University of
WATERLOO

Math 147
Calculus I (advanced)

Final Examination
Fall 2013

Student Name: (please print) _____

Student Identification Number: _____

Instructor: Kenneth R. Davidson
Section: 001
Term: 1139
Materials: No calculators allowed

Date: Wednesday, December 11, 2013
Time: 12:30 a.m. to 3:00 a.m.
Duration of exam: 2.5 hours
Exam type: closed book

Instructions

1. Fill in your Name and ID Number above.
2. Write your initials in the top right corner of every page.
3. This exam consists of 8 questions worth a total of 60 marks.
4. This exam has 9 pages. Make sure that you have all of them!
5. Show all of your work.

1.	/ 6
2.	/ 8
3.	/10
4.	/ 6
5.	/ 7
6.	/ 8
7.	/ 5
8.	/10
Total	/60

[6] 1. Compute the following limits.

(a) $\lim_{x\to\infty}(1-e^{-x})^{e^x}$

(b) $\lim_{x\to 0}\dfrac{1-\cos(x^2)}{x^2\sin^2 x}$

(c) $\lim_{x\to 0}\dfrac{e^x+e^{-x}-2}{1-\cos x}$

[8] 2. (a) Define "$f(x)$ is continuous at $x = a$".

(b) State a characterization of "$f(x)$ is differentiable at $x = a$" that involves the tangent line.

(c) Find the tangent line to $f(x) = x^2 \ln(1 + x^4)$ at $x = 1$.

(d) Prove that a function $f(x)$ which is differentiable at $x = a$ is continuous at $x = a$.

[10] 3. (a) State the Intermediate Value Theorem.

(b) Show that $xe^{1/x} = 4$ has *exactly one* solution in $(0, 1)$.

(c) Prove the Intermediate Value Theorem. (*Continue on the back of this page if needed.*)

[6] 4. (a) State the Mean Value Theorem.

(b) Show that $x \leq \sin^{-1}(x) \leq \dfrac{x}{\sqrt{1-x^2}}$ on $(0,1)$.

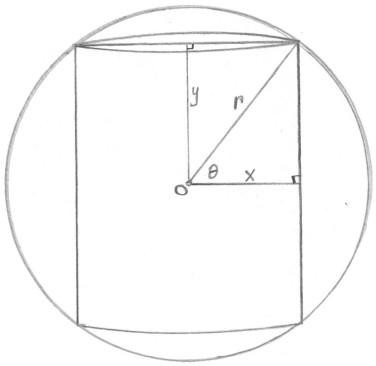
[7] 5. (a) Define a "convex function" and explain the definition geometrically (include a sketch).

(b) Show that if $x_1, \ldots, x_n$ are positive numbers, then $\left(\frac{1}{n}\sum_{i=1}^{n} x_i^5\right)^{1/5} \leq \frac{1}{n}\sum_{i=1}^{n} x_i$.

[8] 6. (a) Find the Taylor polynomial P of degree 3 for $f(x) = \tan x$ at $a = \frac{\pi}{4}$.

(b) Estimate the error $\left|\tan\left(\frac{\pi}{4} - \frac{\pi}{180}\right) - P\left(\frac{\pi}{4} - \frac{\pi}{180}\right)\right|$.
(Give an explicit number here. One digit accuracy is enough.)

[5] 7. Find the maximum volume of a right cylinder inscribed inside a sphere of radius r.



[10] 8. Graph the function $f(x) = e^{1/x}(x + 2)$ exhibiting as many features as possible. Show your calculations. (*Continue on the back of this page if needed.*)