MAT 1330 Practice Final Exam

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First Name:________________________ Family Name:_______________________

Student Number:________________________ Seat Number:_______________

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: <u>you do not</u> have to proceed through the paper in the order given.

- You have 3 hours to complete this exam. You can use the back of the pages to write our solutions.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Question	1	2	3	4	5	6	7	8	9	10	Total (60)
Points											

Question 1. [4 points] Compute the derivatives of the following functions

(a) $f(x) = e^{(10x+12)} \ln(x^{2010} + 1)$.

$f'(x) =$

(b) $g(x) = \cos^2(\sqrt{x})$.

$g'(x) =$

Question 2. [8 points] Determine if the following limits exist. If the limit exists, compute the limit without using table of values.

(a) $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x^2-2x+4} = \boxed{}$

(b) $\lim\limits_{x\to 0} \dfrac{x}{1-\sqrt{1+x}} = \boxed{}$

(c) $\lim\limits_{x\to 0^+} x \ln x = \boxed{}$

(d) $\lim\limits_{x\to \infty} \dfrac{x^2}{e^x} = \boxed{}$

Question 3. [2 points] A golf ball hit with an angle of θ radians and initial velocity of 10m/s will fly for a distance of $d(\theta) = 20.41\sin(\theta)\cos(\theta)$ metres before it lands (neglecting air resistance). Find the angle θ^* between 0 and $\pi/2$ radians that maximizes the distance flown, and find the maximal distance.

Answer: $\theta^* =$ [ ], $d(\theta^*) =$ [ ].

Question 4. [4 points] Find the Taylor polynomial of degree 3 for $f(x) = \sin(2x) + x^2$ with base point $a = 0$. (x in radians!)

(a) $P_3(x) =$ [ ]

(b) Use this polynomial to approximate $f(-0.1)$. $\hat{f}(-0.1) =$ [ ]

Question 5. [4 points]
(a) Give the definition for the function $f(x)$ to be differentiable at the point $x = a$.

(b) Use the definition (first principles) to find the derivative of

$$f(x) = \frac{2x}{x+1}$$

Question 6. [5 points] Consider the DTDS

$$x_{t+1} = \frac{1 + x_t}{1 + x_t^2}, \qquad t = 0, 1, 2, \ldots.$$

(a) The updating function of this DTDS is $f(x) =$

(b) The only positive steady state is $x^* =$

(c) According to the derivative test, is the steady state stable or unstable? Answer:

(d) Starting from $x_0 = 5$ calculate x_1, x_2, x_3. Answer:

Question 7. [12 points] Compute the following indefinite integrals:

(a) $\int (x^3 + \cos x)dx =$

(b) $\int \frac{e^{2x} + 4}{e^{2x}}dx =$

(c) $\int 16x^3 \ln(7x)dx =$

(d) $\int \frac{\sin(\frac{1}{x})}{x^2}dx =$

Question 8. [4 points] Consider the function $f(x) = e^{-x} - x$.

(a) [2 points] Explain why this function has a zero in the interval $[0, 1]$.

(b) [2 points] Calculate the zero to 3 decimal places, i.e. $f(x^*) = 0$ for $x^* =$

Question 9. [7 points] An athlete starts a marathon with a speed of 14 km/h. Due to an existing ankle injury she is forced to slow down and her speed decreases at a constant rate according to the differential equation $v'(t) = -2$ for $t \geq 0$ with $v(0) = 14$. Answer the following questions:

(a) Find the equation for the speed (in km/h) as a function of time (in hours)

$v(t) =$

(b) Solve the pure-time differential equation $\frac{dp}{dt} = v(t),\ p(0) = 0$ for the location p (in km).

$p(t) =$

(c) How long will it take the athlete to complete the marathon (42 km)?

Finish time T^*=

What is her speed when crossing the finishing line? $v(T^*) =$

Question 10. [10 points] Consider the function $f(x) = \frac{1}{x^2} + \frac{1}{2x^3}$. Follow these steps to graph the function.

(a) The domain of f is

(b) The x-intercept(s) of f are

(c) The derivative of f is $f' =$

(d) The critical point(s) of f are

(e) The second derivative of f is $f'' =$

(f) The point(s) of inflection are

(g) In the limit $\lim_{x \to 0^+} f(x) =$

(h) In the limit $\lim_{x \to 0^-} f(x) =$

(i) In the limit $\lim_{x \to \infty} f(x) =$

(j) In the limit $\lim_{x \to -\infty} f(x) =$

(k) The graph of f for $x \in [-2, 2]$ is