Module 7 The Smith Chart and Impedance Matching

(7.1) Smith Chart Construction and Basic Use

There are 3 general relations that describe all lines:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \ , Z(z') = Z_0 \left\{ \frac{Z_L + Z_0 \tanh(\gamma z')}{Z_0 + Z_L \tanh(\gamma z')} \right\} \ , S = \frac{|V|_{\max}}{|V|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Historically, these were tedious to directly employ. In 1939, Philip Smith (Bell Labs) published a graphical technique for analyzing and designing transmission line circuits.

We will only consider loss-less lines.

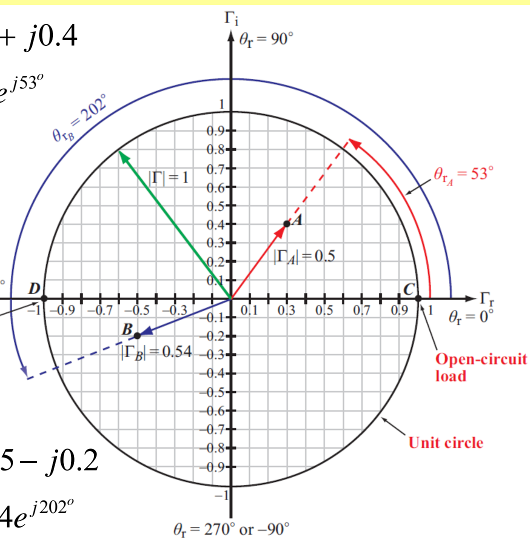
The reflection coefficient Γ is complex: $\Gamma = \Gamma_r + j\Gamma_i$

$$\Gamma_r = |\Gamma| \cos\theta_\Gamma$$

$$\Gamma_i = |\Gamma| \sin\theta_\Gamma$$

The Smith Chart lies in the complex Γ plane.

$$\Gamma_A = 0.3 + j0.4 = 0.5e^{j53^o}$$



$$\Gamma_B = -0.5 - j0.2 = 0.54e^{j202^o}$$

NB. When both Γ_r and Γ_i are negative, θ_r lies in the third quadrant (i.e. add 180°).

NB. Since $|\Gamma| \leq 1$, only that part of the $\Gamma_r - \Gamma_i$ plane that lies with the unit circle has any meaning.

Impedances on a Smith Chart are represented by normalized values:

$$z_L = \frac{Z_L}{Z_0} \quad \text{(dimensionless)}$$

"lowercase" → z_L

We thus write: $$\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

Hence: $$z_L = \frac{1+\Gamma}{1-\Gamma}$$

The normalized load impedance z_L is complex: $z_L = r_L + jx_L$

We may then write: $$r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

normalized load resistance (r_L) *normalized load reactance* (x_L)

Separating this into real and imaginary parts:

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \quad (*) \qquad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2} \quad (**)$$

(*) May be re-written as: $\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$

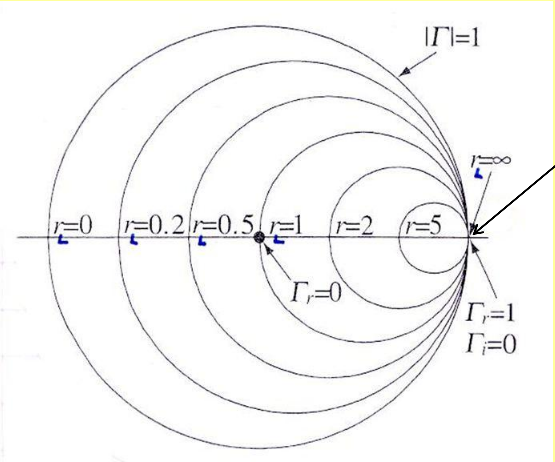
This is the equation of a circle,

$\left[\text{cf. } (x - x_0)^2 + (y - y_0)^2 = a^2 \text{ , centre } (x_{0,} y_0)\text{, radius a}\right]$

centered at: $\Gamma_r = \frac{r_L}{1+r_L},\ \Gamma_i = 0$

with a radius: $\text{radius} = \frac{1}{1+r_L}$

This is a family of circles, of different radii, centred on the real axis:



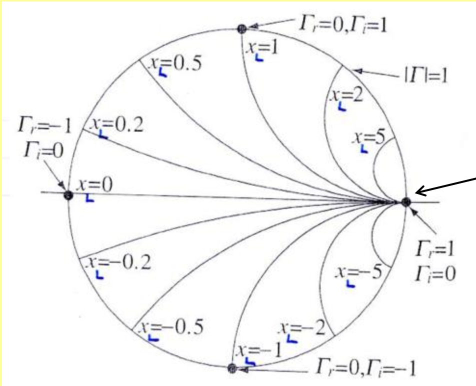
NB. All the circles pass through a common point.

NB. $r_L \geq 0$

(Ida, p.958)

Now consider the other equation (**). It, too, may be rewritten as $(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$

which is also an equation of a circle, centred at $\Gamma_r = 1$, $\Gamma_i = \dfrac{1}{x_L}$



with a radius: radius $= \dfrac{1}{x_L}$

NB. All the circles pass through a common point.

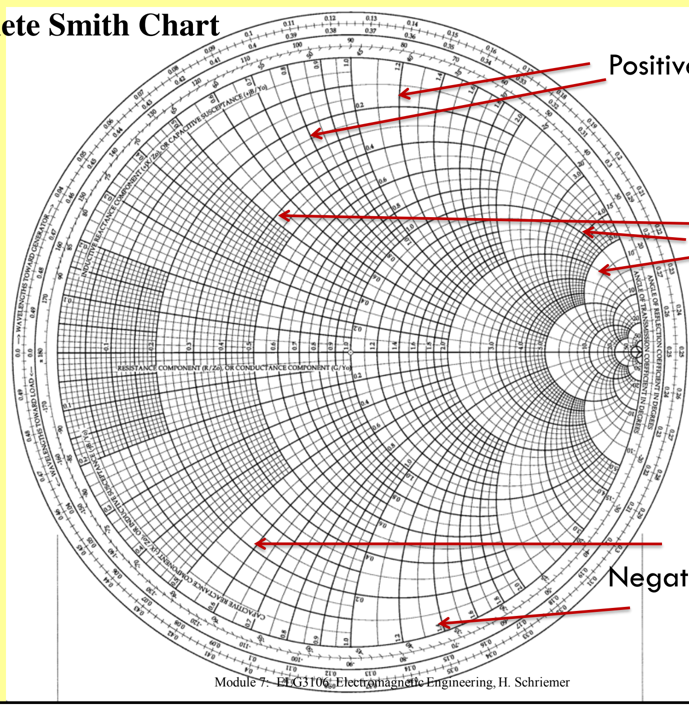
NB. The reactance x_L may be either positive (above the real axis), or negative (below the real axis).

NB. Only part of a circle falls within the bounds of the unit circle.

The following page shows a typical Smith Chart.

How do we use this chart?

Complete Smith Chart



Positive x_L Circles

r_L Circles

Negative x_L Circles

Reflection coefficient at the load

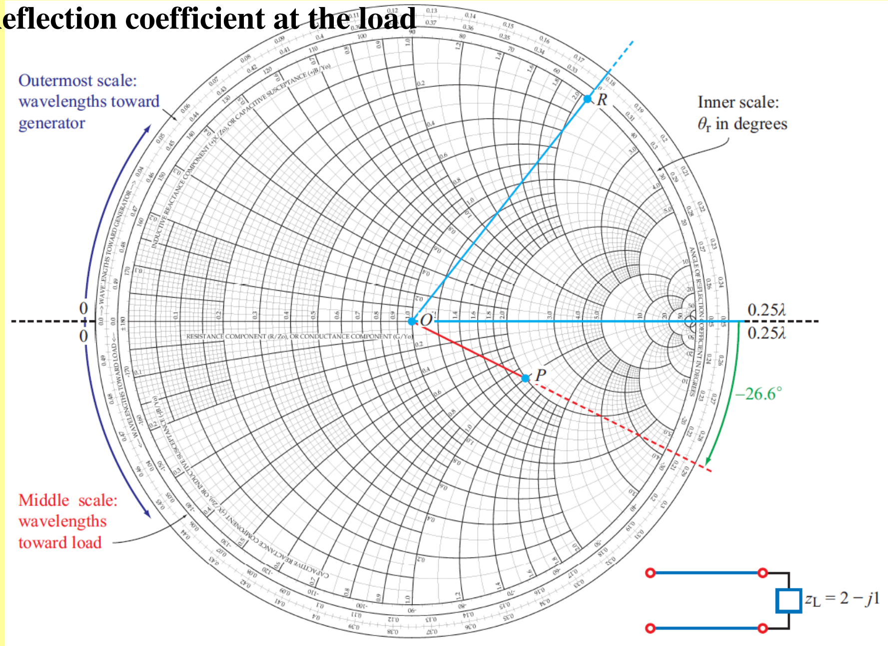


Figure 2-26: Point P represents a normalized load impedance $z_L = 2 - j1$. The reflection coefficient has a magnitude $|\Gamma| = \overline{OP}/\overline{OR} = 0.45$ and an angle $\theta_r = -26.6°$. Point R is an arbitrary point on the $r_L = 0$ circle (which also is the $|\Gamma| = 1$ circle).

Input Impedance

Recall the line equations:

$$V(z') = V_0^+ e^{+\gamma z'} + V_0^- e^{-\gamma z'}$$

$$I(z') = I_0^+ e^{+\gamma z'} + I_0^- e^{-\gamma z'}$$

and the definitions $V_o^- = \Gamma V_0^+$, $I_o^- = -\Gamma I_0^+$

$$V_0^+ = I_0^+ Z_0 \ , \qquad V_0^- = -I_0^- Z_0$$

For lossless lines, this gives

$$V(z') = V_0^+ \left(e^{+j\beta z'} + \Gamma e^{-j\beta z'} \right)$$

$$I(z') = \frac{V_0^+}{Z_0} \left(e^{+j\beta z'} - \Gamma e^{-j\beta z'} \right)$$

Hence, the input impedance looking towards the load is

$$Z_{in}(z'=+l)=\frac{V(z')}{I(z')}=Z_0\left[\frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}\right]$$

Since $\Gamma=|\Gamma|e^{j\theta_r}$ is the voltage reflection coefficient, we can write

$$\Gamma_l=\Gamma e^{-j2\beta l}=|\Gamma|e^{j(\theta_\Gamma-2\beta l)}$$ *phase-shifted voltage reflection coefficient*

We can thus write: $z_{in}=\frac{Z_{in}}{Z_0}=\frac{1+\Gamma_l}{1-\Gamma_l}$,

which is identical in form to: $z_L=\frac{1+\Gamma}{1-\Gamma}$

What does this suggest? $\rightarrow$ If Γ is transformed into Γ_l, then z_L is transformed into z_{in}.

How is this done?

On the Smith chart, to go from $\Gamma\rightarrow\Gamma_l$:

*maintain $|\Gamma|=$ constant
*decrease phase by $2\beta l$
(i.e., a clockwise rotation)

NB. A complete rotation about the Smith chart gives a 2π phase change. The length *l* corresponding to such a change is

$$2\beta l=2\left(2\pi/\lambda\right)l\equiv 2\pi \quad\Rightarrow\quad l=\lambda/2,$$ one complete rotation

On the Smith chart, the outermost scale is called the "wavelengths towards generator" (WTG) scale.

→ denotes movement along the line towards the generator, in units of λ

i.e., l is measured in wavelengths

There is also a "wavelengths towards load" (WTL) scale (the middle scale between the θ_Γ and WTG scales)

→ denotes movement along the line towards the load (phase increases and rotation is in the counter-clockwise direction)

Finding Z_{in} : An Example

Consider a 50-Ω lossless T-line terminated in a load impedance Z_L =(100 - j50) Ω. Find Z_{in} a distance l=0.1 λ from the load.

Input Impedance

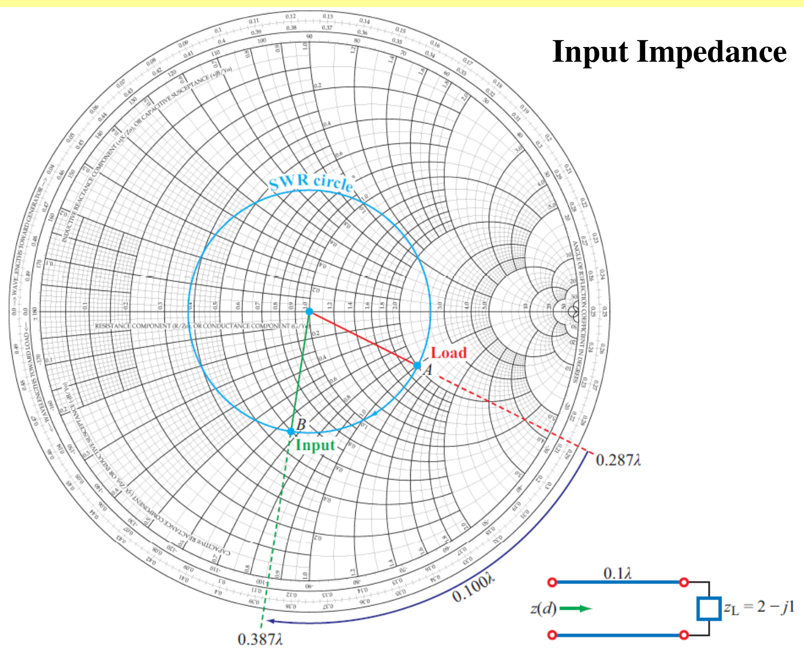


Figure 2-27: Point A represents a normalized load $z_L = 2 - j1$ at 0.287λ on the WTG scale. Point B represents the line input at $d = 0.1\lambda$ from the load. At B, $z(d) = 0.6 - j0.66$.

$$z_L = Z_L / Z_0 = 2 - j1$$

Point A on the above figure.
r_L=2, x_L=-1

On the figure, take the origin as (r=1, x=0).

From the center through point A

$\rightarrow$ draw a straight line to θ_Γ scale $\Rightarrow$ 0.287λ

Now, draw a circle, centred at the origin, that goes through A:

• $|\Gamma|$ is constant on this circle

"Constant-$|\Gamma|$ circle"

or

"SWR circle" (Standing Wave Ratio)

Recall: $S = \frac{1+|\Gamma|}{1-|\Gamma|}$ $\therefore$ constant $|\Gamma| \rightarrow$ constant S

Hence, to transform z_L to z_{in}, we need to keep $|\Gamma|$ constant

$\rightarrow$ stay on the SWR circle

We need to decrease the phase of Γ by $2\beta l$ $(\theta_\Gamma \rightarrow \theta_\Gamma - 2\beta l)$

WTG scale: 0.287λ + 0.1λ = 0.387λ

(Remember: clockwise is negative, counter-clockwise is positive)

We now draw a line form this position on the WTG scale, across the SWR circle, and to the origin.

$\rightarrow$ B is the intersection point

$\Rightarrow z_{in}$: read off r_L and x_L $r_L = 0.6$ and $x_L = -0.66$

$\therefore z_{\min} = 0.6 - j0.66$ $\rightarrow Z_{in} = z_{\min} Z_0 = 50(0.6 - j0.66)$

$= (30 - j33)\Omega$

NB. The points between A and B on the SWR circle represent different points on the transmission line.

The SWR Circle, $|V|_{\max}$ and $|V|_{\min}$

Suppose we have a point A on the Smith chart, eg. $z_L = 2 + j1$; the SWR circle through this point intersects the real axis at two points:

$\rightarrow P_{\max}$ and $P_{\min}$

$\Gamma_i = 0$ at both points $\Rightarrow \Gamma = \Gamma_r$, purely real

Naturally, the load impedance is real on this axis $\rightarrow x_L = 0$

Since: $\Gamma = \dfrac{z_L - 1}{z_L + 1}$ $\rightarrow P_{\max}$ and $P_{\min}$ correspond to the special case

$$\Gamma = \Gamma_r = \frac{r_L - 1}{r_L + 1} \quad (\text{for } \Gamma_i = 0)$$

* When $r_L < 1$, we have $P_{\min}$
* When $r_L > 1$, we have $P_{\max}$

Since: $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} \Rightarrow |\Gamma| = \dfrac{S - 1}{S + 1}$

and: $|\Gamma| = \Gamma_r$ for $P_{\max}$ and $P_{\min}$

$$\therefore \Gamma_r = \frac{S - 1}{S + 1}$$

Comparing the two forms of Γ_r, we conclude $S = r_L$

BUT $S \geq 1$ by definition $\therefore$ the above is true only for P_{max} !

$$e.g.\ \ r_L = 2.6 \text{ at } P_{max} \Rightarrow S = 2.6$$

P_{min} (P_{max}) represents the distance from the load at which the voltage magnitude is a minimum (maximum)

$$\text{i.e. } |V|_{min} \quad (|V|_{max})$$

Recall the "phase-shifted voltage reflection coefficient"

$$\Gamma_l = |\Gamma| e^{j(\theta_r - 2\beta l)} = |\Gamma| e^{-j(2\beta l - \theta_\Gamma)}$$

For P_{max}, $|V|_{max}$ is achieved for $2\beta z' - \theta_\Gamma = 2n\pi$

Similarly, for P_{min}, $|V|_{min}$ is achieved for $2\beta z' - \theta_\Gamma = (2n+1)\pi$

So, for our illustration:

* Draw the SWR circle through A, and identify P_{min} and P_{max}.

* Draw lines from the origin through…

…A to WTG scale $\leftarrow$ Line #1
…P_{max} to WTG scale $\leftarrow$ Line #2
…P_{min} to WTG scale $\leftarrow$ Line #3

* Measure, clockwise, the distance from Line #1 to…

…Line #2 $\rightarrow$ location of $|V|_{max}$ is l_{max}
…Line #3 $\rightarrow$ location of $|V|_{min}$ is l_{min}

So here: $l_{max} = 0.25\lambda - 0.213\lambda = 0.037\lambda$

$l_{min} = 0.5\lambda - 0.213\lambda = 0.287\lambda$

Maxima and Minima

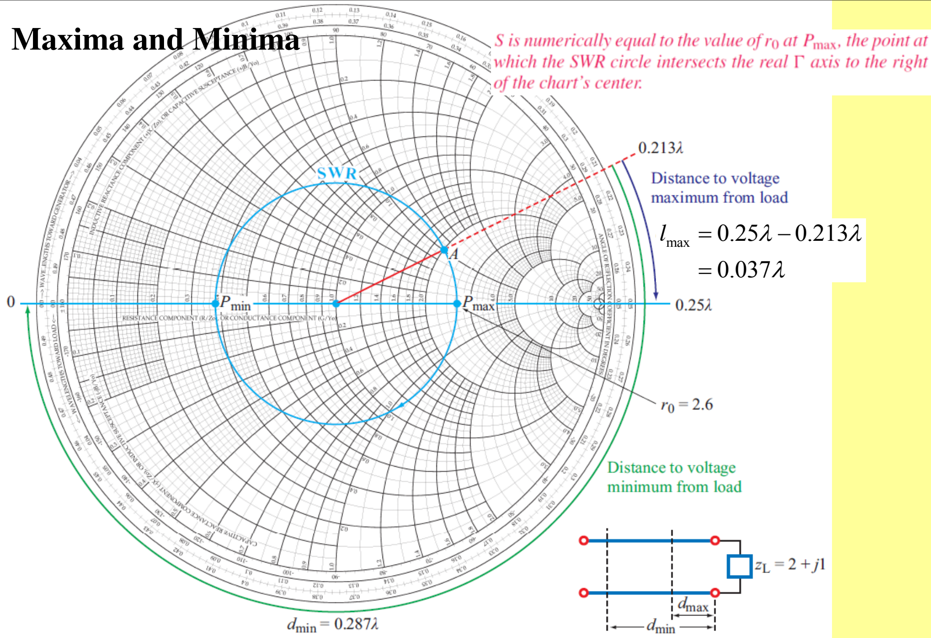


Figure 2-28: Point *A* represents a normalized load with $z_L = 2 + j1$. The standing wave ratio is $S = 2.6$ (at P_{max}), the distance between the load and the first voltage maximum is $d_{max} = (0.25 - 0.213)\lambda = 0.037\lambda$, and the distance between the load and the first voltage minimum is $d_{min} = (0.037 + 0.25)\lambda = 0.287\lambda$.

NB. The location of $|V|_{max}$ is that of $|I|_{min}$

The location of $|V|_{min}$ is that of $|I|_{max}$

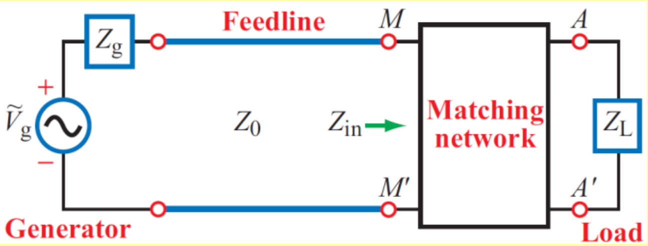
Since the standing wave pattern has a repetition period of λ/2, the Smith chart gives the distances to all maxima and minima on the line.

(7.2) Impedance Matching

Transmission lines transport power/information from a generator to a load. It is critical that this happen with minimal loss. However, it is not always possible to design a load circuit such that $Z_L=Z_0$.

ALTERNATIVE: Impedance matching network...

- Between the load and T-line



The purpose of the matching network is to prevent reflections at M'M. Even though multiple reflections may occur between A'A and M'M , only a forward traveling wave exists on the feedline. This is done by designing the matching network to have an impedance of Z_0 at M'M from the T-line side.

For a lossless T-line/network, all the power then goes to the load.

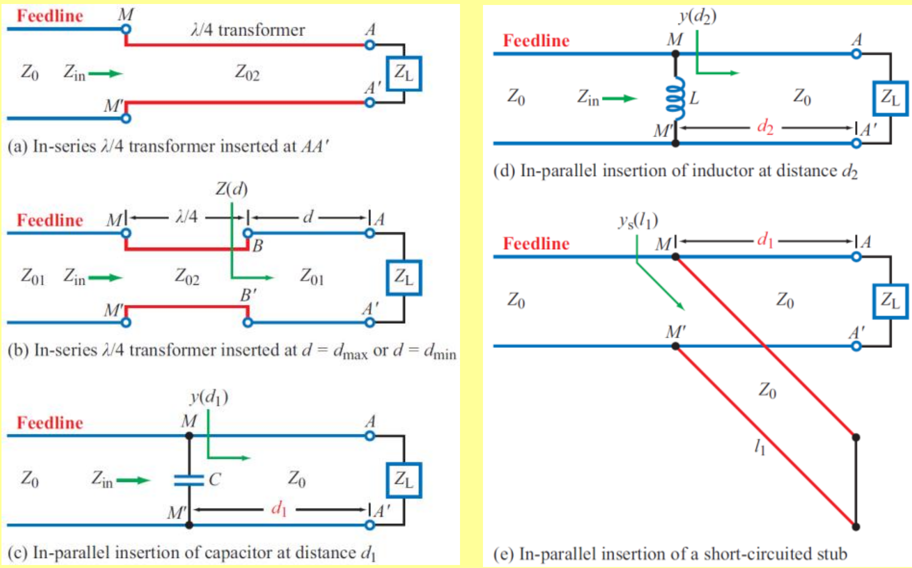
Matching networks may consist of:

*transformers

*lumped elements (capacitors and inductors, to avoid ohmic losses)

*T-line sections of appropriate length and termination

Examples of Matching Networks



For a lossless line, the matching network transforms the load impedance $Z_L=R_L+jX_L$ to the T-line characteristic impedance Z_0.

⇒ transform the real part of the load impedance, R_L, at the load to the real part of the characteristic impedance, R_0, at M'M

⇒ transform the reactive part from X_L at the load to zero (lossless T-line) at M'M

Hence: matching network needs at least two adjustable parameters

In-series Lumped Element Matching Network – Smith Chart Approach

General comments:

- From Z_L, calculate z_L and locate the position on the Smith chart

- Draw the SWR circle through this point

$\Rightarrow$ constant-$|\Gamma|$ circle

Require $Z_{in} = Z_0 \quad \Rightarrow \quad |\Gamma| = 0$

$\Rightarrow$ must "move" to the centre of the circle

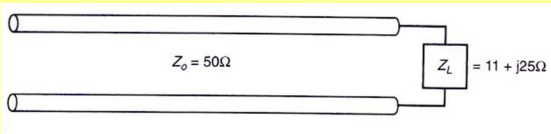
- Follow the SWR circle to the two points where it intersects the $r_L = 1$ circle (i.e., the $1 \pm jx_L$ circle)
- At either of these locations, adding the appropriate reactive element ($\mp jx_L$) moves us to the centre of the SWR circle.

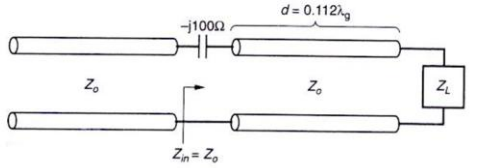
$\rightarrow$ This gives one of the parameters (the value of the reactive element)

- Determine the location (distance from the load) of the reactive element, from the WTG scale, as the difference between the intersection point and the load.

$\rightarrow$ This gives the second parameter

e.g. Construct a simple matching network by adding a reactive element at the appropriate location along a 50 Ω T-line terminated in an (11+j25) Ω load.



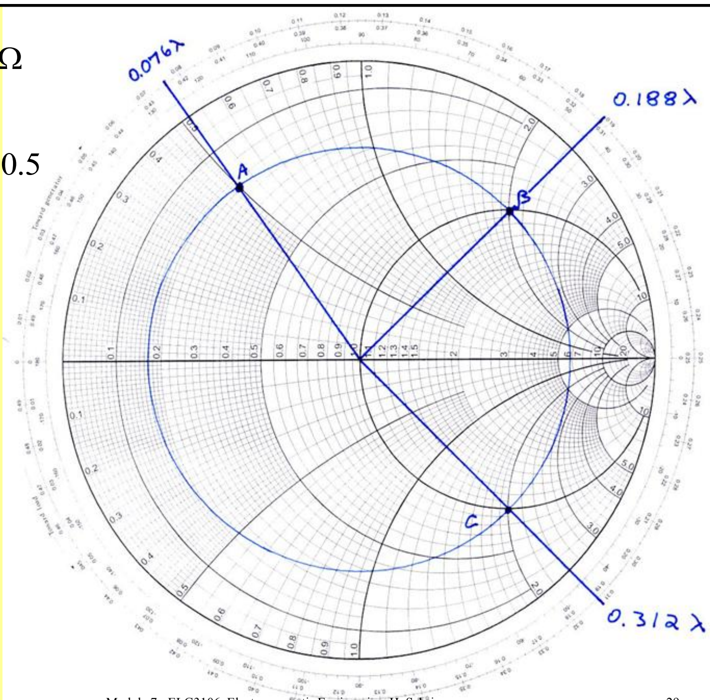


$Z_L = (11 + j25)\ \Omega$

and $Z_0 = 50\Omega$

$\Rightarrow z_L = 0.22 + j0.5$

Point A on Smith chart



Intersection points: B: $1 + j2.0$ C: $1 - j2.0$

For B: require a series capacitive element

$x_L = -2.0 \Rightarrow X_L = -100\Omega$

located at $0.188\lambda - 0.076\lambda = 0.112\lambda$ from the load

$$\Rightarrow -j100 = \frac{-j}{\omega C}$$

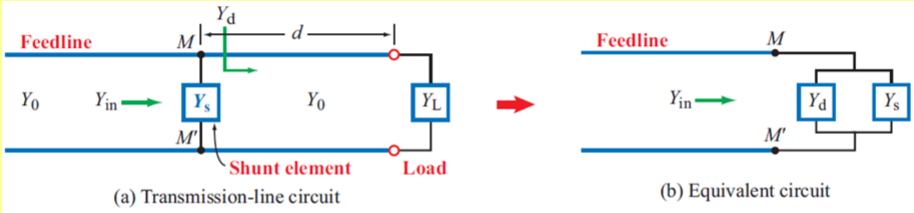
For C: require a series inductive element

$x_L = +2.0 \Rightarrow X_L = +100\Omega$

located at $0.312\lambda - 0.076\lambda = 0.236\lambda$ from the load

$\Rightarrow +j100 = j\omega L$

In-parallel Lumped Element Matching Network – Analytic Approach



Inserting a reactive element with admittance Y_s at MM' modifies Y_d to Y_{in}.

e.g. A load impedance $Z_L = 25 - j50\ \Omega$ is connected to a 50 Ω transmission line. Insert a shunt element to eliminate reflections in the feedline, given that f = 100 MHz. Specify its type, value and location.

Shunt → *In parallel* → Need to work with admittance

$$y_L = \frac{Z_0}{Z_L} = \frac{50}{25 - j50} = 0.4 + j0.8$$

↑ normalized load admittance

Recall: $\Gamma = \frac{1 + z_L}{1 - z_L} \rightarrow \frac{1 - y_L}{1 + y_L}$

↑ Reflection coefficient

$$\Gamma = \frac{1 - (0.4 + j0.8)}{1 + (0.4 + j0.8)} = \frac{0.6 - j0.8}{1.4 + j0.8} = \frac{1\angle -53.13^\circ}{1.612\angle 29.74^\circ} = 0.62\angle -82.9^\circ$$

Recall: $z_{in} = \frac{1 + \Gamma_l}{1 - \Gamma_l}$, $\Gamma_l = |\Gamma| e^{j\theta'}$, $\theta' = \theta_\Gamma - 2\beta l$

↑ Phase shifted reflection coefficient

Hence: $$y_d = \frac{1-\Gamma_d}{1+\Gamma_d} = \left(\frac{1-|\Gamma|e^{j\theta'}}{1+|\Gamma|e^{j\theta'}}\right)\left(\frac{1-|\Gamma|e^{-j\theta'}}{1+|\Gamma|e^{-j\theta'}}\right)$$

$$= \frac{1+|\Gamma|e^{-j\theta'} - |\Gamma|e^{j\theta'} - |\Gamma|^2}{1+|\Gamma|e^{-j\theta'} + |\Gamma|e^{j\theta'} + |\Gamma|^2} = \frac{1-|\Gamma|^2 - j2|\Gamma|\sin\theta'}{1+|\Gamma|^2 + 2|\Gamma|\cos\theta'}$$

Therefore: $$g_d = \frac{1-|\Gamma|^2}{1+|\Gamma|^2+2|\Gamma|\cos\theta'}$$

g_d ← conductance

$$b_d = \frac{-2|\Gamma|\sin\theta'}{1+|\Gamma|^2+2|\Gamma|\cos\theta'}$$

b_d ← susceptance

For an in-parallel network: $Y_{in} = Y_d + Y_S$ ← shunt (Y_S); line (Y_d)

Thus: $y_{in} = g_d + (b_d + b_s)$

Matching at MM' requires... $y_{in} = 1 + j0$

Therefore: $g_d = 1$, $b_s = -b_d$

*For the real-part condition, $$\frac{1-|\Gamma|^2}{1+|\Gamma|^2+2|\Gamma|\cos\theta'} = 1$$

$\Rightarrow \cos\theta' = -|\Gamma|$ ...means that θ' is in the 2nd or 3rd quadrant

Therefore: $\theta' = \cos^{-1}(-0.62) = \pm 128.3^\circ$

→There are **two** values for the location of a shunt element (e.g., either a capacitor or an inductor), d_1 and d_2 ...
where d is the distance from the load (i.e., the line length)

*For d_1: $\theta' = -128.3^\circ = -2.240$ rad

Since... $\theta_\Gamma = -82.9^\circ = -1.446$ rad

Therefore: $\theta' = \theta_\Gamma - 2\beta d \quad \Rightarrow \quad d_1 = \dfrac{\theta_\Gamma - \theta'}{2\beta} = \dfrac{(\theta_\Gamma - \theta')\lambda}{4\pi}$

$$d_1 = \frac{(-1.446 + 2.240)\lambda}{4\pi} = 0.063\lambda$$

Now we use the imaginary part condition to find the value (and type) of the shunt element...

$$b_{S_1} = \frac{+2|\Gamma|\sin\theta'}{1+|\Gamma|^2 + 2|\Gamma|\cos\theta'} = \frac{2(0.62)\sin(-128.3^\circ)}{1+(0.62)^2 + 2(0.62)(-0.62)} = -1.58$$

Hence: $y_{S_1} = -j1.58 \quad \Rightarrow \quad Y_{S_1} = y_{S_1} Y_0$

$$\Rightarrow \quad Z_{S_1} = \frac{Z_0}{y_{S_1}} = \frac{50}{-j1.58} = +j31.62\ \Omega$$

↑ ...inductor

Therefore: $Z_{S_1} = j\omega L \quad \Rightarrow \quad L = \dfrac{j31.62}{j(2\pi \times 10^8)} = 50$ nH

*For d_2: $\theta' = 128.3^\circ = 2.240$ rad

$$d_2 = \frac{(\theta_\Gamma - \theta')\lambda}{4\pi} = \frac{(-1.446 - 2.240)\lambda}{4\pi} = -0.293\lambda \quad ?!$$

...add enough multiples of 0.5λ to make it positive

Therefore: $d_2 = -0.293\lambda + 0.5\lambda = 0.207\lambda$

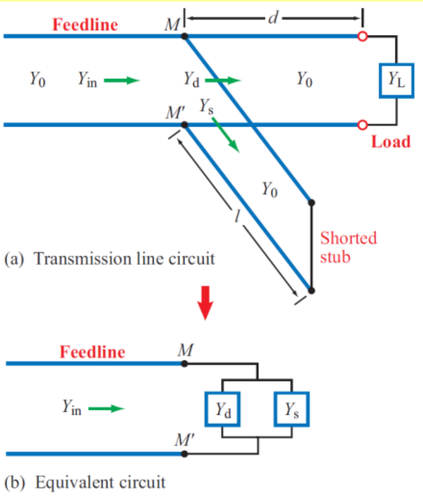
Next, we find: $b_{S_2} = +1.58$ using $\theta' = +128.3^\circ$

So: $Z_{S_2} = \frac{Z_0}{y_{S_2}} = \frac{50}{+j1.58} = -j31.62\,\Omega$ ← ...capacitor

Therefore: $Z_{S_2} = \frac{1}{j\omega C} \Rightarrow C = \frac{1}{j(2\pi \times 10^8)(-j31.62)} = 50$ pF

Single Stub Matching Network – Smith Chart Approach

The idea: To match a load $Z_L=R_L+jX_L$ to a line of characteristic impedance Z_0, we need to change the impedance seen at M'M such that



$$Z_{in} = Z_0$$

↳ $Z_{M'M}$

How? • Move out from the load a distance d at which

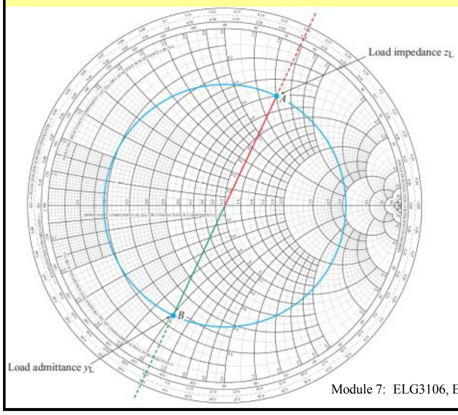
$$Z(z') = Z_0 + jX(z')$$

(NB. Take $Z_0 = R_0$, lossless)

- At d, connect either a shorted or and open T-line such that the $jX(z'=d)$ term cancels. The line now sees a new load.

Parallel lines - admittances are easier to use than impedances

$$\text{Y} \rightarrow \text{ admittance, } y_L = \frac{Y}{Y_0} \rightarrow \text{ normalized admittance}$$



$$y_L = \frac{1}{z_L} = \frac{1-\Gamma}{1+\Gamma} \equiv \frac{1+\Gamma'}{1-\Gamma}$$

with $\Gamma' = -\Gamma$

Locate z_L on the Smith chart and reflect though the centre of the chart to a point diametrically opposite.

$\rightarrow y_L$, the normalized admittance

$$y = \frac{1}{r+jx} = \frac{r-jx}{\sqrt{r^2+x^2}} \equiv g - jb \longleftarrow \text{ susceptance}$$

$\uparrow$ conductance

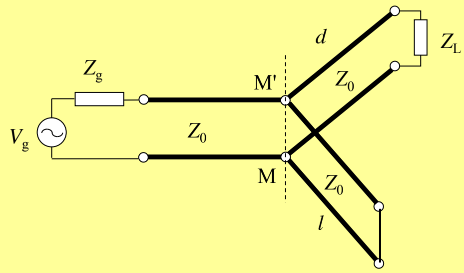
Method and Example: •$Z_L \rightarrow z_L$; locate on Smith chart.

- Locate y_L on Smith chart...

Proceed by example...

Consider an antenna that operates at λ=2 m. Suppose it is not well-designed, and its load impedance after installation is $Z_L = (15 + j60)\Omega$. It is connected to a 75-Ω T-line. What is the length and the location of the shorted stub that matches the resulting load to the line? The line and stub have the same characteristic impedance.

We need to find the location on the line, *d*, and the length of the shorted stub, *l*, by matching the admittances at M'M.

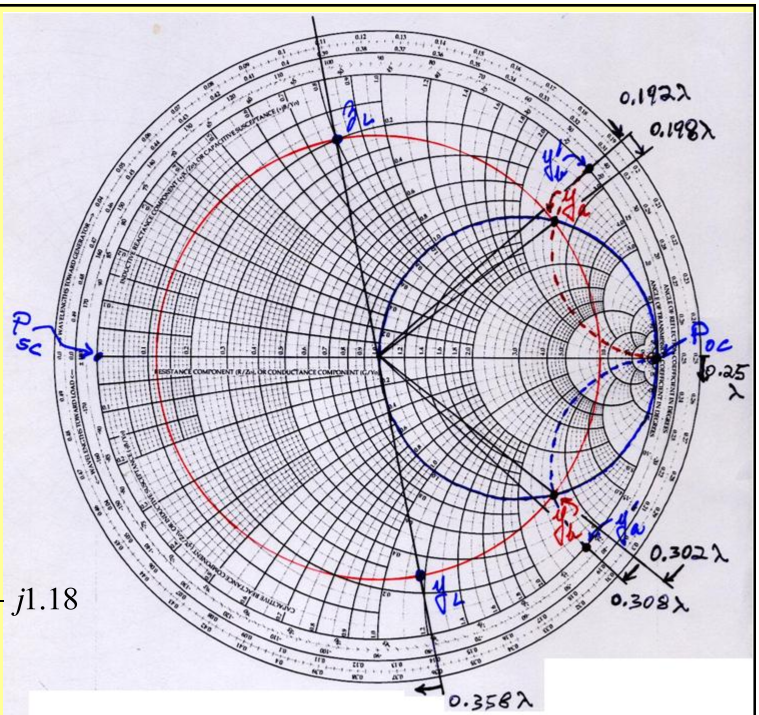


- Calculate the normalized load impedance:

$$z_L = \frac{15 + j60}{75} = 0.2 + j0.8 \qquad \text{...Point } z_L \text{ on Smith chart}$$

- Draw the SWR-circle, around the centre of the chart, through z_L.
- Find the load admittance by drawing a straight line from z_L through the centre and up to the WTG scale

Where this line intersects the SWR-circle is y_L, the normalized load admittance point



$$\Rightarrow y_L = 0.3 - j1.18$$

- As we move around the SWR-circle, the line admittance changes. We want to find the location where the real part of the 'new load' (*i.e.*, at M'M) equals the real part of the characteristic impedance.

$\Rightarrow \text{Re}\{y_{in}\} = 1$ because we work with normalized values

This happens at two locations on the SWR-circle, where it intersects the $g = 1$ circle, at y_a and y_b

At y_a: $y_a = 1 + j2.6$ At y_b: $y_b = 1 - j2.6$ $\Rightarrow$ Two possible solutions.

- Solution "a" $\rightarrow$ Point y_a

$\rightarrow$ "d" is the distance traveled from y_L to y_a, clockwise on the WTG scale: $\Rightarrow y_a - y_L$

$y_a - P_{sc} = 0.198\lambda - 0$ $\qquad$ $P_{sc} - y_L = 0.5\lambda - 0.358\lambda = 0.142\lambda$

$$\Rightarrow y_a - y_L = 0.198\lambda + 0.142\lambda = 0.34\lambda = d$$

At this point the normalized susceptance is b=2.6 (line)

$\Rightarrow$ stub must have b = -2.6 $\qquad$ Point y_a' on the unit circle

The length of the stub is the distance from the open-circuit point P_{oc} (the short has infinite admittance) clockwise to y_a'

$$l_{1a} = y_a' - P_{oc} = 0.308\lambda - 0.25\lambda = 0.058\lambda$$

Hence: $d = 0.68\text{m}$, $l_{1a} = 0.116\text{m}$

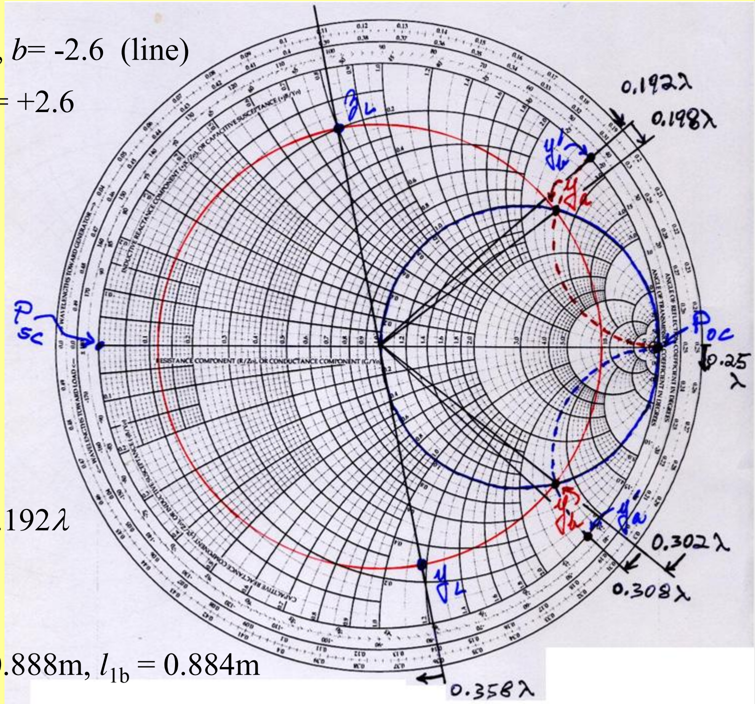
- Solution "b" $\rightarrow$ Point y_b $\rightarrow$ "d" is the distance between y_L and y_b

$$d = y_b - y_L = 0.302\lambda - 0.358\lambda + 0.5\lambda = 0.444\lambda$$

At this point, b= -2.6 (line)

$\Rightarrow$ Stub b = +2.6

Point y_b'



$$l_{1b} = y_b{}' - P_{oc}$$
$$= 0.25\lambda + 0.192\lambda$$
$$= 0.442\lambda$$

Hence: d = 0.888m, l_{1b} = 0.884m