MAT 1302C，Fall 3
Assignment 4
Professor: Hadi Salmasian
Due on December 4, 2012

1,let z=(2+2i)/(1-i), write z in form a+ib
solution:
(2+2i)(1+i)/(1-i)(1+i)= (2+2i+2i+2 $i^{2.}$) / (1- $i^{2.}$)=（2+4i-2) / (1+1)=2i=0+2i
therefore the form is z=0+2i

2, consider the following matrix A= $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(a) . the characteristic polynomial of A

(A- λ I)= $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ - $\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$ = $\begin{bmatrix} 1-\lambda & 0 & 1 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{bmatrix}$

det (A- λ I)=(1-λ) $\begin{vmatrix} 1-\lambda & 1 \\ 0 & -1-\lambda \end{vmatrix}$ = (1-λ)[(1-λ) (-1-λ)] = $(1-\lambda)^2$ (-1-λ).

λ= 1 , -1

(b). List the eigenvalues of A with their multiplicities.
The eigenvalues of A are 1, -1

λ=1 with multiplicities 2
λ=-1 with multiplicities 1

because det (A- λ I)= $(1-\lambda)^2$ (-1-λ)=0
so λ= 1, -1

(c) for the eigenvalues found in part (b) describe the eigenvectors.
When λ=1

A-I = $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 - (-2R_1)} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

therefore, $x_1 = 0, x_3 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

therefore, vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ of A corresponding to λ = 1

when λ= -1

$$(A+I)= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_1 \to \frac{1}{2} R_1]{R_2 \to 2R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{4} R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ therefore, } x_1 = -\frac{1}{2} x_3,\ x_2 = \frac{1}{4} x_3 \ , \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix}$$

therefore vector $\begin{bmatrix} -\frac{1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix}$ of A corresponding to λ= -1

(d) Is the matrix A diagonalizable? Justify your answer.

It not a diagonalizable matrix because there are two vectors $v_{1,} v_2$
it is a 3×3 matrix, so n=3, 2<3 , so it is not a diagonalizable matrix

3.
(a) find the migration matrix and set up a difference equation for this situation.

(From) field	my house	
0.6	0.3	(to) field
0.4	0.7	my house

the initial matrix is $\begin{bmatrix} .7 \\ .3 \end{bmatrix}$

therefore the migration matrix is $\begin{bmatrix} .6 & .3 \\ .4 & .7 \end{bmatrix}$

the equation is $\begin{bmatrix} .6 & .3 \\ .4 & .7 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .51 \\ .49 \end{bmatrix}$

(b) the week 1 is w1= $\begin{bmatrix} .51 \\ .49 \end{bmatrix}$, w2= M *w1= $\begin{bmatrix} .6 & .3 \\ .4 & .7 \end{bmatrix} \begin{bmatrix} .51 \\ .49 \end{bmatrix} = \begin{bmatrix} .453 \\ .547 \end{bmatrix}$

so the rats' distribution after two weeks in my farm is 45.3% in the field and 54.7% in my house.