The excited states of nucleus

When the general mass-energy balance equation of the nucleus presented in SuppNotes#2 (shown below), we deferred the discussion of the energy term Q by assuming the nucleus being at the ground state. In fact, Q is a very important term, called the ***internal excitation energy***, that essentially dictates the type and rate of the neutron-nucleus interaction.

$$m_o c^2 + \frac{1}{2} m_o \mathrm{v}^2 + Q = Constant$$

We learned in high school physics and chemistry courses that electrons can not occupy their orbits around the nucleus arbitrarily. They can only be in discrete orbits, and we used the term ***quantized*** to describe these discrete energy states. Same principle is applied to the nucleus. However, the difference is that the nuclear forces and nuclear energy levels are of significantly bigger magnitudes. As such, Q; the internal excitation energy can only assume specific discrete values.

When a neutron interacts with a nucleus, the first step in that reaction is the formation of a **compound nucleus.** If the neutron energy is such that the compound nucleus is created at one of the discrete energy states, the probability of the reaction; that is the nuclear cross section greatly increases. The neutron and the nucleus are said to be at resonance.

We will further discuss in the subsequent sections the cross section's variation with neutron energy for different interactions.

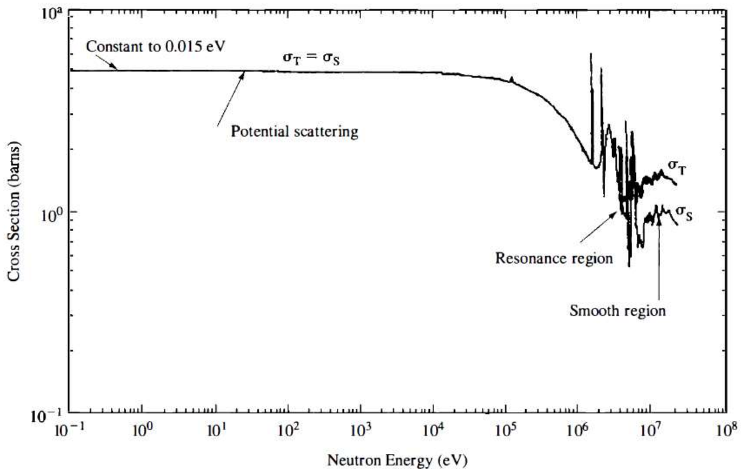
Elastic scattering (σ_e)

The figure next page shows the variation of the elastic scattering cross section (σ_e) against the neutron energy for the carbon nucleus. Three regions can be identified from the figure.

- Region of approximately constant (σ_e). It depends on the atomic mass number in this region and is approximated by $\sigma_e = 4\pi R^2$, where R is the radius of the nucleus. In this equation the cross section equals the surface area of the sphere representing the nucleus. The radius is a direct function of the atomic mass number A, and consequently σ_e depends solely on A in this region, which is called ***potential scattering***.
- At higher neutron energies we can see a region of resonances due to the formation of compound nucleus.
- Beyond the region of resonance, the elastic cross section is a smooth varying function of the neutron speed.

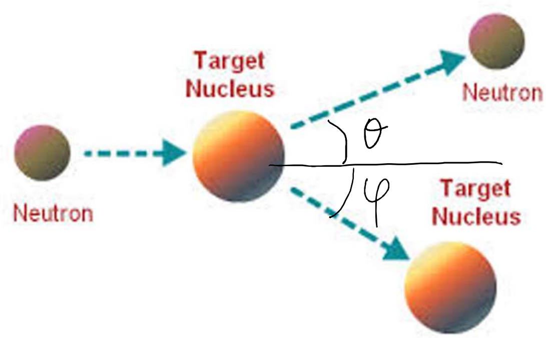
Carbon nucleus is relatively light. For heavier nuclei, resonances start at lower neutron energies.

The elastic scattering reaction is utilized to reduce the energy (***thermalization***) of the neutrons released in a fission reaction. If we know the average energy loss in a scattering reaction, the average number of collisions to thermalize a neutron can easily be determined. The neutron mean free path along with the average number of collisions are the two parameters that dictate the distance between fuel elements and the size of a thermal reactor.



During an elastic scattering reaction, the incident neutron (having kinetic energy E and momentum P) hits the target nucleus (assumed at rest). The two particles leave the reaction site in different directions and with different kinetic energies (momenta) as shown in the figure. We are interested in the energy of the scattered neutron $\acute{E}$. It can be shown purely from classical physics (conservation of momentum and energy) that the minimum energy of scattered neutron $(\acute{E})_{min}$ is given by

$$(\acute{E})_{min} = \left(\frac{A-1}{A+1}\right)^2 E = \alpha E \qquad (1)$$



Where $\alpha = \left(\frac{A-1}{A+1}\right)^2 = \frac{(\acute{E})_{min}}{E}$ is called the collision parameter and A is the mass number of the target nucleus. It is obvious that α=0, and consequently $(\acute{E})_{min} = 0$, when A=1 (i.e. when the target-nucleus is that of the hydrogen). We can also see that as A gets bigger (heavier elements), α approaches 1 and the minimum energy of the scattered neutron $(\acute{E})_{min}$ tends to that of the incident neutron. Effectively, neutrons do not loose their energy by colliding with heavy nuclei. For example, for U-238, α =0.983.

of more interest is the average energy of the scattered neutron. Its calculation includes all possible angles of collision and the probability distribution of post collision neutron energy($\acute{E}$). It can be shown that the average energy $(\acute{E})_{ave}$ of the scattered neutron (for light nuclei) and a wide range of incident neutron energy is given by

$$(\acute{E})_{ave} = \frac{1}{2}(1+\alpha)E \qquad (2)$$

Then the average energy loss in a collision is

$$(\Delta E)_{ave} = E - (\acute{E})_{ave} = \frac{1}{2}(1-\alpha)E \qquad (3)$$

and the average fractional energy loss is given by

$$\frac{(\Delta E)_{ave}}{E} = \frac{1}{2}(1-\alpha) \qquad (4)$$

Based on the last equation, on average, a neutron loses less than 1% of its energy by colliding with a U-238 nucleus while it loses about 50% of its energy by colliding with a hydrogen nucleus. In nuclear reactor applications it is often necessary to slow down fast neutron through successive elastic collisions; a process called ***MODERATION.*** Energy loss through collision with hydrogen nucleus indicates the main reason behind using hydrogen as one of the preferred moderators.

To summarize, elastic scattering is described by three average quantities:

1. The average cosine of the scattering angle $\bar{\mu}$

$$\bar{\mu} = \overline{cos\theta} = \frac{2}{3A} \qquad (5)$$

For A=1 $\bar{\mu} = 2/3$ meaning that on average neutrons scatter in forward direction. For very large A, e.g. U-238, $\bar{\mu} \rightarrow 0$ meaning that there is no preferred direction and scattering is almost equally likely in each direction

2. The natural logarithm of the fractional energy loss is known as ***lethargy***. The average change in lethargy over all possible post collision energies is known as the ***logarithmic energy decrement*** or the ***logarithmic energy loss***, denoted by ξ. It can be derived as. (*The derivation is too lengthy and will not be given here*)

$$\xi = 1 + \frac{\alpha ln\alpha}{1-\alpha} \qquad (6)$$

For hydrogen (A=1), ξ =1; the largest possible value.

3. We are also interested in the average number of collisions (c) required to slow down a neutron from an initial energy E_o to a final energy E_f. It is given by

$$c = \frac{ln\left(\frac{E_o}{E_f}\right)}{\xi} \qquad (7)$$

The collision parameters for commonly used nuclei are shown in the table next page.

Moderator	ξ	Number of collisions
H	1.0	14
D	0.725	20
H_2O	0.920	16
D_2O	0.509	29
He	0.425	43
Be	0.209	69
C	0.158	91
Na	0.084	171
Fe	0.035	411
^{238}U	0.008	1730

Inelastic Scattering (σ_i)

As explained in a previous lecture inelastic scattering takes place when the incident neutron imparts its energy to the target nucleus and the compound nucleus ejects a neutron with lower energy. The difference is being used to place the target nucleus in its first excited state. Therefore, in order for this interaction to take place the neutron energy has to exceed that threshold. As such the inelastic scattering cross section **(σ_i)** is zero over lower energy regions. since the energy needed for the first excited state is lower for heavier nuclei. Or is zero over a wider energy ranges for small nuclear mass number nuclei. Above threshold **(σ_i)** is roughly equal to **(σ_s)**.

Radiative Capture (σ_γ)

variation of the radiative capture cross section with the neutron energy can also be, in general, divided into three regions (there are few exceptions) :

1. At low neutron energies, (σ_γ) varies as $1/\sqrt{E}$ where E is the energy of the incident neutron. When plotted on a log-log scale, (σ_γ) appears as a straight line with a slop of -1/2.
2. Beyond The $1/\sqrt{E}$ region, (σ_γ) shows a region of resonances.
3. At higher neutron energies, (σ_γ) decreases smoothly and rapidly with the energy.

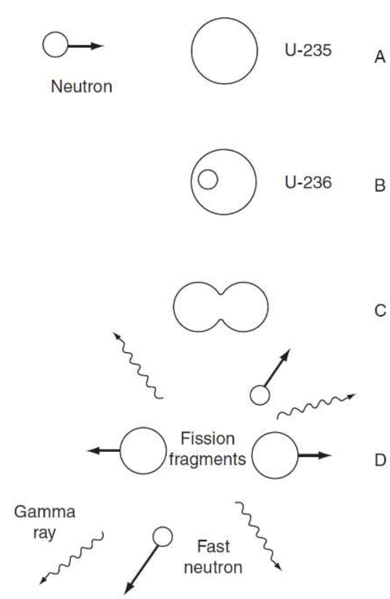
Charged Particle Reaction (σ_a)

As a general rule (n,p) and (n, α) reactions do not take place below certain threshold energy of the incident neutron. cross sections of such reactions are generally small compared to cross section of other types of interactions particularly for heavy nuclei. However, there are some important reactions charged particle reactions for light nuclei in nuclear engineering applications. One of them is the reaction $^{10}B(n,\alpha)\ ^{7}Li$. The cross section of boron to absorb neutron in a charged particle reaction is very high and extends so over a wide range of neutron's energy. That is the reason boron is used as a neutron absorber to control and terminate fission reaction.

The fission event

We have seen through the binding energy curve (SuppNotes#2) that a nucleus is held together essentially through the balance between the electrostatic repulsive forces of the protons and the short range strong attractive nuclear forces. The average binding energy increases steeply until 8.8 Mev (for A≈ 50), then slowly decreases to about 7.4 for Uranium. It was also mentioned in SuppNotes#2 that the binding energy can be thought of as an energy well. The higher the binding energy is, the deeper the nucleus is in the well which also implies a more stable nucleus.

The absorption of neutron to form a compound nucleus by most isotopes involves radiative capture where the excitation energy appears as gamma ray. However, for certain nuclei notably U-235 and some Pu isotopes, the added excitation energy of the last neutron is adequate to cause distortion of the nucleus and the two bits shown in c to come out of the grip of the nuclear forces. Eventually the will violently separate into fission fragments shown in d. The minimum energy needed to cause the fission of a heavy nucleus is called **critical energy of fission** E_{crit}. if the last (incident) neutron binding energy is more than E_{crit}, then the nucleus is said to be fissile. For example E_{crit} for U-235 is 5.3 Mev, while the B.E. of the last neutron is 6.4 Mev.



That means U-235 can fission even with a near zero K. E. neutron. A nucleus that can fission when interact with a near zero-energy neutron is said to be ***fissile.*** U-235 is the only naturally occurring fissile material. When the energy needed to cause the fission of a heavy nucleus exceeds the binding energy of the last neutron of the compound nucleus then the heavy nucleus is called ***fissionable.*** That means; for the fission event to occur, the incident neutron has to be at or above certain energy threshold. For example, the BE. of the last neutron in U-239 is 4.9 Mev. while E_{crit} is 5.5 Mev. It is obvious in this case that the fission reaction would not occur unless the incident neutron has enough energy to bring the internal excitation energy of U-239 above E_{crit}.

The Fission Cross section (σ_f)

Due to its importance we will consider U-235 as a typical example for fission cross section. Three regions of σ_f variation with the energy of incident neutron can be identified. At low energy neutrons, σ_f shows $1/\sqrt{E}$ behavior. This is followed by a region of resonances, and finally σ_f is smoothly decreasing function of neutron energy. For non-fissile but fissionable nucleus σ_f is zero up until certain threshold. that threshold typically occurs above the resonance region. Consequently, σ_f is relatively smooth over the whole energy spectrum.

Even when all the condition mentioned above are met, fission event is not certain for fissile and fissionable nuclei. Depending on the nucleus cross sections of different reactions, there are many possible outcomes when a neutron collides with a fissile or fissionable nucleus, namely scattering (elastic and inelastic), radiative capture, and fission. Since the scattering cross section σ_s for heavy nuclei is typically much smaller than the other two, then fission and radiative capture are more likely outcomes, A parameter α called ***capture to fission ratio*** is defined as

$$\alpha = \frac{\sigma_\gamma}{\sigma_f} \qquad (8)$$

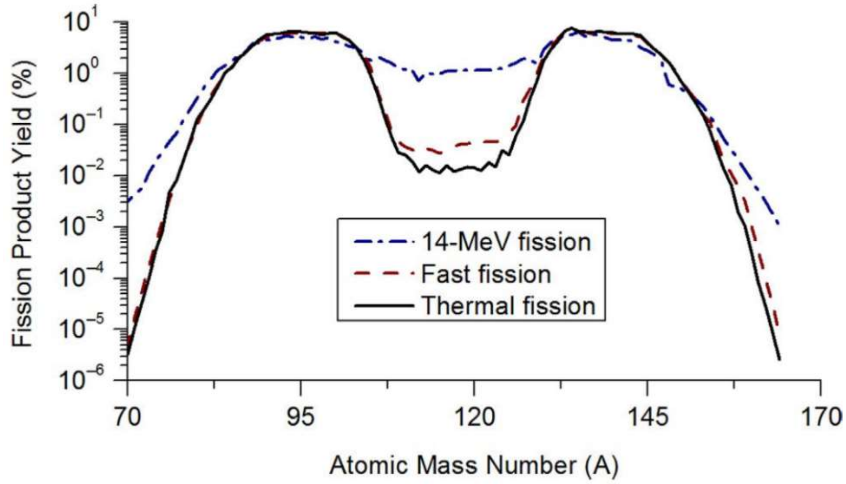
which is a function of the neutron energy. It is an important parameter for certain reactor designs.

Fission products and fission energy

The nuclear reaction equation for the fission of U-235 nucleus may be written in the general form as

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow ^{A_1}_{Z_1}F_1 + ^{A_2}_{Z_2}F_2 + \nu ^{1}_{0}n + Energy$$

Where F_1 and F_2 are general labels for the fission fragments, and ν represents the average number of neutrons coming out of the fission reaction. F_1 and F_2 indicate many possible ways of splitting (fission) the target nucleus. In general, the fragments of a thermal neutron-induced fission are asymmetric. That means the masses of the two fragments are significantly different. This is indicated in the figure next page. This figure shows the fission products yield for U-235; that is the percentage of the different fragments produced as a function of the atomic mass number. As shown in the figure, the asymmetry is more pronounced when the fission is induced by thermal neutrons.



Let us Consider a few more general characteristics when fissions and subsequently radioactive decay takes place in a reactor.

- Fission and radioactive decay can produce completely different materials. For example the solid radium decays to radon, which is a gas.

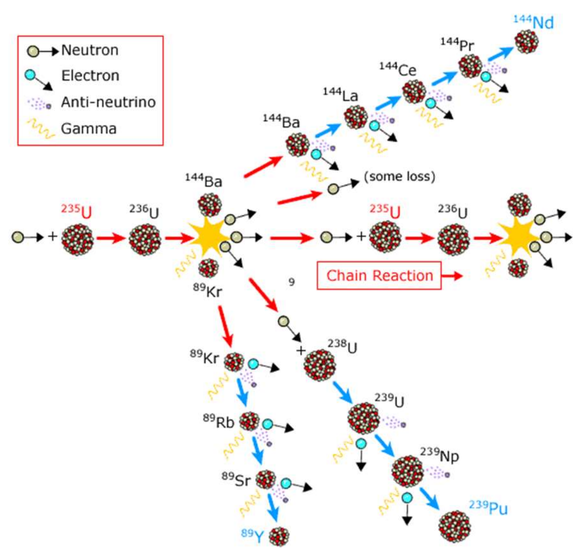
$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \longrightarrow ^{90}_{38}Sr + ^{144}_{54}Xe + 2\ ^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \longrightarrow ^{87}_{35}Br + ^{146}_{57}La + 3\ ^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \longrightarrow ^{96}_{37}Rb + ^{137}_{55}Cs + 3\ ^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \longrightarrow ^{137}_{52}Te + ^{97}_{40}Zr + 2\ ^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\ ^{1}_{0}n$$



- As might be expected the decay history of this large variety of radioactive nuclides is extremely complex. Fortunately, except for few nuclides which are of major importance for biological or neutron absorption reasons, it is not necessary to closely follow the decay of all radioactive nuclides in the reactor core environment. Specifically Iodine (I) and strontium (Sr) for biological reasons and Xenon (Xe) and Samarium (Sm) for their high neutron absorption cross-section.

- The most important feature of the fission fragments is that they contain the largest fraction of the fission energy mostly manifested as kinetic energy. Fission fragments are heavy and highly charged. Consequently, they can not move far from the fission site in spite of their K.E. Fuel sheath thickness is adequate to stop them. The average distance traveled by a fission fragment is about 10^{-3} Cm. The table below gives the break down of the energy generated from the fission of a U-235 nucleus. Typically, for engineering calculations, 200 Mev is considered as the recoverable energy from a fission event

TABLE 3.6 EMITTED AND RECOVERABLE ENERGIES FOR FISSION OF ^{235}U

Form	Emitted Energy, Me V	Recoverable Energy, Me V
Fission fragments	168	168
Fission-product decay		
β-rays	8	8
γ-rays	7	7
neutrinos	12	—
Prompt γ-rays	7	7
Fission neutrons (kinetic energy)	5	5
Capture γ-rays		3–12
Total	207	198–207

Fission Neutrons

In a fission reaction, 99% of the neutrons are emitted at the instant of the fission and are called ***prompt neutrons***, the remaining 1% are emitted at a relatively later time as a result of the decay of fission products and called **delayed neutron**. These neutrons play an important role in the control of the fission reaction (reactor power) as we will see later. The average number of fneutrons (prompt and delayed) per fission is denoted by $\boldsymbol{\nu}$. It is convenient also to introduce the parameter $\boldsymbol{\eta}$ which is defined as the average number of neutrons emitted per neutron absorbed by a fissile nucleus. These parameters are related through the following equations. $\boldsymbol{\alpha}$, $\boldsymbol{\eta}$ and $\boldsymbol{\nu}$ values are shown in the table below.

$$\eta = \nu \frac{\sigma_f}{\sigma_a} = \nu \frac{\sigma_f}{\sigma_\gamma + \sigma_f} = \nu \frac{1}{1+\alpha} \qquad (9)$$

For a mixture

$$\eta = \frac{1}{\Sigma_a} \sum_i \nu_i \Sigma_{f_i} \qquad (10)$$

TABLE 3.4 THERMAL (0.0253 eV) DATA FOR THE FISSILE NUCLIDES*

	$\sigma_a{}^{\dagger}$	σ_f	α	η	ν
^{233}U	578.8	531.1	0.0899	2.287	2.492
^{235}U	680.8	582.2	0.169	2.068	2.418
^{239}Pu	1011.3	742.5	0.362	2.108	2.871
^{241}Pu	1377	1009	0.365	2.145	2.917

*From *Neutron Cross-Sections*, Brookhaven National Laboratory report BNL-325, 3rd ed., 1973.
$^{\dagger}\sigma_a = \sigma_\gamma + \sigma_f$.

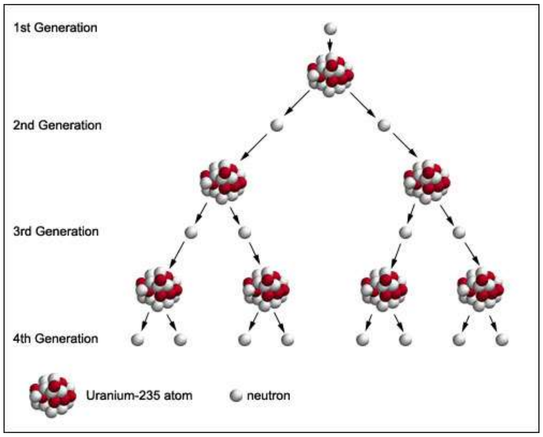
The Chain reaction

The energy released in a fission reaction can be utilized only when we have a sustained chain reaction. The neutron emitted by a fission reaction should be able to induce the next generation of fissions and so on. Quantitatively, the chain reaction is described by the multiplication factor (k), which is defined as the ratio of the number of fissions in one generation to the number of fissions in the preceding generation.

$$k = \frac{Number\ of\ fissions\ in\ one\ generation}{Number\ of\ fission\ in\ the\ preceding\ generation}$$

Equivalently k is defined as

$$k = \frac{Number\ of\ neutrons\ in\ one\ generation}{Number\ of\ neutrons\ in\ the\ preceding\ generation}$$



If *k>1* then the energy released from the fission is increasing from one generation to the next and the nuclear system is said to be ***supercritical.*** If *k<1* energy decreases between generation and the nuclear system is said to be ***subcritical***. In special case when *k=1*, the chain reaction proceeds at constant rate and the energy released is constant.

A device that is designed to maintain a controlled chain reaction is called nuclear reactor. To show the importance of the control of chain reaction let us assume a reactor operates at power P, and due to certain transient k becomes 1.05. The next generation power would be 1.05 P, and after two generation becomes $(1.05)^2$P, and after 100 generation, the power is $(1.05)^{100}$P=131.5P. knowing that, roughly, the life cycle of a prompt neutron in a CANDU reactor is about 0.001 sec, the power generated could well be 131 times in 0.1 second is.

In practice we use the term reactivity (ρ) which is defined as the deviation of k from 1with respect to the value of k:

$$\rho = \frac{k-1}{k} = 1 - \frac{1}{k} \qquad (11)$$

Then

$$k < 1 \ \Rightarrow \rho < 0 \quad subcritical,$$
$$k = 1 \ \Rightarrow \rho = 0 \quad critical$$
$$k > 1 \ \Rightarrow \rho > 0 \quad supercritical$$

And

$$P_n = P_o(1+\rho)^n \qquad (12)$$

where *n* is the number of generations.
The time *t* required for *n* generations to elapse is $t = l \times n \quad or \quad n = t/l$, where *l* is the average time for one neutron generation. To make the arithmetic easier, we can write equation (12) in a different way:

$$P = P_o e^{t/\tau} \qquad (13)$$

It can be shown that equations (12) and (13) are the same thing for small values of reactivity (ρ) encountered in normal operation. The constant τ is the reactor period as $\tau = l/\rho$. This equation gives the power conveniently in terms of the elapsed time *t* and reactor period τ. Note that the larger ρ is, the shorter the reactor period becomes, and the faster the power changes will be.

<u>The effects of delayed neutrons on the reactor's power change</u>

For fission of U-235, 99.35% of the neutrons produced are prompt neutrons, and 0.65% are delayed neutrons emitted by fission products. The time for one generation of prompt neutrons is 0.001 s. The average lifetime of the delayed neutrons is almost 13 seconds. The average lifetime *l* for all the neutrons, prompt and delayed is then, *l*= 0.9935 x 0.001 s + 0.0065 x 13 s = 0.085 seconds.
For simplicity, we usually round off the value of to 0.1 sec. Although delayed neutrons represent a small fraction (0.65%) of neutrons generated by fission, they increase the average lifetime of all

neutrons from 0.001 s to 0.085 s, that is, by a factor of 85. This reduces the initial rate of power rise by a factor of 85. In summary, the effect of the delayed neutrons is to make the rate of power changes reasonably slow for small additions of positive reactivity.

The six-factor formula (neutron life cycle)

As we have just seen, the power generated in a reactor could spike up effectively in a very short time. It is therefore important to control the fission reaction through tracking the life cycle of the neutrons within the reactor. We are going to do that by dividing the neutron life cycle into a sequence of steps which are characterized by some dominant interactions (resonance, absorption, leakage ...etc.). From each of these steps, the probability of neutron survival or multiplication is calculated. Computation of these probabilities (factors) is extremely difficult and beyond the scope of this course. They depend on the specifics of each reactor.

As we have discussed, in thermal reactors most of the fissions are induced by the absorption of thermal neutrons in the fissile isotopes. The thermal fission generates a number of high energy (Fast) neutrons. let's assume that N fast neutron are just generated as a result of thermal fissions:

Step#1 (Fast Fission Factor): Although Fission cross section (likelihood) is much higher for slow (thermal) neutrons, It is possible that few of these fast neutrons induce fission given that, nuclear fuel typically contains significant amount of U-238. These fast fissions will add to the already exist N fast neutrons. This contribution is included by the fast fission factor ε which is defined as

$$\varepsilon = \frac{total\ number\ of\ fast\ neutron\ produce}{fast\ neutron\ produced\ by\ thermal\ fission}$$

Step#2 (Resonance escape probability): The generated fast neutrons will immediately start to interact with the surroundings (absorbed, loose energy, or move out of the reactor). Let's consider the absorbed fraction. We have seen that, in general, the absorption cross-section is generally low for higher energy neutrons, however the absorption cross section shows resonances (spikes) at certain energy levels. A prominent resonance absorber is U-238 which is typically available in significant amounts in reactor cores. Most of the absorptions occur while the neutrons are slowing down. we are interested in the number of neutrons that survive the resonance absorption. The probability that a neutron will not be caught in a resonance absorption is denoted by p

$$p = \frac{Neutrons\ manage\ to\ escape\ resonance\ absorption}{Total\ number\ of\ fast\ neutrons}$$

Therefore, the number of neutrons that manage to slow down successfully past the resonance absorption is $N\,\varepsilon\,p$. p can also be expressed in terms of the proper macroscopic cross sections. if Σ_a is the absorption macroscopic cross section for fast neutron, Σ_e is the slow down (scattering) cross section, and ϕ represents the fast neutron flux, then

$$p = \frac{\Sigma_a \phi}{(\Sigma_e + \Sigma_a)\phi} = \frac{\Sigma_a}{(\Sigma_e + \Sigma_a)} \qquad (14)$$

Step#3 Fast neutron leakage

During the thermalization (slowing down) of neutrons, they move around through collisions and scattering. Since the total cross section for fast neutrons is small, the mean free path $1/\Sigma_{total}$ is relatively large. i.e., these neutrons tend to move through relatively large distances before collision. Since the reactors have a finite dimension, some of these neutrons may escape out of the reactor before they make it to the thermal spectrum range (slow down). We are interested in the neutrons that survive the leakage and reach the thermal energy range. let P_1 represents its probability then,

$$P_1 = \frac{number\ of\ neutrons\ that\ reach\ the\ thermal\ energy\ spectrum}{total\ number\ of\ fast\ neutrons\ that\ could\ leak}$$

Step# 4 (Thermal neutron leak)
The generated thermal neutron population, could again be absorbed or leaked out. Similar to what we discussed in step 3, let P_2 be the probability of non-leakage of thermal neutrons. The difference is that due to higher total cross section for thermal neutrons , the mean free path $1/\Sigma_{total}$ is much less and consequently the non-leakage probability (P_2) is much higher. The total number of neutrons available for absorption is now $N\,\varepsilon\, pP_1P_2$

Step# 5 Thermal utilization factor

The remaining neutrons that do not leak will eventually get absorbed. For the most part, they will be absorbed by the fuel, yet a small fraction will be absorbed by the structural material, moderator... etc. The fraction of neutrons that are absorbed by the fuel is called the thermal utilization factor *(f)*. The thermal utilization factor can be calculated from the appropriate cross-sections let Σ_{af} represents the macroscopic absorption cross-section, Σ_{am} is that of all other materials, and let ϕ be the thermal neutron flux, then

$$f = \frac{Rate\ of\ neutron\ absorption\ in\ the\ fuel}{Total\ absorption\ rate\ of\ thermal\ neutrons} = \frac{\Sigma_{af}\phi}{(\Sigma_{af} + \Sigma_{am})\phi} = \frac{\Sigma_{af}}{(\Sigma_{af} + \Sigma_{am})}$$

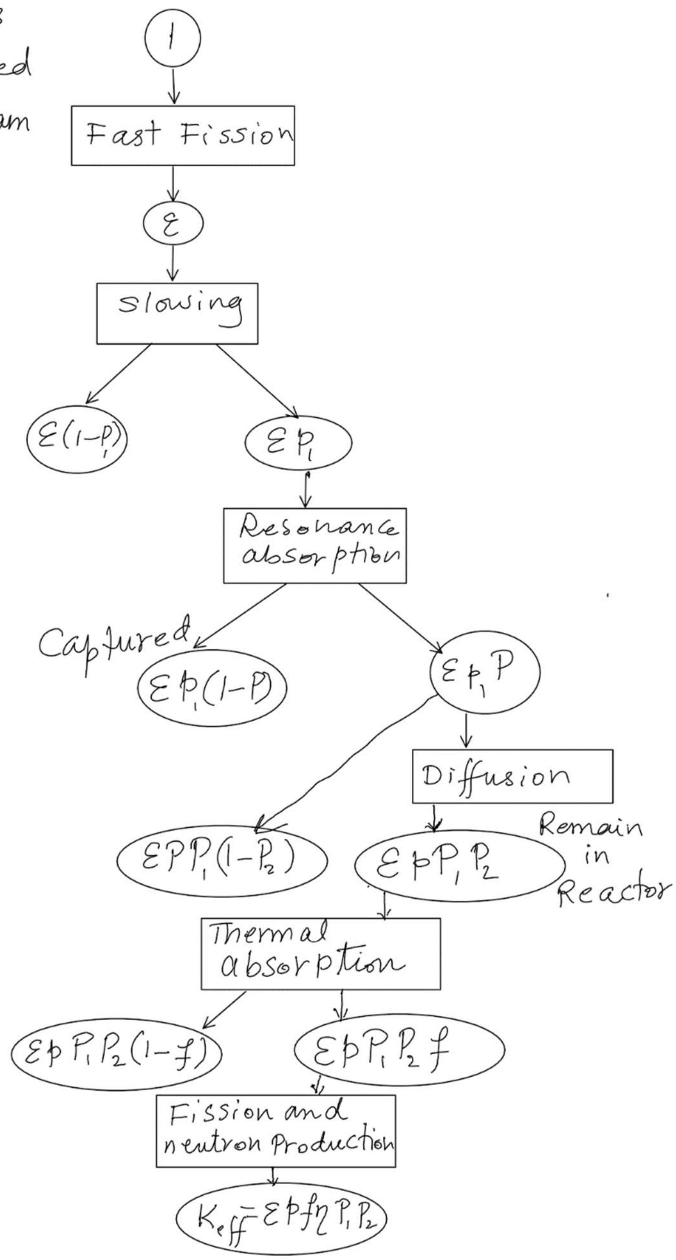
Step#6

Not all neutrons absorbed by the fuel will result in a fission. The number of neutrons generated from a fission per neutron absorbed is (as we have seen) is denoted by η

$$\eta = \frac{Number\ of\ neutron\ produced}{Thermal\ neutron\ absorbed\ in\ the\ fuel} = \frac{\nu\Sigma_f\phi}{\Sigma_a\phi} = \frac{\nu\Sigma_f}{\Sigma_a}$$

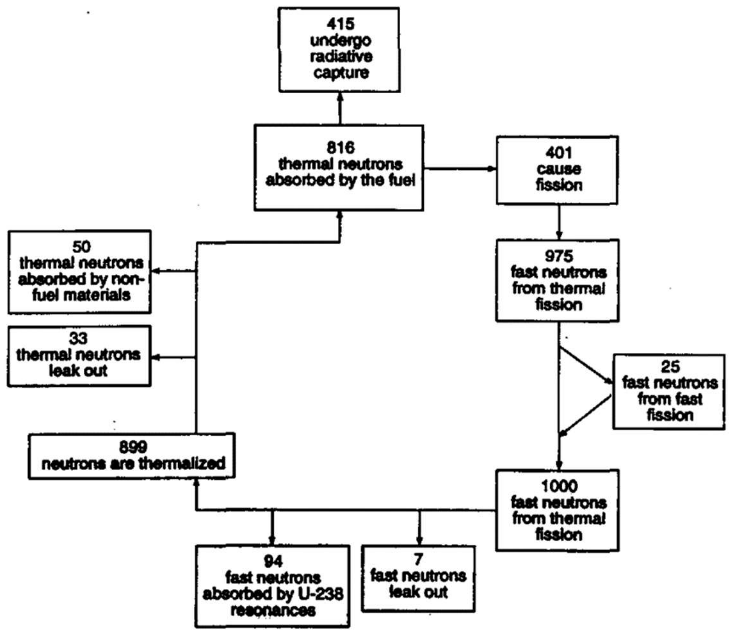
Based on these factors the number of fast neutron available for the next generation is $N\varepsilon pf\eta P_1P_2$ and the multiplication factor is k= $\varepsilon pf\eta P_1P_2$. The last two factors depends on the size and geometry of the reactor and they (P_1P_2) approach 1 as the reactor becomes infinitely large. This limit is used to define the multiplication factor for infinitely large reactor (k_∞). it is called the four-factor formula $k_\infty = \varepsilon p f \eta$

The six steps can be depicted in this diagram



Examples

1. Calculate the six factors, k, and k_∞ from the diagram below.



2. The table below gives the thermal neutron cross section of U-235 and U-238 for scattering, radiative capture and fission. The table also gives the average number of neutrons per fission ν. Calculate the value of k_∞ for natural uranium assuming that all the neutrons involved (those causing fission and those produced by fission) are at the thermal spectrum. The gram density of natural uranium is 19 g/cm^3 and its composition is 0.7% U-235, and the rest is U-238.

	σ_s (barns)	σ_c (barns)	σ_f (barns)	v
U-235	17.6	98.3	580.2	2.43
U-238	10.0	2.71	0	0

3. Compare the number of collisions in going from the fission energy of 2 Mev to the thermal energy of 0.025 ev in both hydrogen and carbon.

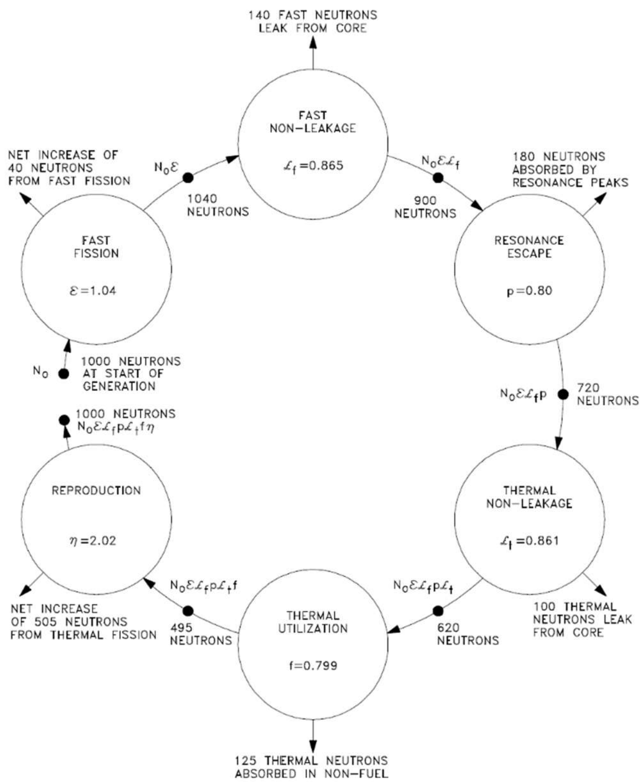
4. Show that the energy decrement can be written in terms of the atomic mass number A, as

$$\xi = 1 - \frac{(A-1)^2}{2A} ln\left(\frac{A+1}{A-1}\right)$$

5. Calculate the thermal utilization factor f for a homogeneous reactor. The macroscopic absorption cross section of the fuel is 0.2028 cm^{-1}. The macroscopic absorption cross section of the moderator is 0.0110 cm^{-1}, and the macroscopic absorption cross section of the absorber is 0.0218 cm^{-1}.

6. What are the values of the average logarithmic energy change ξ and the average cosine of the scattering angle $\bar{\mu}$ for neutrons in beryllium, A=9? How many collisions are needed to slow neutrons from 2 MeV to 0.025 eV in Be-9?

7. Calculate the six factors, k, and k_∞ from the diagram below.



8. Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. The microscopic absorption cross section for uranium-235 is 694 barns. The cross section for uranium-238 is 2.71 barns. The microscopic fission cross section for uranium-235 is 582 barns. The number density of uranium-235 is 4.83 x 1021 atoms/cm3. The number density of uranium-238 is 4.35 x 10^{22} atoms/cm^3. ν is 2.42.

9. A nuclear power plant operates at a net electric output of 470 megawatts. The overall efficiency of the plant is 32.3%. Approximately 60% of the plant's power comes from fissions in U-235, the remainder from fissions in converted plutonium, mostly Pu-239. If the plant were operated at full power for 1 year, how many kilograms of U-235 and Pu-239 would be (a) fissioned? (b) consumed? Consider 200 Mev per fission, and the atomic weight of U-235and Pu-239 are 235 and 239, respectively. Consider the values of the following table

	σ_a	σ_f	α	η	ν
U-235	680.8	582.2	0.169	2.068	2.418
Pu-239	1011 .3	742.5	0.362	2.108	2.871