NAME: ______________________________ STUDENT ID #: ____________________

WILFRID LAURIER UNIVERSITY
WATERLOO, ONTARIO

BUSINESS/ECONOMICS 275
MIDTERM EXAMINATION, Winter 2008

TIME: 3 hours (180 minutes)

PAGES: 15 (including cover page)

INSTRUCTOR: Dr. Ron Craig
Dr. Sapna Isotupa
Dr. Robert Jefferson
Professor Trent Tucker

INSTRUCTIONS:

1. Total Mark Value: 100
   Number of questions: 6

2. 1 page (single sided) notes are permitted.

3. Please use pen only (blue or black ink).

4. Calculators are permitted.

5. Budget your time carefully.

6. Read each question and instruction carefully.

7. Answers are to be given in the space provided. However, should you require additional space for a complete answer, use the blank page (page 15) attached for this purpose.

8. For all problems where calculation space is provided, ***show your reasoning and work***. Unsubstantiated answers will usually receive ***no marks***.

9. You are to stop writing immediately upon being told that the exam is over.

MARKS RECEIVED

Question 1	/40
Question 2	/10
Question 3	/20
Question 4	/10
Question 5	/10
Question 6	/10
TOTAL	/100

Q1. Multiple Choice (20 x 2 marks each)

For each question choose the "best answer" by clearly circling your choice.

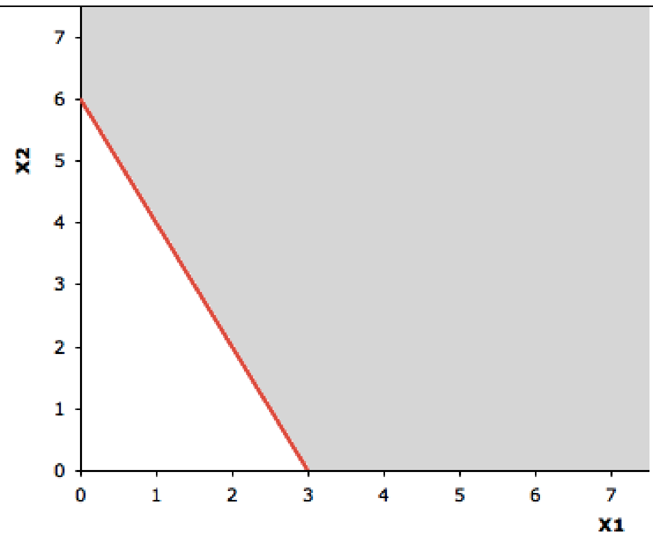
1. A furniture manufacturer is trying to decide how many of each of four styles of wooden chairs to produce. Each chair type requires differing amounts of three main resources to produce: wood, cushion materials, and time spent assembling the chair. The decision variables in this example are:

   A. The style of chairs to produce.
   B. The amount of wood and cushion material to purchase.
   C. The amount of labour hours to schedule for assembly.
   D. The number of each style of chair to produce.

2. A furniture manufacturer is trying to decide how many of each of four styles of wooden chairs to produce. Each chair type requires differing amounts of three main resources to produce: wood, cushion materials, and time spent assembling the chair. Excluding non-negativity, this problem has how many constraints?

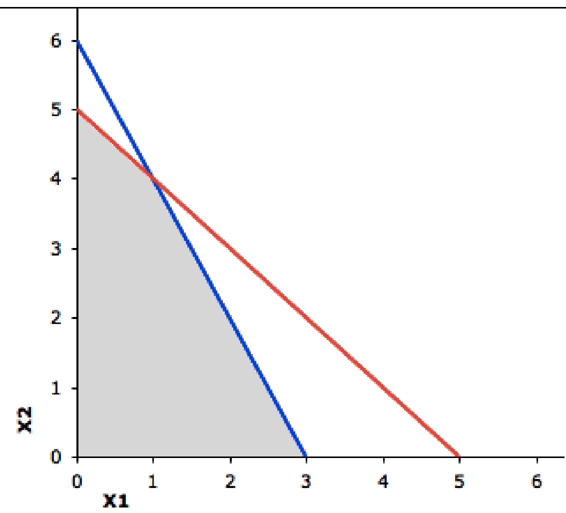
   A. One
   B. Three
   C. Four
   D. Seven

3. You are solving a two-variable problem graphically. You draw a constraint line and shade in a half-plane (as the feasible region). The constraint that corresponds to your diagram is…



   A. $10X_1 + 5X_2 \leq 30$
   B. $3X_1 + 6X_2 \geq 10$
   C. $10X_1 + 5X_2 \geq 30$
   D. $6X_2 \geq 3X_1$

4. The feasible region represented by the shaded area in the graph below is bounded by the X1 and X2 axes and which two constraints?



   A. $X_1 + X_2 \geq 5$ and $10X_1 + 5X_2 \geq 30$
   B. $X_1 + X_2 \geq 5$ and $10X_1 + 5X_2 \leq 30$
   C. $X_1 + X_2 \leq 5$ and $10X_1 + 5X_2 \geq 30$
   D. $X_1 + X_2 \leq 5$ and $10X_1 + 5X_2 \leq 30$

5. Which of the following is ***not*** a valid LP constraint?

A. $2X_1 - 4X_2 - 4X_3 \geq -12$
B. $2X_1 / 4X_2 - 4X_3 > 16$
C. $2X_1 + 5X_2 + 3X_3 \leq 14$
D. $2X_1 - 3X_2 - 3X_3 \leq 0$

6. Consider a linear programming problem with two decision variables and three or more constraints (not including non-negativity). The objective function has the same slope as that of one of the binding constraints.

Which of the following will characterize the results obtained from attempting to solve this LP problem (using the simplex method or Excel's Solver)?

A. Infeasible
B. Multiple optimal solutions
C. Unbounded
D. Unique optimal solution

7. A solution that satisfies all the constraints of a linear programming problem except the non-negativity constraints is called

A. optimal.
B. feasible.
C. infeasible.
D. semi-feasible.

8. XYZ Inc. produces two types of paper towels—regular and super-soaker. Let X_1 be the number of units of regular produced per month and X_2 the number of units of super-soaker produced per month. Marketing has imposed a constraint that the total monthly production of regular should be no more than twice the monthly production of super-soakers. Which of the following is the correct expression for this constraint?

A. $X_1 - 2X_2 \leq 0$
B. $2X_1 - X_2 \leq 0$
C. $X_1 - 0.5X_2 \leq 0$
D. $X_1 - 0.5X_2 \geq 0$

9. Let X_1, X_2, and X_3 denote the amount of money invested in three increasingly risky stocks. In order to reduce risk, the mutual fund manager purchasing these stocks requires that at least 25% of the money can be invested in low risk stock #1 (X_1). The expression for this constraint is:

A. $0.25X_1 \geq X_1 + X_2 + X_3$
B. $0.75X_1 - 0.25X_2 - 0.25X_3 \geq 0$
C. $-0.75X_1 + 0.25X_2 + 0.25X_3 \geq 0$
D. $X_1 + X_2 + X_3 \geq 0.25X_1$

10. Which of the following statements about redundant constraints is ***NOT TRUE***?

A. A redundant constraint does not affect the optimal solution.
B. A redundant constraint does not affect the feasible region.
C. Recognizing a redundant constraint is easy with the graphical solution method.
D. At the optimal solution, a redundant constraint will have zero slack.

11. Consider the following LP problem :
Max $z = X_1 + X_2$
Constraint A: $2X_1 + X_2 \leq 90$
Constraint B: $3X_1 + 2X_2 \leq 120$
Constraint C: $X_1 - X_2 \geq 0$

Which of the constraints is redundant, if any?

A. constraint A
B. constraint B
C. constraint C
D. none of the constraints is redundant

12. In formulating a coffee blending problem where there are three types of coffee beans, the objective is to find a recipe to make 1 pound of blended coffee that satisfies a set of properties at the least cost. The decision variables are X_1, X_2, and X_3, representing pounds (actually fractional pounds) of coffee beans used per pound of blended coffee.

Which of the following constraints must be included in the statement of the problem?

A. $X_1 + X_2 + X_3 < 1$
B. $X_1 + X_2 + X_3 > 1$
C. $X_1 + X_2 + X_3 = 1$
D. None of the above constraints is required

13. An objective function reflects the relevant cost of labour hours used in production rather than treating them as a sunk cost. What is the interpretation of the shadow price associated with the labour hours constraint?

A. the maximum additional amount the company would be willing to pay for an additional hour of labour.
B. the upper limit on the total hourly wage the company would pay.
C. the reduction in hours that could be sustained before the solution would change.
D. the number of hours by which the right-hand side can change before there is a change in the solution point.

14. Consider the Excel Solver Sensitivity Report.
The amount that the objective function coefficient of a decision variable would have to improve before that variable would enter the optimal solution with a positive value is the

A. shadow price.
B. value of the slack variable.
C. reduced cost.
D. allowable increase.

15. When a constraint is binding, which of the associated values **must** equal zero?

A. Slack/Surplus variable.
B. Reduced cost.
C. Shadow Price.
D. Allowable increase/allowable decrease.

16. A bicycle manufacturer used Solver to determine the profit-maximizing number of each model of bicycle to produce. The sensitivity report stated the “reduced cost” associated with variable X_3 is \$150. The current profit per unit of X_3 is \$450.

There is also a storage-related constraint limiting the production of X_3 to 35 ($X_3 \leq 35$). What is the shadow price associated with this storage constraint?

A. \$450
B. \$150
C. \$ 35
D. \$ 0

17. In the standard linear programming formulation of the transportation problem, the cost of transporting one unit of the material from a supply point to a demand point appears in

A. the objective function only
B. the constraints only
C. both objective function and constraints
D. neither objective function nor constraints

18. A transportation problem has total demand equal to 900 and total supply equal to 1200. To convert this to a balanced problem, we add a _______________ _____________ with a quantity equal to ___________.

A. dummy supply, 300
B. dummy supply, 1200
C. dummy demand, 300
D. dummy demand, 1200

19. In an assignment problem, there are 5 projects and a potential list of 6 managers, each of whom could be the project manager for any one of the projects. The problem can be formulated by adding a __________ __________, thus converting it to a standard assignment problem.

A. dummy manager
B. dummy cost
C. dummy constraint
D. dummy project

20. Consider a plant location problem with five potential locations. There are five 0-1 decision variables X_i (X_1, X_2, X_3, X_4, X_5), each of which takes a value of 1 if a plant is built in location ‘i’ and 0 otherwise. Management has specified that, if a plant is built in location 2, then a plant must be built in location 4.

Identify the correct specification of this constraint:

A. $X_2 - X_4 \leq 0$
B. $-X_2 + X_4 \geq 0$
C. $X_2 - X_4 = 0$
D. $-X_2 + X_4 \leq 0$

Q2. (10 marks)

The table below gives a list of potential courses that a BBA student can register in for the next semester. The second column contains the course codes. The course code MATH denotes math courses, the code PSYC denotes psychology courses, the code MUS denotes music courses and the code GEO denotes geography courses. The third column denotes the number of lecture hours per week for the course. The fourth column denotes the minimum number of hours that a student needs to spend doing homework and preparing for the class if he/she registers in the course.

	Course Code	Lecture hours	Minimum prep. hours
1	MATH101	3	6
2	MATH102	3	6
3	MATH203	4	8
4	MATH205	4	10
5	MATH303	3	5
6	MUS110	3	6
7	MUS113	4	7
8	MUS214	6	10
9	PSYC121	3	8
10	PSYC221	2	4
11	PSYC225	3	5
12	GEO141	3	6
13	GEO143	4	7
14	GEO247	3	3

Write down constraints for each of the following conditions, independent of the other conditions.
Let **Xi = 1 if the student registers for course i [i=1,2,..., 14] and 0 otherwise**

A. If a student registers in MATH101, the student must also register in MATH102.

B. A student can register in MATH205 only if the student has registered in MATH102 and MATH203.

C. A student must register for at least one course in each subject.

D. A student must have at least 15 lecture hours per week.

E. A student must not have more than 40 preparation hours per week.

F. A student **must** register in MUS214 if they have not registered in both MUS110 and MUS113. Further, the student **may** register in MUS214 only if they have not registered in both MUS110 and MUS113.

G. A student must register in at least 4 courses.

Q3. (20 marks)

Digital Controls Inc.(DCI) manufactures two models of a radar gun used by the police to monitor speed of automobiles. Model A has an accuracy of plus or minus 1 km/hour whereas the smaller model B has an accuracy of plus or minus 3 km/hour. For the next week, the company has orders for 100 units of model A and 150 units of model B. Although DCI purchases all the electronic components in both models, the plastic cases for both models are manufactured at a DCI plant in Toronto. Each model A case requires 4 minutes of injection-molding time and 6 minutes of assembly time. Each model B case requires 3 minutes of injection-molding time and 8 minutes of assembly time. For next week, the Toronto plant has 600 minutes of injection-molding time available and 1080 minutes of assembly time available. The manufacturing cost is $10 per case of model A and $6 per case of model B. Depending on demand and the time available at the Toronto plant, DCI occasionally purchases cases for one or both models from an outside supplier to fill customer orders that could not be filled otherwise. The purchase cost is $14 for a case for model A and $9 for a case for model B. Management wants to develop a minimum cost plan for next week.

The linear programming formulation for this problem is as follows:

Decision Variables
AM = number of cases of model A manufactured in Toronto
BM = number of cases of model B manufactured in Toronto
AP = number of cases of model A purchased
BP = number of cases of model B purchased

Objective Function
Minimize z = 10AM + 6BM + 14AP + 9BP

Constraints
Demand for Model A: AM + AP = 100
Demand for Model B: BM + BP = 150
Injection molding time: 4AM + 3BM $\leq$ 600
Assembly time: 6AM + 8BM $\leq$ 1080

AM,BM,AP,BP $\geq$ 0

This model is put into Solver and solved. The sensitivity report is given below.
The objective function value at optimality is 2170.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Decision Variables AM	100	0	10	1.75	1E+30
$C$9	Decision Variables BM	?a	0	6	3	2.333333333
$D$9	Decision Variables AP	0	?b	14	?c	1.75
$E$9	Decision Variables BP	90	0	9	2.333333333	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$4	Demand for A LHS	?d	12.25	100	11.42857143	100
$F$5	Demand for B LHS	?e	9	150	1E+30	90
$F$6	Injection molding time LHS	?f	0	600	?g	20
$F$7	Assembly time LHS	?h	-0.375	1080	53.33333333	480

Answer the following questions based on the sensitivity report. Show your work and thinking, in the space provided for each question.

(i) Fill in the spaces in the sensitivity report which have “?” signs.

?a =		?e =	
?b =		?f =	
?c =		?g =	
?d =		?h=	

In parts (ii) to (ix) in each case you have two options for ‘Optimal Solution’ – unchanged DVs (decision variables remain optimal; their optimal value may change) or re-solve (to find new decision variables). Circle the correct one. Also, you have three options for the Objective Function Value: (i) calculate a new value for the objective function, (ii) state ‘OFV unchanged’, or (iii) state ‘re-solve’.

ii. If there is no demand for model A next week, what is the optimal solution and objective function value?

Optimal Solution: unchanged DVs or re-solve

Objective Function Value:

iii. If the demand for B decreases to 50, what is the new optimal solution and objective function value?

Optimal Solution: unchanged DVs or re-solve

Objective Function Value:

iv. If there are only 590 hours available for injection molding, what is the new optimal solution and objective function value?

Optimal Solution: unchanged DVs or re-solve
Objective Function Value:

v. If demand for model A is only 90 and there are 1100 hours of assembly time available, what is the new optimal solution and objective function value?

Optimal Solution: unchanged DVs or re-solve

Objective Function Value:

vi. There is an additional requirement that at least 80 cases should be purchased from an outside supplier. What is the new optimal solution and objective function value?

Optimal Solution: unchanged or re-solve Objective Function Value:

vii. If the cost of manufacturing a single case of Model A increases to $11, what is the new optimal solution and objective function value?

Optimal Solution: unchanged or re-solve Objective Function Value:

viii. If the cost of manufacturing a single case of Model B decreases to $3, what is the new optimal solution and objective function value?

Optimal Solution: unchanged or re-solve Objective Function Value:

ix. If the cost of purchasing a case of Model B increases by $2, what is the new optimal solution and objective function value?

Optimal Solution: unchanged or re-solve Objective Function Value:

Q4. (10 marks)

Buy Direct Motherboards (BDM Inc.) produces two computer chipsets for motherboards, which in turn go into desktop computers (PCs) or laptop computers. The PC chipsets are sold to customers at 35 $/unit, while the laptop chipsets are sold at 50 $/unit. The cost to produce a PC chipset is 20 $/unit; the laptop chipsets are double the cost to produce, i.e. 40 $/unit. The following business rules are also in effect:

[A] BDM wants to produce at least 3 times as many laptop chipsets as PC chipsets.

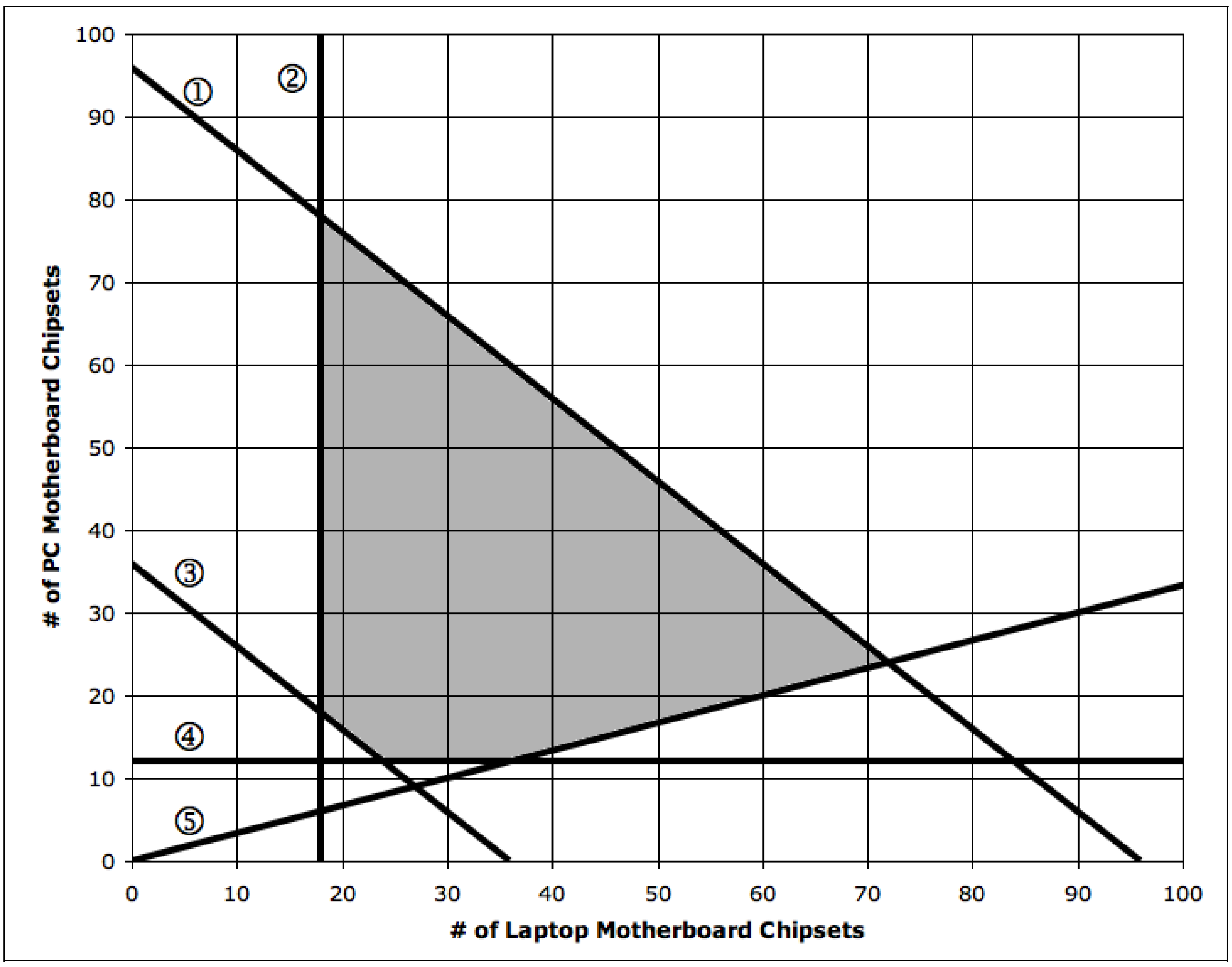
[B] BDM has enough raw materials on hand to produce a maximum of 96 chipsets TOTAL.

[C] BDM has been contracted to produce at least 12 PC chipsets.

[D] BDM must produce at least 36 chipsets TOTAL in order to prevent oxide build up on equipment.

[E] BDM has been contracted to produce at least 18 laptop chipsets.

Use the following ***PARTIAL Graphical Solution*** (constraint lines + feasible region) to answer the questions that follow…



1) **Match** the five business rules [A] … [E] to the five constraint lines shown ① … ⑤.

Business Rule:	Constraint Line #:
[A] BDM wants to produce at least 3 times as many laptop chipsets as PC chipsets.	
[B] BDM has enough raw materials on hand to produce a maximum of 96 chipsets TOTAL.	
[C] BDM has been contracted to produce at least 12 PC chipsets.	
[D] BDM must produce at least 36 chipsets TOTAL in order to prevent oxide build up on equipment.	
[E] BDM has been contracted to produce at least 18 laptop chipsets.	

2) If BDM's business objective is to minimize the costs of chipset production, what is:

2A) The objective function?	
2B) The optimal solution? (*)	
2C) The objective function value?	

What if, instead, BDM's business objective is to maximize its profit? What is:

2D) The objective function?	
2E) The optimal solution? (*)	
2F) The objective function value?	

*Note: In order to receive credit for the optimal solution parts, you need to show some work on the previous page illustrating your thought process about how you found the optimal solutions…

3) What is the impact to your answer in part 2B above if the cost of the laptop chipset drops to $20 (the same as the cost of the PC chipset)?

4) BDM Inc. has been approached by Research In Motion (RIM) to produce a chipset for the BlackBerry Pearl™. Due to the miniaturization required, the chipsets will cost 75$/unit. However, RIM is willing to pay 100 $/unit for the chipsets. If we only consider the cost minimization objective (again, part 2B above), what is the impact on your solution above with this new information?

5) The above graph was created using Excel's line charting capabilities. Assuming each line is defined by two end points (and ignoring any labels), how many rows and columns of the spreadsheet are used to capture the data to create the lines on the chart?

________ ROWS & ________ COLUMNS

Q5. (10 marks)

Buy Da Milk (BDM) is a corner store that sells three brands of milk in 2 litre cartons: its own brand (profit: 97¢/carton), organic milk from a local dairy (profit: 83¢/carton), and the leading national brand (profit: 69¢/carton). The "footprint" of each container is 16 square inches and there is a total of 36 square feet of refrigerated space available. (Recall: there are 12 inches in 1 foot). The local dairy can only supply up to 10 dozen cartons per week. The store manager has observed that each week, Buy Da Milk always sells more of the national brand than the local dairy and its own brand combined, and at least three times as much of the national brand as its own brand. The manager wishes to maximize the store's weekly profit and has begun translating the following linear program into Excel.

Let X1, X2, and X3 denote the number of cartons of each brand of milk (BDM, local, and national)

MAX Z = \$0.97X1 + \$0.83X2 + \$0.69X3
s.t.

$X1 + X2 + X3 \leq 324$	(Refrigerator space)
$X2 \leq 120$	(Local dairy maximum supply)
$X3 \geq X1 + X2$	(National brand outsells others combined)
X3 ≥ 3X1	(Three times as much national as own brand)
$X1, X2, X3 \geq 0$	(Non-negativity)

Buy Da Milk.xls

	A	B	C	D	E	F	G
1		X1	X2	X3	LHS	ineq	RHS
2	Objective Function	0.97	0.83	0.69	Part (A)		
3							
4	Constraints						
5	Refridgerator Space	1	1	1	Part (B)	≤	Part (C)
6	Local Dairy Supply	0	1	0	0	≤	120
7	National Outsells...		Part (D)		0	≥	0
8	National vs. Own...	-3	0	1	0	≥	0
9							
10	Decision Variables	0	0	0			
11							
12							

Question | Original | Sheet2 | Sheet3
Ready   Sum=0

(A) Suppose the manger of Buy Da Milk enters their objective function in Cell E2 as:

```
=(0.97)*B2+(0.83)*C2+(0.69)*D2
```

While technically correct, what principle of "spreadsheet engineering" would be violated?

(B) Which Excel formula should be entered into cell E5 so that it can be copied down through the range E5:E8 to calculate the LHS's of the constraints?

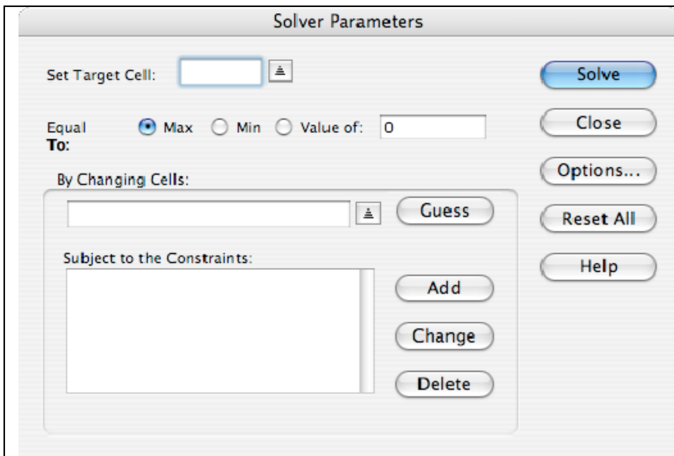
(a) `=SUMPRODUCT(B5:D5,B10:D10)`
(b) `=SUMPRODUCT($B$5:$D$5,B10:D10)`
(c) `=SUMPRODUCT(B5:D5,$B$10:$D$10)`
(d) `=SUMPRODUCT($B$5:$D$5,$B$10:$D$10)`

(C) The value for this cell (according to the LP formulation) should be 324. Describe how this value was derived? What units of measure should be associated with it?

(D) What values should be entered into cells B7:D7 for this constraint?

Cell B7	**Cell C7**	**Cell D7**

(E) When the manager of Buy Da Milk finally gets to launching Solver to come up with the profit maximizing solution, what cells / cell ranges would he enter into the Solver Parameters dialog box for…

<table>
<tr>
<td rowspan="2">
</td>
<td>Set Target Cell:</td>
</tr>
<tr>
<td>By Changing Cells:</td>
</tr>
</table>

Q6. (10 marks)

Waterloo Investment Advisors (WIA) are putting together a 4 year investment plan for a new client. The client wants to invest \$250K, and maximize her return. She has talked with WIA about investment options, expected returns, and the risks she is willing to take on. Together, they have decided she should invest in stock indexes, medium term bond funds, and mortgage funds. Based on expected annual returns, compounded returns over the four year period will be 36%, 17%, and 26% (money will be invested immediately, for the full 4 year period).

WIA and their client have agreed on the following restrictions for the investment:

(1) at least 20% of the investment will be in each of the investment options (stocks, bonds, mortgages);
(2) no more than 50% of the investment will be in stock indexes;
(3) mortgage funds cannot exceed bond funds;
(4) the amount in bonds must be at least two and a half times the amount in stocks;
(5) no more than \$250K will be invested.

Set up the LP formulation for this investment situation.

(A) Define the Decision Variables, along with their units:

(B) State the full Objective Function:

(C) List all Constraints for this problem:

(C1) Does it matter if constraint (5) is written as '=' or '<=' ? Why?

(D) Does it make sense to define annual Idle Cash as a decision variable in this problem? Why or why not?

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