Example # 1 :-

- starting with 975 neutrons generated in fission, 25 neutron are produced by the fast fission makes 1000 fast neutrons, Therefore the fast fission factor ε is

$$\varepsilon = \frac{975+25}{975} = \frac{1000}{975} = 1.0256$$

- Probability of leakage escape (non-leakage) of fast neutron P_1

$$P_1 = \frac{1000-7}{1000} = \frac{993}{1000} = 0.993$$

- 993 neutrons survive fast leakage. 94 of them are captured through the moderation (thermalization), passing through resonances. The probability of resonance escape P will be

$$P = \frac{993-94}{993} = \frac{899}{993} = 0.905$$

- 899 neutrons make it to the thermal energy spectrum. 33 of them are lost to thermal leakage. The probability (Fraction) of escaping thermal leakage P_2 is therefore

$$P_2 = \frac{899-33}{899} = \frac{866}{899} = 0.963$$

- 866 neutrons survive thermal leakage, 50 of which are absorbed by reactor structures other than fuel. Therefore thermal utilization factor f is

$f \quad 866-50 \quad 816$

fuel. Therefore thermal utilization factor f is

$$f = \frac{866-50}{866} = \frac{816}{866} = 0.942$$

- The remaining 816 neutrons have to be absorbed by the fuel. 401 of them cause fission and the remaining are involved in other types of reactions (mainly radiative capture). The fission of 401 neutrons result in 975 neutrons (Back to the first block of the cycle). The reproduction factor η is

$$\eta = \frac{975}{816} = 1.195$$

$$K_\infty = \varepsilon P f \eta = 1.0447$$

$$K = K_\infty P_1 P_2 = 0.9999 \simeq 1$$

NOTE If a similar question came up in a test, you do not need to write the explanations above, just plug in the numbers into equations, and get the results

Example 2

Given in the example

(a) All neutrons are thermal $\longrightarrow$ No fast fission, No resonance capture i.e. $P = \varepsilon = 1$

(b) Infinite reactor size $\Rightarrow$ No leakage i.e. $P_1 = P_2 = 1$

(c) There is no material other than natural Uranium, i.e. All neutrons will eventually be absorbed by the fuel, Then the Thermal utilization factor $f = 1$

$\therefore K_\infty = \eta$, the reproduction factor

$$\eta = \frac{\nu \Sigma_{fission}}{(\Sigma_a)_{fuel}}$$

$$(\Sigma_a)_{fuel} = (\sigma_a N)_{U-235} + (\sigma_a N)_{U-238}$$

$$\eta = \frac{\nu [\sigma_f N] U-235}{(\sigma N)_{U-235} + (\sigma_a N) U-238}$$

Devide the numerator and denominator by $(N)_{U-235}$

$$\eta = \frac{\nu (\sigma_f) U-235}{(\sigma_a)_{U-235} + \frac{(\sigma_a N) U-238}{(N) U-235}}$$

For natural uranium, it is given

$$\frac{N_{U-238}}{N_{U-235}} = \frac{0.993}{0.007} = 141.85$$

Using the values given in the table

$$K_\infty = \eta = \frac{2.43 * 580.2}{(580.2 + 98.3) + (2.71 * 141.85)} = 1.326 \#$$

Example#3

Collision parameter $\alpha = \frac{(A-1)^2}{(A+1)^2}$

For hydrogen $\alpha = 0$

For Carbon $\alpha_{Carbon} = \frac{(12-1)^2}{(12+1)^2} = 0.716$

number of Collisions c is given by

$$c = \frac{\ln E_0/E_f}{\xi}$$

The energy decrement $\xi = 1 + \frac{\alpha \ln \alpha}{1-\alpha}$

For hydrogen $\xi = 1$

$0.716 \ln 0.716$

For hydrogen $\xi = 1$

For Carbon $\xi = 1 + \dfrac{0.716 \ln 0.716}{1 - 0.716} = 0.158$

$$(C)_{hydrogen} = \frac{\ln \frac{2\times10^6}{0.025}}{1} = 18 \ \#$$

$$(C)_{Carbon} = \frac{\ln \frac{2\times10^6}{0.025}}{0.158} = 115 \ \#$$

Note This example shows that the average number of collisions to thermalize (slow down) a neutron is much smaller when water is used as a moderator. This is directly translated into the size of the reactor. For same power, graphite moderated reactors are much bigger than water moderated reactors. Equivalently, we can say water-moderated reactors have higher power density (power/volume).

Example #4

$$\xi = 1 + \frac{\alpha \ln \alpha}{1 - \alpha}, \quad \alpha = \frac{(A-1)^2}{(A+1)^2}$$

$$\xi = 1 + \frac{\frac{(A-1)^2}{(A+1)^2} \ln \frac{(A-1)^2}{(A+1)^2}}{1 - \frac{(A-1)^2}{(A+1)^2}}$$

Multiplying numerator and denominator by $(A+1)^2$

$$\xi = 1 + \frac{(A-1)^2 \ln \frac{(A-1)^2}{(A+1)^2}}{(A+1)^2 - (A-1)^2}$$

$$= 1 + \frac{2(A-1)^2 \ln \left(\frac{A-1}{A+1}\right)}{4A}$$

$$= 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right) \leftarrow \ln x = -\ln \frac{1}{x}$$

Example # 5

$$f = \frac{(\Sigma a) fuel}{(\Sigma a)_{fuel} + (\Sigma a)_{mod.} + (\Sigma a)_{absor}} = \frac{0.2028}{0.2028 + 0.0218 + 0.011}$$

$$f = 0.861 \#$$

Example #6

$$\bar{\mu} = \overline{\cos\theta} = \frac{2}{3A} = \frac{2}{3*9} = \frac{2}{27} \#$$

$$\alpha = \left(\frac{A-1}{A+1}\right)^2 = \left(\frac{9-1}{9+1}\right)^2 = 0.64$$

$$\xi = 1 + \frac{\alpha \ln \alpha}{1-\alpha} = 1 + \frac{0.64 \ln 0.64}{1-0.64} = 0.206$$

$$C = \frac{\ln E_0/E_f}{\xi} = \frac{\ln \frac{2\times10^6}{0.025}}{0.206} \simeq 88 \text{ Collisions}$$

Example # 8

By definition η is the average number of fission neutrons per captured neutron

$$\eta = \frac{(\Sigma_f)_{U-235} * \nu}{(N\sigma_a)_{U-235} + (N\sigma_a)_{U-235}} = \frac{4.83\times10^{21} * 582\times10^{-24} * 2.42}{4.83\times10^{21} * 694\times10^{-24} + 4.35\times10^{22} * 2.7\times10^{-24}}$$

$$= 1.96$$

Example # 9

Thermal power of the plant $= \frac{470}{\text{efficiency}} = \frac{470}{0.323} = 1455\,MW$

Based on 200 Mev of energy restored per fission, The mass of U-235 fissioned per MW-day is shown to be 1.05 g.

$$\therefore \text{The mass of fissioned U-235} = \frac{0.6 * 1455 * 1.05 * 365 \frac{\text{day}}{\text{yr}}}{1000\ g/kg}$$

$$= 223.05\ kg/yr$$

$$\text{The mass of U-235 Consumed} = 223.05 * (1+\alpha)$$

$$= 223.05(1+0.169) = 260.75\ kg/yr$$

similarly mass of Pu fissioned = 340.95 kg/yr

Mass of Pu Consumed = $340.95\,(1+\alpha)$

For Pu, $\alpha = 0.362$

Pu Consumed = 464.4 kg/yr

Some of you asked, where did this 1.05 g/MW-day Came from:-

$$1\ \text{MW-day} = 1 \times 10^{6}\ \frac{J}{s} * \frac{(24 \times 3600)\ s}{day} = 8.64 \times 10^{10}\ \frac{J}{day}$$

we know that each fission (i.e. each atom fissioned) generates 200 Mev.

Convert 200 Mev to Joules

$$200\ Mev = 200\ Mev * 1.602 \times 10^{-13}\ \frac{J}{Mev}$$

$$= 3.2044 \times 10^{-11}\ J$$

Now, the number of fissions (i.e. U-235 atoms) needed to generate $8.64 * 10^{10}$ J should be

$$\frac{8.64 \times 10^{10}}{3.2044 \times 10^{-11}} = 2.7 \times 10^{21} \text{ atoms fission per day}$$

The mass of This number atmos es

$$\frac{235 * 2.7 * 10^{21}}{6.023 \times 10^{23}} = 1.05 \, g/day$$

So, if p MW of power is to be generated Continuously by U-235 Fission, Then The mass of U-235 fissioned each day is

$$\boxed{1.05 \, P} \, g/day$$