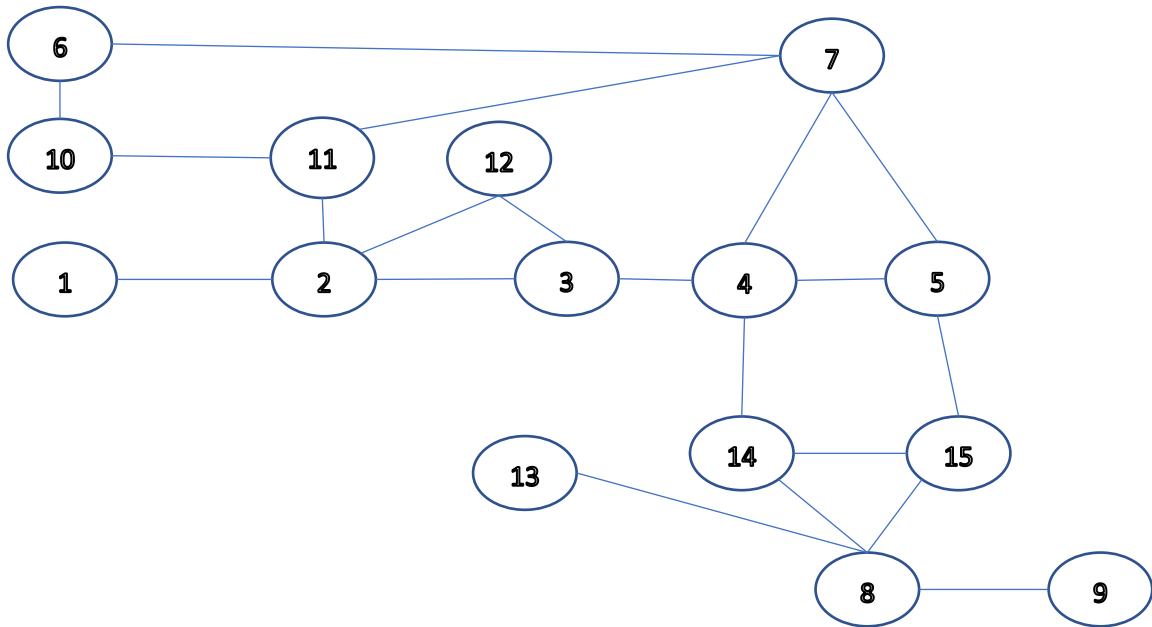
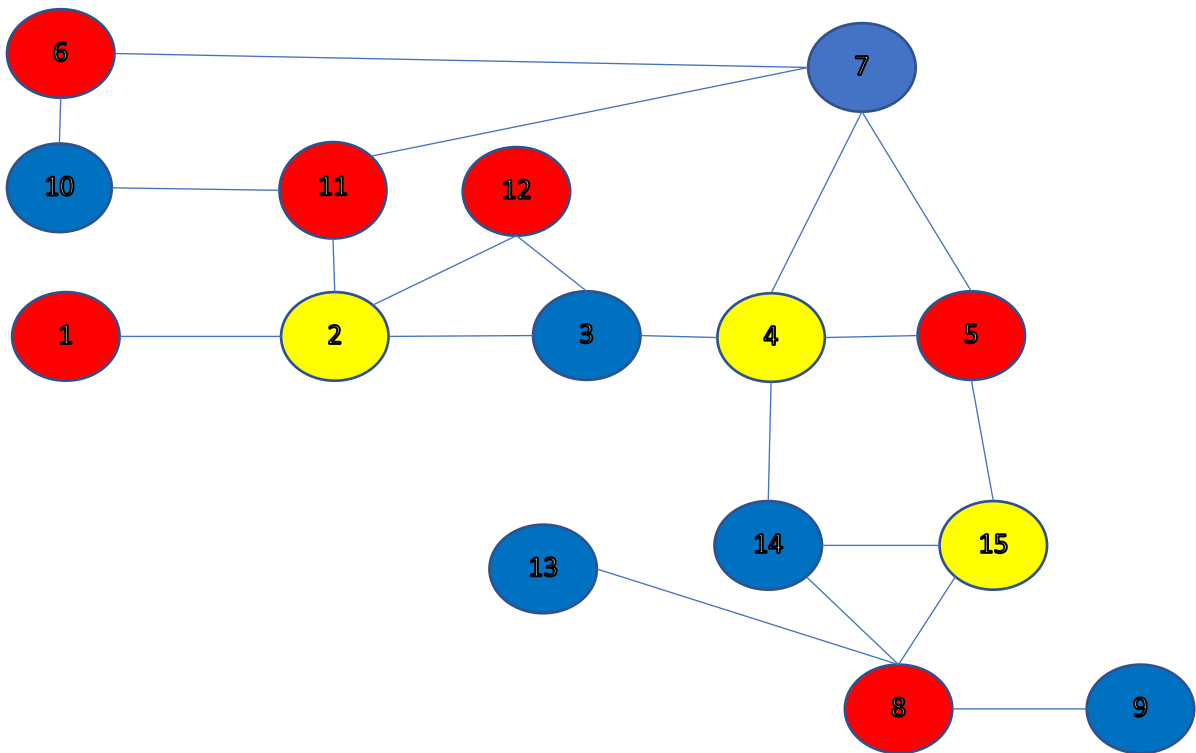


COMP 1805S19 ASSIGNMENT 4

1.



2. The chromatic number for this planar graph is 3. I use the K_3 sub-graph $V' = \{4, 5, 7\}$, $E' = \{\{4, 5\}, \{5, 7\}, \{7, 4\}\}$ to determine the number of colours require. Since the 3 vertices are adjacent to one another, I must use 3 colours.



3. There is no Euler circuit for the graph. Some vertices have odd degrees, each vertex must have an even degree for a Eulerian circuit to exist. No Euler path either, there are more than one vertex with an odd degree which contradicts the theorem that there must be exactly 2 vertices with odd degrees.
4.
 - a. No
 - b. No
 - c. No
5. Bubble sort

	i = 1	i = 2	i = 3	i = 4	i = 5
j = 1	15, 25, 5, 30, 20, 10	15, 5, 25, 20, 10, 30	5, 15, 20, 10, 25, 30	5, 15, 10, 20, 25, 30	5, 10, 15, 20, 25, 30
j = 2	15, 25, 5, 30, 20, 10	5, 15, 25, 20, 10, 30	5, 15, 20, 10, 25, 30	5, 15, 10, 20, 25, 30	
j = 3	15, 5, 25, 30, 20, 10	5, 15, 25, 20, 10, 30	5, 15, 20, 10, 25, 30		
j = 4	15, 5, 25, 30, 20, 10	5, 15, 20, 25, 10, 30			
j = 5	15, 5, 25, 20, 30, 10				

The sorted list is therefore: 5, 10, 15, 20, 25, 30

NOTE:

Blue: Compared and no swapping

Red: Compared and swapped

Orange: sorted

6. True or not
 - a. $(2n + 1)^2$ is $\Theta(n^2)$ this is true

We need to show that $f(n)$ is $O(n^2)$ and $f(n)$ is $\Omega(n^2)$ for $f(n)$ to be $\Theta(n^2)$

Showing $(2n + 1)^2$ is $O(n^2)$

$$= 4n^2 + 4n + 1$$

$$|4n^2 + 4n + 1| \leq |4n^2 + 4n^2 + n^2|$$

$$|4n^2 + 4n + 1| \leq |9n^2| \text{ for } \forall n \geq 1$$

$k = 1, c = 9$, $(2n + 1)^2$ is therefore $O(n^2)$

Showing that $(2n + 1)^2$ is $\Omega(n^2)$

$$|4n^2 + 4n + 1| \geq |4n^2| \text{ for } \forall n$$

Since $(2n + 1)^2$ is both $O(n^2)$ and $\Omega(n^2)$ it must be the case that it is $\Theta(n^2)$

b. $3n^2 - 1$ is $O(n^2)$ is true

$$\begin{aligned} 3n^2 - 1 &\leq 3n^2 \\ &\leq 3n^2 \text{ for } \forall n \geq 1 \\ &= 3n^2 \\ &= 3(g(n^2)), c = 3, k = 1, \text{ therefore } 3n^2 - 1 \text{ is } O(n^2) \end{aligned}$$

c. $\frac{3 \log(n+1)}{5}$ is $O(n)$ false

$$\begin{aligned} \frac{3}{5}(\log(n + 1)) &\leq \frac{3}{5}\log(n + n) \text{ using log base 2} \\ &\leq \frac{3}{5}\log(2n) \text{ for } \forall n \geq 1 \\ &\frac{3}{5}\log(2n) \text{ which is not the same as } O(n) \end{aligned}$$

d. $9n + 2n \log n$ is $\Omega(n)$ true

$$\begin{aligned} 9n + 2n \log_2 n &\geq 9n \\ &\geq 9n \text{ for } \forall n \geq 1 \\ &= 9(g(n)), c = 9, k = 1 \end{aligned}$$

e. $\log\left(\frac{1}{n} + n^2\right)$ is $O(\log n)$ false

$$\begin{aligned} \log\left(\frac{1}{n} + n^2\right) &\geq \log(n^2) \\ &\geq 2 \log n, \text{ for } n \geq 1 \\ &= 2 \log n \text{ } c = 2, k = 1, \text{ this should be } \Omega(\log n) \end{aligned}$$

7. Linear search will locate the element rapidly compared to binary search. Linear search requires 4 iterations at most, and binary search requires 5 iterations at most.

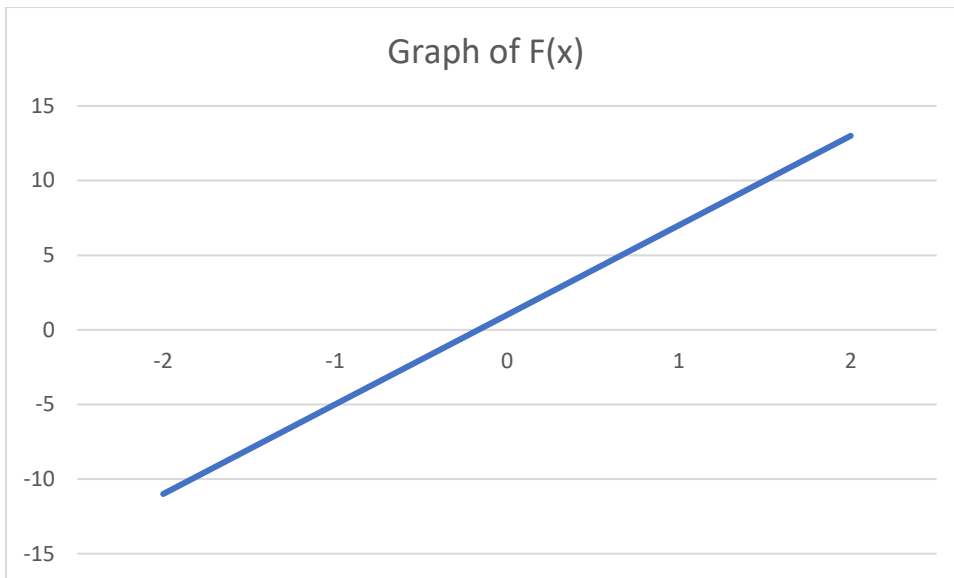
8.

a. **Linear search:** linear search will compare an element to every element in the list starting from 1, 2, 3, ..., n. This will create n comparisons. This number will double to **2n** when the size of the list is double

- b. **Binary search:** the list is dived into half every time an element is check from the list. This means the number of comparisons would be $\log_2 n$. If the size of the list doubles to $2n$, the new number of comparisons would be $\lceil \log_2(2n) \rceil = \log_2 2 + \log_2 n$ (by decomposition of logathrm of a product)
 $= 1 + \log_2 n$ comparisons. The number of comparisons therefore increase by 1.

9. Plot $f(x) = 6x + 1$ $\{x|x \in \mathbb{Z}, -2 \leq x \leq 2\}$

x	-2	-1	0	1	2
f(x)	-11	-5	1	7	13



10. Function composition

My student ID is ****84

$$a = 4,$$

$$c = 2a = 2(4) = 8$$

$$b = 8$$

$$d = 3b = 3(8) = 24$$

$$f : \mathbb{Z} \rightarrow \mathbb{Z}; f(x) = ax + c = 4x + 8$$

$$g : \mathbb{Z} \rightarrow \mathbb{Z}; g(x) = c^2 - d = 8^2 - 24 = 64 - 24 = 40$$

a. $g \circ f$

$$g \circ f = g(f(x))$$

$$= 40$$

b. $f \circ g$

$$\begin{aligned} &= f(g(x)) \\ &= f(40) \\ &= 4(40) + 8 \\ &= 160 + 8 = 168 \end{aligned}$$

c. $(f \circ g) \circ g$

$$\begin{aligned} &= (4(40) + 8) \circ g(x) \\ &= 168 \circ (g(x)) \\ &= 168 \end{aligned}$$

11. Countability is a term used to describe sets that are either finite or countably finite. An example of a countable set is a set of rational numbers. An example of uncountable set is the set of real numbers.