CALCULUS REVIEW

Chapter 1: Introduction to Calculus

Concepts:
Slope of a Secant & Tangent, Slope as a Limit, AROC & IROC, Limits, Properties, & Strategies to calculate

Chapter 1 Review Extra Practice

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1. Rationalize each denominator.
   a. $\dfrac{3\sqrt{3}}{\sqrt{14} - \sqrt{7}}$
   b. $\dfrac{3\sqrt{2}}{2\sqrt{3} - \sqrt{2}}$
   c. $\dfrac{5\sqrt{3} + 3\sqrt{2}}{5\sqrt{2} - 4\sqrt{3}}$
   d. $\dfrac{2\sqrt{3} - 3\sqrt{2}}{6\sqrt{6} - 5\sqrt{7}}$

2. Consider the graph of the function $f(x) = -2x^2 + 4x + 3$.
   a. Find the slope of the secant that joins the points on the graph given by $x = -1$ and $x = 0$.
   b. Determine the average rate of change as x changes from -2 to 3.
   c. Find the equation for the line tangent to the graph of the function at $x = 2$.

3. Calculate the slope of the graph of
   $$f(x) = \begin{cases} -3x - 2, \text{ if } x \leq -3 \\ x^2 - 2, \text{ if } x > -3 \end{cases}$$
   a. at $P(-4, 10)$
   b. at $P(2, 2)$

4. The height (in metres) of an object that has fallen from a height of 250 m is given by the position function $s(t) = -5t^2 + 250$, where $t \geq 0$ and t is in seconds.
   a. Find the average velocity of the object between the times $t = 1$ and $t = 4$.
   b. Find the average velocity of the object when $t = 3$.
   c. At what velocity will the object hit the ground?

5. Complete the table and use the results to estimate the limit. Then determine the limit using an algebraic technique and compare the answer with the estimate.
   $$\lim_{x \to 4} \frac{x - 4}{x^2 - x - 12}$$

x	$\dfrac{x-4}{x^2-x-12}$
3.9	
3.99	
3.999	
4.001	
4.01	
4.1	

6. a. Sketch the graph of the following function.
   $$f(x) = \begin{cases} x + 2, \text{ if } x < -3 \\ -x + 3, \text{ if } -3 \leq x < 2 \\ x + 3, \text{ if } x \geq 2 \end{cases}$$
   b. Find all values at which the function is discontinuous.
   c. Find the limits at those values, if they exist.

7. Calculate each limit.
   a. $\lim_{x \to 1} (-2x)$
   b. $\lim_{x \to -3} (3x - 4)$
   c. $\lim_{x \to 4} (x^2 - 2x + 3)$
   d. $\lim_{x \to 0} 3^{\frac{x}{2}}$

8. Find constants a and b such that $f(x)$ is continuous for all values of x.
   $$f(x) = \begin{cases} ax + 1, \text{ if } x > 2 \\ 3, \text{ if } x = 2 \\ x^2 + bx - a, \text{ if } x < 2 \end{cases}$$

Chapter 1 Review Extra Practice Answers

1. a. $\dfrac{3(\sqrt{42} + \sqrt{21})}{7}$

   b. $\dfrac{3(\sqrt{6} + 1)}{5}$

   c. $\dfrac{37\sqrt{6} + 90}{2}$

   d. $\dfrac{36\sqrt{2} + 10\sqrt{21} - 36\sqrt{3} - 15\sqrt{14}}{41}$

2. a. The slope of the secant is 6.

   b. The average rate of change is 2.

   c. $y = -4x + 11$

3. a. -3

   b. 4

4. a. -25 m/s

   b. -30 m/s

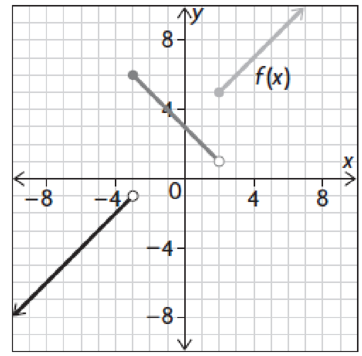
   c. -70.71 m/s

5.

x	$\dfrac{x - 4}{x^2 - x - 12}$
3.9	0.1449
3.99	0.1431
3.999	0.1429
4.001	0.1428
4.01	0.1427
4.1	0.1408

The algebraic limit is $\frac{1}{7}$ and the values in the table confirm this.

6. a.



   b. The function is discontinuous at the points $x = -3$ and $x = 2$.

   c. The limit of $f(x)$ does not exist at either $x = -3$ or $x = 2$.

7. a. -2

   b. -13

   c. 11

   d. 1

8. $a = 1, b = 0$

Chapter 2: Derivatives

Concepts:
Derivative at a Point, Der. Function, Leibnitz Notation, Der. Of Polynomials Rules; Product & Quotient Rules, Der. Of Composite Functions – Chain rule.

Chapter 2 Review Extra Practice

STUDENT BOOK PAGES 110–113

1. Use the definition of the derivative to determine $f'(x)$ for each of the following functions.
   a. $f(x) = 3x + 2$
   b. $f(x) = x^2 + 2x - 1$
   c. $f(x) = \sqrt{x + 1}$
2. Differentiate each of the following functions.
   a. $f(x) = 6x^2 + 3x - 1$
   b. $f(x) = 2\sqrt{x^2 + 1}$
   c. $f(x) = (x + 3)(2x^2 + 3x)$
   d. $f(x) = (5x^2 + 1)^3$
   e. $f(x) = \dfrac{2x + 3}{x - 1}$
   f. $f(x) = 2x(3x - 2)^3$
3. Determine the derivative of the given function.
   a. $f(x) = \dfrac{1}{\sqrt{x}}$
   b. $f(x) = \dfrac{x}{x^3 - 1}$
   c. $f(x) = (x + 4)(x^2 + 3x + 2)$
   d. $f(x) = (3x^2 + x - 1)^4$
   e. $f(x) = \dfrac{x + \pi}{2x - \pi}$
   f. $f(x) = (x - 1)^3(x - 4)^2$
4. Determine the slope of the tangent line to the curve at the given value of x.
   a. $f(x) = 5x^2 + x^3 - 2x, x = 1$
   b. $f(x) = (x + 2)^3(x - 4)^2, x = 0$
   c. $f(x) = \sqrt{x^2 + x + 2}, x = 1$
   d. $f(x) = 4 - \dfrac{3}{2}x, x = 7$
   e. $f(x) = \left(\dfrac{2x + 1}{x + 2}\right)^3, x = 0$
   f. $f(x) = \dfrac{x^2 + 3}{2 - x}, x = 1$
5. If f is a differentiable function, find an expression for the derivative of each of the following functions.
   a. $g(x) = f(x^2 + 1)$
   b. $g(x) = (x + 1)f(3x)$
6. Use the chain rule, in Leibniz notation, to determine $\frac{dy}{dx}$ at the given value of x.
   a. $y = 3u^2 + 1, u = x^2 + 1, x = -1$
   b. $y = u + \sqrt{u}, u = x - 2, x = 4$
   c. $y = (u + 1)^3, u = (x + 1)^2, x = 0$
   d. $y = \dfrac{1}{u + 1}, u = \dfrac{1}{x}, x = 1$
7. Determine the slope of the tangent to the curve $y = \sqrt{(1 + x)^3}$ at $(3, 8)$.
8. Determine the value(s) of x where the graph of each function has a horizontal tangent.
   a. $f(x) = (x - 1)^2(x + 3)^2$
   b. $f(x) = \sqrt{x^2 + 4x - 2}$
9. Determine the equation of the normal to $y = x^3 + x - 1$ at $(1, 1)$.
10. Determine the equation of the tangent to $y = x(2x + 1)^3$ when $x = 1$.

Chapter 2 Review Extra Practice Answers

1. a. 3
   b. $2x + 2$
   c. $\frac{1}{2\sqrt{x + 1}}$
2. a. $12x + 3$
   b. $\frac{2x}{\sqrt{x^2 + 1}}$
   c. $6x^2 + 18x + 9$
   d. $30x(5x^2 + 1)^2$
   e. $\frac{-5}{(x - 1)^2}$
   f. $4(3x - 2)^2(6x - 1)$
3. a. $-\frac{1}{2\sqrt{x^3}}$
   b. $-\frac{2x^3 + 1}{(x^3 - 1)^2}$
   c. $3x^2 + 14x + 14$
   d. $4(6x + 1)(3x^2 + x - 1)^3$
   e. $\frac{-3\pi}{(2x - \pi)^2}$
   f. $(x - 1)^2(x - 4)(5x - 14)$
4. a. 11
   b. 128
   c. $\frac{3}{4}$
   d. $-\frac{3}{2}$
   e. $\frac{9}{16}$
   f. 6
5. a. $2xf'(x^2 + 1)$
   b. $3(x + 1)f'(3x) + f(3x)$
6. a. -24
   b. $\frac{\sqrt{2}}{4} + 1$
   c. 24
   d. $\frac{1}{4}$
7. 3
8. a. $x = -3, x = -1, x = 1$
   b. $x = -2$
9. $x + 4y - 5 = 0$
10. $81x - y - 54 = 0$

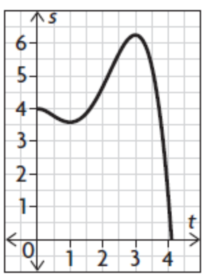
Chapter 3: Derivatives and their Applications

Concepts:
Higher Order Derivatives: Velocity, Acceleration; Min. & Max. values on a Interval; Optimization; Optimization in Science & Economics

Chapter 3 Review Extra Practice

STUDENT BOOK PAGES 156–159

1. For the following position versus time functions determine the functions for velocity and acceleration.
   a. $s(t) = t(t - 5)^2$
   b. $s(t) = \dfrac{6}{(t - 4)^2}$
   c. $s(t) = \sqrt{t^2 - 5}$
   d. $s(t) = \dfrac{t^4}{16} + t^3 + t^{\frac{4}{5}}$
   e. $s(t) = \dfrac{t^2}{\sqrt{t + 2}}$
2. For $0 \leq t \leq 4.1$ use the following position versus time graph for the motion of an object moving horizontally.



   a. When is the velocity zero?
   b. What is the maximum value on the given interval?
   c. What is the minimum value on the given interval?
   d. When is the object moving in the positive direction?
   e. When is the object moving in the negative direction?
3. Determine the absolute extreme values of each function of the given interval.
   a. $f(x) = x^4 - 15x^3 + 50x, x \in [-1, 5]$
   b. $f(x) = \dfrac{x^2 - 1}{x - 2}, 0 \leq x \leq 1$
   c. $f(x) = 2x^2 + \dfrac{13}{x}, x \in [1, 4]$
4. A man is building a fence around a portion of his backyard. He wants the area inside the fence to be 600 m^2. If his house accounts for 20 m of the fence, what is the minimum amount of fencing he needs to purchase?
5. At noon a car is driving west at 55 km/h. At the same time, 15 km due north another car is driving south at 85 km/h. At what time are the two cars closest, and what is the distance between them?
6. A company is afraid they are spending too much on the production of some small cans they use. The current cans hold 80 ml. If the bottom of the can costs \$0.002 per cm^2, the top costs \$0.01 per cm^2 and the side of the can costs \$0.0009 per cm^2, how much is the cheapest can that holds the required amount?

Chapter 3 Review Extra Practice Answers

1. a. $v(t) = 3t^2 - 20t + 25$
   $a(t) = 6t - 20$

   b. $v(t) = -\dfrac{12}{(t-4)^3}$

   $a(t) = \dfrac{36}{(t-4)^4}$

   c. $v(t) = \dfrac{t}{\sqrt{t^2 - 5}}$

   $a(t) = \dfrac{1}{\sqrt{t^2-5}} - \dfrac{t^2}{(t^2-5)^{\frac{3}{2}}}$

   d. $v(t) = \dfrac{1}{4}t^3 + 3t^2 + \dfrac{4}{5}t^{-\frac{1}{5}}$

   $a(t) = \dfrac{3}{4}t^2 + 6t - \dfrac{4}{25}t^{-\frac{6}{5}}$

   e. $v(t) = \dfrac{2t}{(t+2)^{\frac{1}{2}}} - \dfrac{t^2}{2(t+2)^{\frac{3}{2}}}$

   $a(t) = \dfrac{2}{(t+2)^{\frac{1}{2}}} - \dfrac{2t}{(t+2)^{\frac{3}{2}}} + \dfrac{3t^2}{4(t+2)^{\frac{5}{2}}}$

2. a. $t = 1$ and $t = 3$
   b. Max: (3, 6.25)
   c. Min: (4.1, 0)
   d. $t \in (1, 3)$
   e. $t \in (0, 1)$ and $t \in (3, 4.1)$

3. a. Max: (1.11, 36.5)
   Min: (5, −1000)
   b. Max: (0.27, 0.54)
   Min: (1, 0)
   c. Max: (4, 35.25)
   Min: (1.48, 13.16)

4. 77.98 m

5. 8.15 km away, 7 minutes and 28 seconds after noon

6. $0.17

Chapter 4: Curve Sketching

Concepts:
Intervals of Increase & Decrease; Critical Points, Local Minima & Maxima; Asymptotes: VA, HA, OA; Concavity & Points of Inflection; Algorithm for C-Sketching.

Chapter 4 Review Extra Practice

STUDENT BOOK PAGES 216–219

1. For each of the following functions, determine algebraically:
   i. the intervals where the function is increasing
   ii. the intervals where the function is decreasing
   iii. the x-coordinates of the points where the tangent to the function is horizontal

   a. $f(x) = x^3 - x + \frac{5}{2}$

   b. $f(x) = x + \frac{9}{x}$

   c. $f(x) = x^5 + 1$

2. For each function, find the critical numbers. Then, use the first derivative test to determine whether the point corresponding to the critical number is a local maximum, a local minimum, or neither.

   a. $f(x) = 1 - 6x^2$

   b. $f(x) = 3x^3 + 3x^2 + 3x + 3$

   c. $f(x) = \frac{4x^3}{x^2 - 8}$

3. For each of the following functions, determine the equations of any vertical asymptotes and horizontal asymptotes.

   a. $f(x) = \frac{x + 2}{x - 2}$

   b. $f(x) = \frac{x}{x^2 - 11x}$

   c. $f(x) = \frac{3x^2 + 10}{x^2 - 16}$

   d. $f(x) = \frac{x^3 - 7}{6x^2}$

   e. $f(x) = \frac{x^3 - 64}{2x^2 - 3x - 20}$

4. Determine the critical numbers for each of the following, and use the second derivative test to decide if the corresponding point is a local maximum, a local minimum, or neither.

   a. $f(x) = (x + 6)(x + 4)$

   b. $f(x) = x^3 + \frac{1}{2}x^2 - 5x - 4$

   c. $f(x) = \frac{5}{x^2 - 4}$

   d. $f(x) = x - \frac{100}{x}$

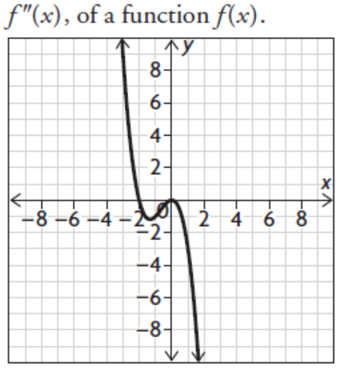
5. Use the algorithm for curve sketching to sketch the graph of each of the following functions.

   a. $f(x) = x^4 - x^3 - x^2$

   b. $f(x) = \frac{1}{x^2 - 9x}$

   c. $f(x) = x^2 - \frac{1}{5}x^4$

6. The following graph represents the second derivative, $f''(x)$, of a function $f(x)$.



   a. On which intervals is the graph of $f(x)$ concave up, and on which is the graph concave down?

   b. List the x-coordinates of any points of inflection.

   c. Make a rough sketch of a possible graph of $f(x)$.

Chapter 4 Review Extra Practice Answers

1. a. i. $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$

   ii. $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

   iii. $x = -\frac{\sqrt{3}}{3}, x = \frac{\sqrt{3}}{3}$

   b. i. $(-\infty, -3)$ and $(3, \infty)$

   ii. $(-3, 0)$ and $(0, 3)$

   iii. $x = -3, x = 3$

   c. i. $(-\infty, 0)$ and $(0, \infty)$

   ii. never decreasing

   iii. $x = 0$

2. a. $x = 0$: local maximum

   b. no critical points or local extrema

   c. $x = 0, x = \sqrt{24}, x = -\sqrt{24}$; local maximum at $x = -\sqrt{24}$, local minimum at $x = \sqrt{24}$

3. a. vertical asymptote $x = 2$; horizontal asymptote $y = 1$

   b. vertical asymptote $x = 11$; horizontal asymptote $y = 0$

   c. vertical asymptotes $x = 4, x = -4$; horizontal asymptote $y = 3$

   d. vertical asymptote $x = 0$; no horizontal asymptote

   e. vertical asymptotes $x = -\frac{5}{2}, x = 4$; no horizontal asymptote

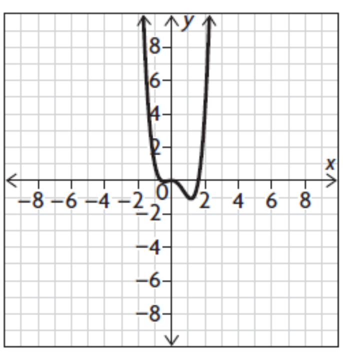
4. a. $x = -5$: local minimum

   b. $x \doteq -1.47$: local maximum, $x \doteq 1.13$: local minimum

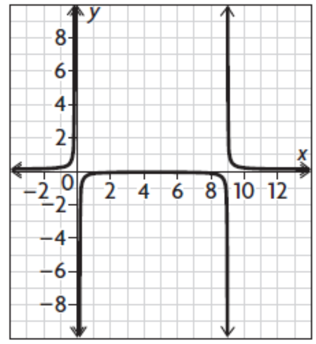
   c. $x = 0$, local maximum at $x = 0$

   d. There are no critical points; no local extrema

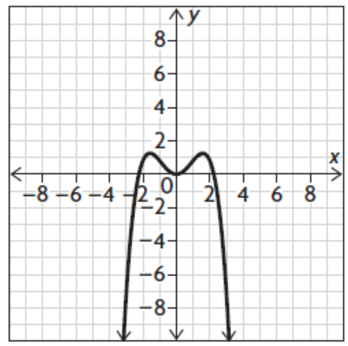
5. a.



b.



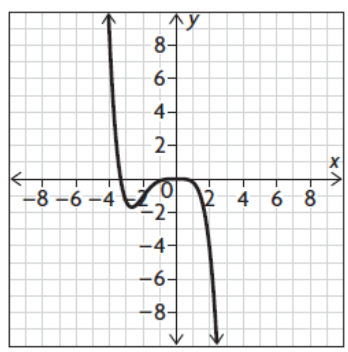
c.



6. a. concave up on $(-\infty, -2)$; concave down on $(-2, 0)$ and $(0, \infty)$

   b. $x = -2$ (Note: For $x = 0$, concavity does not switch signs on each side of $x = 0$, so $x = 0$ is not an inflection point.)

   c.



Chapter 5: Derivatives of Exponential and Trigonometric Functions

Concepts:

$(e^x)'$; $(e^{g(x)})'$; $(b^x)'$; $(b^{g(x)})'$; Optimization w/ Exponentials; $(\ln(x))'$; $(\ln(g(x)))'$; $(\log_b(x))'$; $(\log_b(g(x)))'$; $(\sin(x))'$; $(\cos(x))'$; $(\tan(x))'$

Chapter 5 Review Extra Practice

STUDENT BOOK PAGES 263–265

1. Determine $\frac{dy}{dx}$ for each of the following.
   a. $y = -e^{-x}$
   b. $y = e^{x^2}$
   c. $y = e^{-5x}$
   d. $y = 2xe^{2x}$
   e. $y = \frac{e^x}{e^{-x}}$

2. Determine $f'(x)$ for each of the following functions.
   a. $f(x) = 6^x$
   b. $f(x) = 3^{\sqrt{x}}$
   c. $f(x) = (x^3)3^x$
   d. $f(x) = \frac{3^x}{x^3}$

3. Determine $\frac{dy}{dx}$ for each of the following.
   a. $y = 8 \sin 8x$
   b. $y = -\sin x + \frac{1}{4}\cos 4x$
   c. $y = \frac{1}{3}\cos^3 x$
   d. $y = 2x (\tan x)$
   e. $y = \sin^2(e^x)$
   f. $y = \tan(2^x)$

4. For the following functions, first determine the x-coordinate of any critical points, if they exist. Then, determine all maximum and minimum values.
   a. $f(x) = -2xe^{-x}$
   b. $f(x) = xe^{x+2} - 4$
   c. $f(x) = -3^x + 5x$

5. For each of the following functions, determine the slope of the tangent to the curve at the point with the given x-coordinate.
   a. $f(x) = \tan 5x,\ x = \pi$
   b. $f(x) = 0.5x - 3\cos x,\ x = \frac{\pi}{6}$
   c. $f(x) = e^{\sin x},\ x = 3\pi$

6. Determine the local maximum point and the local minimum point on the curve $y = 4x - \tan x$ in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

7. Determine $\frac{d^2y}{dx^2}$ for each of the following.
   a. $y = 4e^{5x}$
   b. $y = \sin(e^{-x})$
   c. $y = \tan(2x^2)$

8. Determine y' for each of the following.
   a. $y = \cos(7^x)$
   b. $y = \tan(5 + x^3)$
   c. $y = \frac{\sin x}{e^x}$

9. For each of the following functions, determine an equation for the tangent to the curve at the point with the given x-coordinate.
   a. $f(x) = \cos(2x - \pi),\ x = \frac{\pi}{3}$
   b. $f(x) = e^{\cos x},\ x = \pi$

Chapter 5 Review Extra Practice Answers

1. a. $\frac{dy}{dx} = e^{-x}$
   b. $\frac{dy}{dx} = 2xe^{x^2}$
   c. $\frac{dy}{dx} = -5e^{-5x}$
   d. $\frac{dy}{dx} = 2e^{2x}(2x + 1)$
   e. $\frac{dy}{dx} = 2e^{2x}$

2. a. $f'(x) = (6^x)\ln 6$
   b. $f'(x) = \frac{1}{2\sqrt{x}}(3^{\sqrt{x}})\ln 3$
   c. $f'(x) = 3^x(x^3\ln 3 + 3x^2)$
   d. $f'(x) = \frac{x(3^x)\ln 3 - 3(3^x)}{x^4}$

3. a. $\frac{dy}{dx} = 64 \cos 8x$
   b. $\frac{dy}{dx} = -\cos x - \sin 4x$
   c. $\frac{dy}{dx} = -\cos^2 x \sin x$
   d. $\frac{dy}{dx} = 2x \sec^2 x + 2 \tan x$
   e. $\frac{dy}{dx} = 2e^x \sin (e^x) \cos (e^x)$
   f. $\frac{dy}{dx} = \ln 2(2^x) \sec^2(2^x)$

4. a. critical point at $x = 1$; minimum value: -0.74 at $x = 1$
   b. critical point at $x = -1$; minimum value: -6.72 at $x = -1$
   c. critical point at $x = 1.38$; maximum value: 2.35 at $x = 1.38$

5. a. 5
   b. 2
   c. -1

6. maximum: (1.05, 2.46); minimum: $(-1.05, -2.46)$

7. a. $\frac{d^2y}{dx^2} = 100e^{5x}$
   b. $\frac{d^2y}{dx^2} = -e^{-2x}(\sin (e^{-x})) + e^{-x}(\cos (e^{-x}))$
   c. $\frac{d^2y}{dx^2} = 32x^2\sec^2(2x^2)\tan (2x^2) + 4\sec^2(2x^2)$

8. a. $-\ln 7(7^x)\sin (7^x)$
   b. $3x^2\sec^2(5 + x^3)$
   c. $\frac{\cos x - \sin x}{e^x}$

9. a. $-\sqrt{3}x + y - \left(\frac{1}{2} - \frac{\pi}{3}\right) = 0$
   b. $y = \frac{1}{e}$

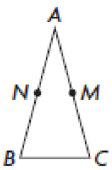
Chapter 6: An Introduction to Vectors

Concepts:
Vectors: Definition; Operations (Addition, Subtraction, Multiplication by a Scalar); Properties of Operations w/ Vectors;
Algebraic Vectors in R^2, R^3 and their components; Operations w/ Algebraic Vectors in R^2, R^3; Linear Combinations & Spanning Sets

Chapter 6 Review Extra Practice

STUDENT BOOK PAGES 344–347

1. Given the following diagram, N and M are midpoints of their respective sides. Determine a vector that represents each of the following.



   a. $2\overrightarrow{MC}$
   b. $\overrightarrow{BC} - \overrightarrow{NM}$

2. Simplify each of the following vector expressions.
   a. $\overrightarrow{XY} + (\overrightarrow{YZ} + \overrightarrow{ZW}) + \overrightarrow{WP}$
   b. $\overrightarrow{MN} + \overrightarrow{NH} + \overrightarrow{HM} + \overrightarrow{AM}$
   c. $(2\overrightarrow{AB} + 2\overrightarrow{CE}) + 2(\overrightarrow{BC} + \overrightarrow{EF})$
   d. $\overrightarrow{XY} + 3(\overrightarrow{YX} - 2\overrightarrow{YZ})$

3. If $\vec{a} = \vec{i} + \vec{j}$ and $\vec{b} = 3\vec{i} - 2\vec{j}$, determine if the following vector pairs are collinear.
   a. $\vec{a}$ and $\vec{b}$
   b. $\vec{a}$ and $-\vec{b}$
   c. $5\vec{a} + 5\vec{b}$ and $\frac{1}{2}\vec{b} + \frac{1}{2}\vec{a}$
   d. $-\vec{b} + 5\vec{a}$ and $\vec{a} + 7\vec{b}$
   e. $\vec{b} - 5\vec{a}$ and $\vec{a} - \frac{1}{5}\vec{b}$

4. Simplify each of the following vector expressions.
   a. $\vec{a} + 5(\vec{a} + \vec{b})$
   b. $12(\vec{a} - \vec{b}) - 2(\vec{b} + \vec{a})$
   c. $3\vec{c} - (\vec{a} + 2\vec{b}) - 5(2\vec{c} - \vec{b})$
   d. $2\vec{a} - \vec{b} + 3\vec{a} + 5\vec{b} + 3(6\vec{a} - 2\vec{b})$
   e. $5(\vec{x} + \vec{y} + \vec{z}) + 7(\vec{x} + 4\vec{y} - \vec{z}) - (3\vec{x} + 2\vec{z})$

5. Determine the axis that the point falls on. Be specific with positive and negative, for example, "the positive x-axis".
   a. (3, 0, 0)
   b. (0, −5, 0)
   c. (0, 12, 0)
   d. (0, 0, −8)
   e. (0, 0, 1)
   f. (−9, 0, 0)

6. Given the points $A(-1, 0)$ and $B(2, 5)$, determine
   a. $\overrightarrow{AB}$
   b. $\overrightarrow{BA}$
   c. $|\overrightarrow{AB}|$
   d. $|\overrightarrow{BA}|$
   e. $|\overrightarrow{AB} + \overrightarrow{BA}|$
   f. $|\overrightarrow{AB}| + |\overrightarrow{BA}|$

7. Given $\vec{m} = (-1, 2, -1)$ and $\vec{n} = (0, -2, 3)$, determine the following:
   a. $|\vec{m}|$
   b. $|-\vec{n}|$
   c. $|\vec{n} + \vec{m}|$
   d. $|\vec{m} - 2\vec{n}|$
   e. $|-\vec{m} - 5\vec{n}|$
   f. $2|\vec{m} - \vec{n}|$

8. Determine which of the following vectors span R^2.
   a. (0, 0) and (0, 1)
   b. (−3, 0) and (−4, 0)
   c. (12, 14) and (6, 7)
   d. (6, 0) and (4, 1)
   e. (−11, 5) and (1, 2)
   f. (6, 12) and (12, 24)

Chapter 6 Review Extra Practice Answers

1. a. $\overrightarrow{AC}$
b. $\overrightarrow{NM}$

2. a. $\overrightarrow{XP}$
b. $\overrightarrow{AM}$
c. $2\overrightarrow{AF}$
d. $-2\overrightarrow{XY} - 6\overrightarrow{YZ}$ or $-2\overrightarrow{XZ} - 4\overrightarrow{YZ}$

3. a. No
b. No
c. Yes
d. No
e. Yes

4. a. $6\vec{a} + 5\vec{b}$
b. $10\vec{a} - 14\vec{b}$
c. $-\vec{a} + 3\vec{b} - 7\vec{c}$
d. $23\vec{a} - 2\vec{b}$
e $9\vec{x} + 33\vec{y} - 4\vec{z}$

5. a. positive x-axis
b. negative y-axis
c. positive y-axis
d. negative z-axis
e. positive z-axis
f. negative x-axis

6. a. (3, 5)
b. $(-3, -5)$
c. $\sqrt{34}$
d. $\sqrt{34}$
e. 0
f. $2\sqrt{34}$

7. a. $\sqrt{6}$
b. $\sqrt{13}$
c. $\sqrt{5}$
d. $\sqrt{86}$
e. $3\sqrt{29}$
f. $2\sqrt{33}$

8. a. No
b. No
c. No
d. Yes
e. Yes
f. No

Chapter 7: Applications of Vectors

Concepts:
Forces as Vectors (Resultant, Equilibrant, Force Decomposition or Resolution, Equilibrium); Velocities as Vectors (Composition & Decomposition of Velocities); Dot Product of Geometric & Algebraic Vectors; Scalar & Vectors Projections; Cross Product; Apps of DOT & CROSS.

Chapter 7 Review Extra Practice

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1. Solve the triangles determined by the points A, B, and C below. (Determine all side lengths and angle measures for $\triangle ABC$.) Calculate all inexact answers to two decimal places accuracy.
   a. $A(1, -1, 3)$, $B(2, 7, -3)$, $C(-1, 2, 6)$
   b. $A(5, 1, -2)$, $B(3, -2, 2)$, $C(1, -2, 3)$
   c. $A(-2, -3, 2)$, $B(-4, 10, 6)$, $C(2, -1, 3)$
   d. $A(-3, 2, 1)$, $B(1, 8, -7)$, $C(-5, 1, 4)$
   e. $A(-1, -2, 1)$, $B(2, 1, -1)$, $C(2, -2, 1)$
   f. $A(1, 8, -2)$, $B(-2, 5, 3)$, $C(-1, 4, 1)$

2. Compute the area of the triangle determined by the following pairs of vectors. Calculate all inexact answers to two decimal places accuracy.
   a. $(-1, 3, -2)$ and $(4, -2, -3)$
   b. $(3, -1, 2)$ and $(2, 1, 2)$
   c. $(4, -3, 5)$ and $(3, -6, 1)$
   d. $(3, 1, -2)$ and $(-2, 5, -6)$
   e. $(1, -1, -1)$ and $(2, 2, 5)$
   f. $(2, 3, -2)$ and $(-3, 4, -1)$

3. Compute the scalar and vector projections of $\vec{a}$ on $\vec{b}$ for each pair of vectors below.
   a. $\vec{a} = (1, 2)$, $\vec{b} = (-1, 3)$
   b. $\vec{a} = (-3, 4)$, $\vec{b} = (2, 2)$
   c. $\vec{a} = (1, 2, 3)$, $\vec{b} = (-2, 4, 1)$
   d. $\vec{a} = (-2, 2, 3)$, $\vec{b} = (-1, 2, 4)$
   e. $\vec{a} = (5, 2, 1)$, $\vec{b} = (1, 2, 2)$
   f. $\vec{a} = (-3, 2, 4)$, $\vec{b} = (-4, 3, 3)$

4. Compute the direction angles α, β, and γ, formed with the positive x-, y-, and z-axes, respectively, for each of the following vectors. Report all inexact answers to two decimal places accuracy.
   a. $(0, -1, 5)$
   b. $(2, -1, 0)$
   c. $(-3, 0, 4)$
   d. $(-1, 1, 4)$
   e. $(2, 3, 6)$
   f. $(4, 1, -2)$

5. For each of the following vector computations, say whether the result will be a scalar, a vector, or if the computation is meaningless.
   a. $|\vec{a} \times \vec{b}| - |\vec{c} \times \vec{d}|$
   b. $((\vec{a} \times \vec{b}) \times \vec{c}) \times \vec{d} - \vec{a} \times \vec{b}$
   c. $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{a} \cdot \vec{b}$
   d. $\vec{a} \cdot (\vec{b} \times \vec{c}) - \vec{b} \cdot \vec{c}$
   e. $((\vec{a} \times \vec{b}) \cdot \vec{c}) \times \vec{d}$
   f. $(((\vec{a} \times \vec{b}) \times \vec{c}) \cdot \vec{d}) \cdot \vec{a} - |\vec{a} \times \vec{b}|\vec{c}$

6. A tow truck driver pulls a car closer to the back of his truck with a hydraulic winch that forms an angle of 15° with the horizontal. If the winch on his truck pulls the car with a force of 2500 N, and the car rolled a total of 7 m horizontally to reach the back of the truck, how much work, in J, has the winch done up to this point? Report your answer to two decimal places accuracy.

7. Suppose that a certain jar-opener has a handle that is 30 cm in length, and that a jar of pickles requires 10 J of torque to open. If we were to apply 40 N of force to the end of the jar opener handle at an angle of 45° to the handle, would the jar open? Explain, and report any inexact computations to two decimal places accuracy.

Chapter 7 Review Extra Practice Answers

1. a. side lengths: $\sqrt{22}$, $\sqrt{101}$, $\sqrt{115}$
   angle measures: 25.84°, 69.03°, 85.13°
   b. side lengths: $\sqrt{5}$, $\sqrt{29}$, $5\sqrt{2}$
   angle measures: 13.67°, 34.70°, 131.63°
   c. side lengths: $\sqrt{21}$, $\sqrt{166}$, $3\sqrt{21}$
   angle measures: 19.47°, 69.56°, 90.97°
   d. side lengths: $\sqrt{14}$, $2\sqrt{29}$, $\sqrt{206}$
   angle measures: 4.98°, 14.47°, 160.55°
   e. side lengths: 3, $\sqrt{13}$, $\sqrt{22}$
   angle measures: 39.76°, 50.24°, 90°
   f. side lengths: $\sqrt{6}$, $\sqrt{29}$, $\sqrt{43}$
   angle measures: 20.85°, 51.50°, 107.65°

2. a. 9.87 square units
   b. 3.35 square units
   c. 16.39 square units
   d. 14.04 square units
   e. 4.30 square units
   f. 9.72 square units

3. a. $\frac{5}{\sqrt{10}}, \left(-\frac{1}{2}, \frac{3}{2}\right)$
   b. $\frac{2}{\sqrt{8}} = \frac{1}{\sqrt{2}}, \left(\frac{1}{2}, \frac{1}{2}\right)$
   c. $\frac{9}{\sqrt{21}}, \left(-\frac{6}{7}, \frac{12}{7}, \frac{3}{7}\right)$
   d. $\frac{18}{\sqrt{21}}, \left(-\frac{6}{7}, \frac{12}{7}, \frac{24}{7}\right)$
   e. $\frac{11}{3}, \left(\frac{11}{9}, \frac{22}{9}, \frac{22}{9}\right)$
   f. $\frac{30}{\sqrt{34}}, \left(-\frac{60}{17}, \frac{45}{17}, \frac{45}{17}\right)$

4. a. $\alpha = 90°, \beta \doteq 101.31°, \gamma \doteq 11.31°$
   b. $\alpha \doteq 26.57°, \beta \doteq 116.57°, \gamma = 90°$
   c. $\alpha \doteq 126.87°, \beta = 90°, \gamma \doteq 36.87°$
   d. $\alpha \doteq 103.63°, \beta \doteq 76.37°, \gamma \doteq 19.47°$
   e. $\alpha \doteq 73.40°, \beta \doteq 64.62°, \gamma \doteq 31.00°$
   f. $\alpha \doteq 29.21°, \beta \doteq 77.40°, \gamma \doteq 115.88°$

5. a. scalar
   b. vector
   c. meaningless
   d. scalar
   e. meaningless
   f. meaningless

6. $7(2500 \cos 15°)$ J $\doteq$ 16 903.70 J

7. No; The magnitude of the torque is only $40(0.3 \sin 45°)$ J $\doteq$ 8.49 J, which is not enough to open the jar.

Chapter 8: Equations of Lines and Planes

Concepts:
Vector, Parametric, and **Cartesian** Equations of a Line in $\mathbf{R}^2$; **Vector, Parametric,** and **Symmetric** Equations of a Line in $\mathbf{R}^3$; **Vector, Parametric,** and **Cartesian** Equations of a Plane (obviously in $\mathbf{R}^3$ - ☺);

Chapter 8 Review Extra Practice

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1. a. Determine the vector, parametric, and symmetric equations (if possible) for the line passing through the points $A(0, 2, 3)$ and $B(3, -1, 3)$.
   b. Determine the vector and parametric equations for the plane containing the points $A(0, 2, 3)$, $B(3, -1, 3)$, and $C(2, 0, 1)$.
2. Determine the Cartesian equation of the plane passing through the point $P(2, -1, 2)$ and containing the line $\vec{r} = (2, -1, 4) + t(0, 3, -5)$, $t \in \mathbf{R}$.
3. Find the vector and parametric equations for the plane parallel to the xy-plane and passing through the point $Q(2, 3, 3)$.
4. A plane has Cartesian equation $4x - 3y + 6z - 12 = 0$.
   a. Determine vector and parametric equations for this plane.
   b. Determine the equation of a line that lies in the plane.
   c. Determine the equation of a plane that is perpendicular to the first plane and contains the line in 4.b.
5. Write a brief explanation of the plane represented by the equation $2x - z = 0$.
6. Sketch the following planes:
   a. π_1: $2x + 3y + 6z - 12 = 0$
   b. π_2: $2x + 3y - 12 = 0$
   c. π_3: $3y - 12 = 0$
7. A line is defined by the parametric equations $x = -1 + 3t$, $y = 2 + 4t$, $z = 2t$, $t \in \mathbf{R}$. If the point $(a, 0, b)$ lies on the line, determine a and b.
8. Two lines L_1 and L_2 are defined by the vector equations L_1: $\vec{r} = (1, -1, 1) + t(0, 3, -5)$, $t \in \mathbf{R}$ and L_2: $\vec{r} = (1, 2, -4) + t(0, 5, 3)$, $t \in \mathbf{R}$.
   a. Do L_1 and L_2 intersect?
   b. Are L_1 and L_2 perpendicular?
   c. Determine the vector and parametric equations of a plane that contains both L_1 and L_2.
9. Calculate the angle formed by the intersection of the lines L_1: $\vec{r} = (1, 0, 3) + t(1, 2, -5)$, $t \in \mathbf{R}$ and L_2: $\vec{r} = (1, 0, 3) + t(1, 3, 3)$, $t \in \mathbf{R}$.
10. Which of the following lines is parallel to the plane $2x - y + 5z - 13 = 0$?
    a. $\vec{r} = (2, -1, 4) + t(0, 5, 1)$, $t \in \mathbf{R}$
    b. $x = -1 + 3t$, $y = 2 + 11t$, $z = t$, $t \in \mathbf{R}$
    c. $\dfrac{x-2}{3} = \dfrac{y+1}{-2} = \dfrac{z}{4}$
11. Does the origin lie in the plane $\vec{r} = (2, -1, 4) + s(1, -2, 3) + t(0, 5, 1)$, $s, t \in \mathbf{R}$?
12. a. Determine the Cartesian equation for the plane that has normal vector $(3, 1, -5)$ and passes through the point $A(0, 1, -1)$.
    b. Determine the Cartesian equation for the plane parallel to the above plane that passes through the origin.
13. Determine the point of intersection of the lines L_1: $x = 2 + 3t$, $y = 3t$, $t \in \mathbf{R}$ and L_2: $x = 3s$, $y = 3 - 2s$, $s \in \mathbf{R}$.

Chapter 8 Review Extra Practice Answers

1. a. $\vec{r} = (0, 2, 3) + t(3, -3, 0), t \in \mathbf{R}$
   $x = 3t, y = 2 - 3t, z = 3, t \in \mathbf{R}$
   no symmetric equations
   b. $\vec{r} = (0, 2, 3) + s(3, -3, 0) + t(2, -2, -2), s, t \in \mathbf{R}$
   $x = 3s + 2t, y = 2 - 3s - 2t, z = 3 - 2t, s, t \in \mathbf{R}$

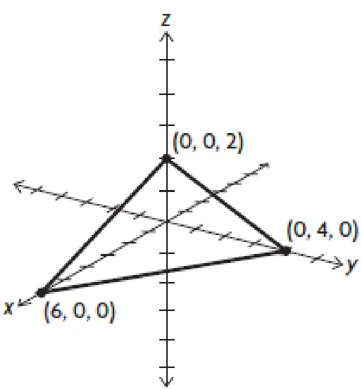
2. $x = 2$

3. $\vec{r} = (2, 3, 3) + s(1, 0, 0) + t(0, 1, 0), s, t \in \mathbf{R}$
   $x = 2 + s, y = 3 + t, z = 3, s, t \in \mathbf{R}$

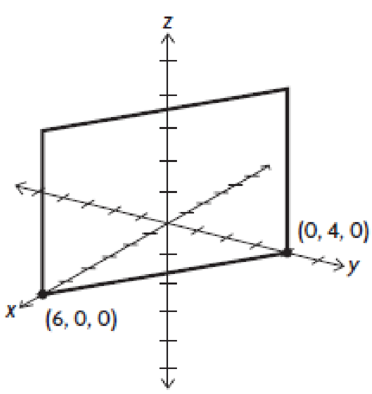
4. a. Answers may vary. For example:
   $\vec{r} = (3, 0, 0) + s(3, 4, 0) + t(3, 0, -2), s, t \in \mathbf{R}$
   $x = 3 + 3s + 3t, y = 4s, z = -2t, s, t \in \mathbf{R}$
   b. Answers may vary. For example:
   $\vec{r} = (3, 0, 0) + s(3, 4, 0), s \in \mathbf{R}$
   c. Answers may vary. For example:
   $\vec{r} = (3, 0, 0) + s(3, 4, 0) + t(4, -3, 6), s, t \in \mathbf{R}$

5. This plane contains the y-axis and goes through the point (1, 0, 2).

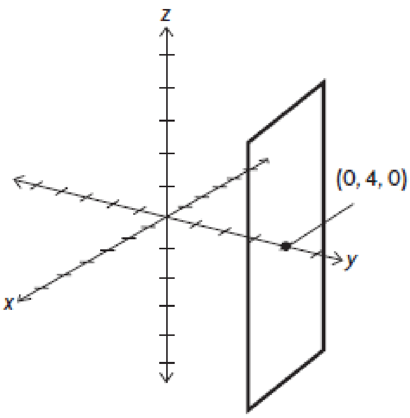
6. a.



   b.



   c.



7. a. $a = -\frac{5}{2}, b = -1$

8. a. yes
   b. yes
   c. $\vec{r} = (1, -1, 1) + s(0, 3, -5) + t(0, 5, 3), s, t \in \mathbf{R}$

9. 70.4° (or 109.6°)

10. a. and b.

11. no

12. a. $3x + y - 5z - 6 = 0$
    b. $3x + y - 5z = 0$

13. (3, 1)

Chapter 9: Relationships between Points, Lines and Planes

Concepts:
Intersection of a Line w/ a Plane (in R^3); Intersection of Two Lines (in R^3); Solving Systems of Equations [for determining the Intersection of TWO LINES in R^2 & TWO PLANES in R^3]; Intersection of Two Planes;

Chapter 9 Mid-Chapter Review Extra Practice

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1. Determine the point of intersection between the given line and plane:
   a. $L: \vec{r} = (7, 3, 6) + t(-5, 1, 4), t \in \mathbf{R}$,
   $\pi: 2x + 4y - 4z = 9$
   b. $L: \vec{r} = (6, 8, -7) + s(-2, 7, 0), s \in \mathbf{R}$,
   $\pi = 3x - 4y - 7z - 3 = 0$
   c. $L: \vec{r} = (-8, 7, 9) + t(5, -9, 3), t \in \mathbf{R}$,
   $\pi = 5x - y + 5z = 0$
   d. $L: \vec{r} = (4, 5, -7) + s(-2, -8, -5), s \in \mathbf{R}$,
   $\pi = 4x - 8y + 2z = 8$
   e. $L: \vec{r} = (6, 7, 1) + t(3, 4, -1), t \in \mathbf{R}$,
   $\pi = x + 2y + 5z = 1$
2. Determine the point of intersection between the following pairs of lines:
   a. $L_1: \vec{r} = (8, 9, -1) + s(-2, 8, 3), s \in \mathbf{R}$,
   $L_2: \vec{r} = (-1, 8, -6) + t(3, -1, 1), t \in \mathbf{R}$
   b. $L_1: \vec{r} = (7, 8, -6) + s(6, -2, 5), s \in \mathbf{R}$,
   $L_2: \vec{r} = (-3, 7, -5.97) + t(7, 4, -6.39), t \in \mathbf{R}$
   c. $L_1: \vec{r} = (-2, 1, 5) + s(7, 2, -3), s \in \mathbf{R}$,
   $L_2: \vec{r} = (5, -3, -10) + t(3, 1, -1), t \in \mathbf{R}$
   d. $L_1: \vec{r} = (-8, -3, 4) + s(6, -5, 4), s \in \mathbf{R}$,
   $L_2: \vec{r} = (5, -8, 7) + t(1, 5, -5), t \in \mathbf{R}$
   e. $L_1: \vec{r} = (8, 1, -1) + s(-2, 1, 9), s \in \mathbf{R}$,
   $L_2: \vec{r} = (-6, 2, -4) + t(6, -2, -16), t \in \mathbf{R}$
3. Determine which of the following pairs of lines are skew lines:
   a. $L_1: \vec{r} = (3, -6, -1) + s(9, -6, -5), s \in \mathbf{R}$,
   $L_2: \vec{r} = (4, -9, -3) + t(-8, 7, 4), t \in \mathbf{R}$
   b. $L_1: \vec{r} = (-4, 2, 5) + s(7, 3, -8), s \in \mathbf{R}$,
   $L_2: \vec{r} = (9, -3, 2) + t(7, 3, -8), t \in \mathbf{R}$
   c. $L_1: \vec{r} = (8, 3, -7) + s(8, -1, -8), s \in \mathbf{R}$,
   $L_2: \vec{r} = (-1, 8, -3) + t(-2, 4, -7), t \in \mathbf{R}$
   d. $L_1: \vec{r} = (1, -3, -4) + s(-7, 6, 3), s \in \mathbf{R}$,
   $L_2: \vec{r} = (1, -3, -4) + t(-4, -3, -3), t \in \mathbf{R}$
   e. $L_1: \vec{r} = (5, -9, -5) + s(-4, 3, -3), s \in \mathbf{R}$,
   $L_2: \vec{r} = (5, -6, -3) + t(1, -3, -4), t \in \mathbf{R}$
4. Solve the following systems of equations using elementary operations:
   a. ① $x + y + z = 5$
   ② $x - y = 1$
   ③ $y - z = 2$
   b. ① $2x - 3y + z = 7$
   ② $x + y + 4z = 4$
   ③ $x - y - z = 5$
   c. ① $x + y = 1$
   ② $y + z = 3$
   ③ $x + z = 2$
   d. ① $x - y + 3z = 2$
   ② $6x + 6y - 3z = 10$
   ③ $3x - 3y + 2z = 8$
5. In the following systems of equations involving two planes, determine the nature of their intersections. That is, state whether they intersect, and if they do intersect, specify if their intersection is a line or a plane.
   a. ① $15x + 5y + 10z = 20$
   ② $3x + y + 2z = 4$
   b. ① $3x - 2y + 6z - 18 = 0$
   ② $-6x + 4y - 12z + 36 = 0$
   c. ① $x + y + z = 3$
   ② $x - y + z = -3$
   d. ① $x + y + 3z = 6$
   ② $x + y = 3$
   e. ① $4x - 3y + 2z = 9$
   ② $-x + 3y - z = 6$
   f. ① $x + 2y - 3z = 4$
   ② $z = 7$
   g. ① $-3x + 3y - 4z = 4$
   ② $x - 6y - z = 2$
   h. ① $x - 6y - 6z = 3$
   ② $-9x + 7y + 5z = -3$

Chapter 9 Mid-Chapter Review Extra Practice Answers

1. a. point: $\left(\frac{189}{22}, \frac{59}{22}, \frac{52}{11}\right)$
   b. point: $\left(\frac{70}{17}, \frac{248}{17}, -7\right)$
   c. point: $\left(\frac{382}{49}, \frac{325}{49}, \frac{447}{49}\right)$
   d. point: $(2, -3, -12)$
   e. point: $(-6, -9, 5)$

2. a. point: $\left(\frac{100}{11}, \frac{51}{11}, -\frac{29}{11}\right)$
   b. point: $\left(\frac{34}{19}, \frac{185}{19}, -\frac{393}{38}\right)$
   c. point: $(131, 39, -52)$
   d. point: $(4, -13, 12)$
   e. point: $(30, -10, -100)$

3. a. skew lines; do not intersect
   b. not skew
   c. skew lines; do not intersect
   d. not skew
   e. skew lines; do not intersect

4. a. $x = 3, y = 2, z = 0$
   b. $x = \frac{60}{11}, y = \frac{12}{11}, z = -\frac{7}{11}$
   c. $x = 0, y = 1, z = 2$
   d. $x = \frac{46}{21}, y = -\frac{2}{3}, z = -\frac{2}{7}$

5. a. plane
   b. plane
   c. line
   d. line
   e. line
   f. line
   g. line
   h. line