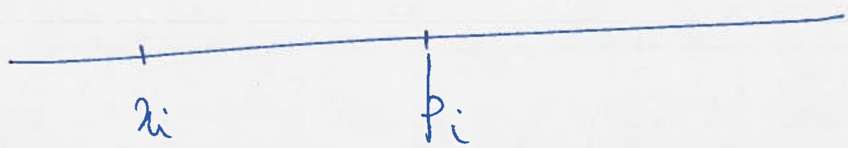
Let $b_i = \min_{j \neq i} \{x_j\}$.

There are two cases to consider.

(a) ~~bidding~~ voting for $x_i < p_i$

(b) ~~for~~ voting for $x_i > p_i$

Case (a)



(i) If $b_i < x_i < p_i$ then

$$\min\{p_i, b_i\} = \min\{x_i, b_i\} = b_i$$

i.e. the payoff of agent i does not change

(ii) $x_i < p_i < b_i$

$$\min\{p_i, b_i\} = p_i$$

Observe in this case.

So by ~~bidding~~ voting p_i, the voters' payoff is the highest.

iii $x_i < b_i < p_i$.

For this case $\min\{p_i, b_i\} = b_i$.

& $\min\{x_i, b_i\} = x_i$.

So if the voter votes for p_i her payoff is

$$-(p_i - b_i)^2 > -(p_i - x_i)^2 =$$

payoff if she votes ~~for~~ x_i.

Case b is analogous.