Q1

A) A PHENOMENA IS OBSERVED MANY TIMES UNDER IDENTICAL CONDITIONS. THE PROPORTION OF TIMES A PARTICULAR EVENT IS RECORDED. THIS PROPORTION REPRESENTS:
THE PROBABILITY OF THE EVENT

B) THE COLLECTION OF ALL POSSIBLE OUTCOMES OF A RANDOM PHENOMENA IS:
SAMPLE SPACE

Q2 A TEST CONTAINS 8 TRUE/FALSE QUESTIONS. ASSUMING YOU ATTEMPT EACH QUESTION, IN HOW MANY DIFFERENT WAYS COULD YOU ANSWER THE TEST?
$2^8 = 256$

Q3 A MAN HAS 5 TIES, 6 SHIRTS, AND 5 PAIRS OF PANTS. HOW MANY DIFFERENT WAYS DOES HE DRESS HIMSELF?
5 TIES × 6 SHIRTS × 5 PANTS = 150 TIE/SHIRT/PANT COMBINATIONS

Q4 A PRESIDENT, A TREASURER, AND A SECRETARY ARE TO BE CHOSEN FROM A COMMITEE OF 40 MEMBERS. IN HOW MANY WAYS COULD THE THREE JOBS BE CHOSEN? A MEMBER CANNOT SERVE MORE THAN 1 POSITION

CORRECT: WITH REPLACEMENT: PERMUTATION $N = 40$, $n = 3$ $\Rightarrow \dfrac{N!}{(N-n)!} = \dfrac{40!}{37!} = 59{,}280$

WITHOUT REPLACEMENT: COMBINATION $N = 40$, $n = 3$ $\Rightarrow \dfrac{N!}{n!(N-n)!} = \dfrac{40!}{3!37!} = 9{,}880$

Q5 WHEN TOSSING A COIN 3 TIMES, THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE:
$2^3 = 8$ OUTCOMES $\Rightarrow$ S: {HHH, TTT, HTH, THH, HHT, TTH, THT, THT}

Q6 AN ELECTRICAL SYSTEM CONSISTS OF 2 COMPONENTS IN SERIES. BOTH COMPONENTS MUST WORK FOR THE SYSTEM TO WORK. LET F_1 BE THE EVENT THAT COMPONENT 1 FAILS. LET F_2 BE THE EVENT THAT COMPONENT 2 FAILS.

$$\rightarrow \boxed{1} \rightarrow \boxed{2} \rightarrow$$

A) THE EVENT THAT THE SYSTEM FAILS DURING THE DAY:

SINCE BOTH COMPONENTS NEED TO WORK, USE "or" $\cup$

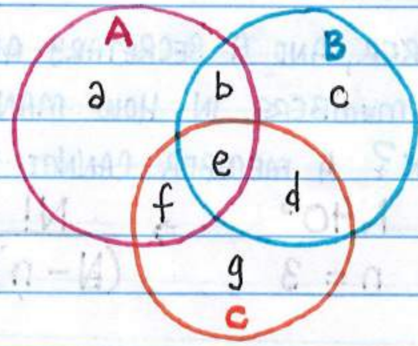
$F_1 \cup F_2$

B) THE EVENT THAT THE SYSTEM WORKS DURING THE DAY:

SINCE BOTH COMPONENTS NEED TO WORK, USE "and" $\cap$

$F_1^c \cap F_2^c$

Q7 HELEN IS BUYING A NEW CAR. SET A CONTAINS TOYOTAS. SET B ARE BLUE CARS. SET C ARE CARS WITH A MANUAL GEARBOX.



A) TOYOTA CARS THAT ARE BLUE WITHOUT MANUAL GEARBOX:

b

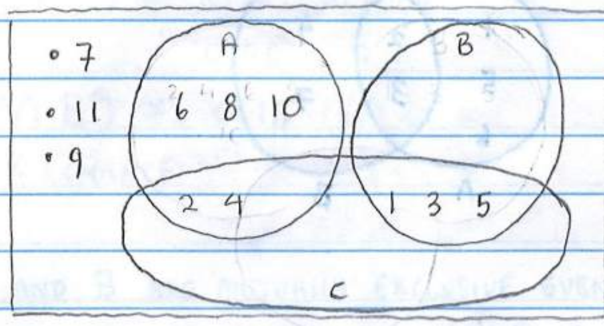
B) BLUE CARS THAT ARE NOT TOYOTAS

EITHER c OR d: $c \cup d$

C) CARS THAT ARE NEITHER TOYOTAS NOR HAVE A MANUAL GEARBOX

$a^c \cap b^c \cap d^c \cap e^c \cap f^c \cap g^c$

Q8 PLAYERS ON SOCCER TEAM HAVE SHIRTS LABELLED WITH NUMBERS 1 TO 11. SET A CONTAINS PLAYERS WITH EVEN NUMBERS. SET B HAS PLAYERS WEARING ODD NUMBERS LESS THAN 7. SET C CONTAINS DEFENDERS, WEARING NUMBERS LESS THAN 6.



A) $A \cap (B \cup C)$

$A : \{2, 4, 6, 8, 10\}$ $\Rightarrow B \cup C : \{1, 2, 3, 4, 5\}$

$B : \{1, 3, 5\}$ $\Rightarrow A \cap (B \cup C) : \{2, 4\}$

$C : \{1, 2, 3, 4, 5\}$

B) $(A^c \cup B^c) \cap C^c$

SINCE $A^c : \{7, 11\}$ $\Rightarrow A^c \cup B^c : \{7, 9, 11\}$

$B^c : \{\phi\}$

$C^c : \{6, 8, 10\}$

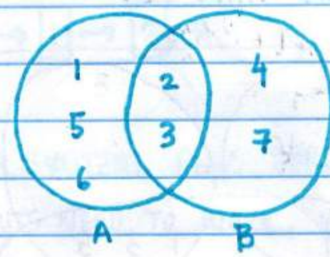
SO $(A^c \cup B^c) \cap C^c : \{6, 7, 8, 9, 10, 11\}$

$Q2$ A SAMPLE SPACE CONTAINS 7 SAMPLE POINTS. LET:

$P(1) = P(2) = P(3) = P(7) = 0.1$

$P(4) = P(5) = 0.05$

$P(6) = 0.5$



A) $P(A)$

$A: \{1, 2, 3, 5, 6\}$

$P(A) = P(1) + P(2) + P(3) + P(5) + P(6)$

$= 0.1 + 0.1 + 0.1 + 0.05 + 0.5$

$P(A) = 0.85$

$\therefore$ 85% CHANCE

B) $P(A \cap B)$

$A: \{1, 2, 3, 5, 6\}$ AND $B: \{2, 3, 4, 7\}$

$A \cap B: \{2, 3\}$

$P(A \cap B) = P(2) + P(3)$

$= 0.1 + 0.1$

$P(A \cap B) = 0.20$

$\therefore$ 20% CHANCE

C) $P(A \cup B)$

$A: \{1, 2, 3, 5, 6\}$ OR $B: \{2, 3, 4, 7\}$

$A \cup B: \{1, 2, 3, 4, 5, 6, 7\}$

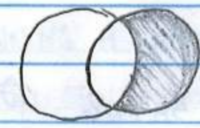
$P(A \cup B) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$

$= 0.1 + 0.1 + 0.1 + 0.05 + 0.05 + 0.5 + 0.1$

$P(A \cup B) = 1$

$\therefore$ 100% CHANCE

D) $P(A^c \cap B)$



$A^c : \{4, 7\}$

$B : \{2, 3, 4, 7\}$

$A^c \cap B : \{4, 7\}$

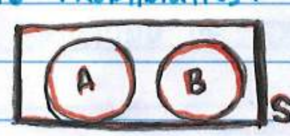
$P(A^c \cap B) = P(4) + P(7)$

$= 0.05 + 0.1$

$P(A^c \cap B) = 0.15$

$\therefore$ 15% CHANCE

Q^{10} IF A AND B ARE MUTUALLY EXCLUSIVE EVENTS WITH $P(A) = 0.45$ AND $P(B) = 0.45$, FIND THE FOLLOWING PROBABILITIES:



A) $P(A \cap B)$

SINCE A AND B ARE MUTUALLY EXCLUSIVE EVENTS, THE PROBABILITY OF BOTH HAPPENING AT ONCE IS 0

$P(A \cap B) = 0$

B) $P(A \cup B)$

FOR MUTUALLY EXCLUSIVE EVENTS, SUM PROBABILITIES FOR A AND B

$P(A \cup B) = P(A) + P(B) = 0.45 + 0.45$

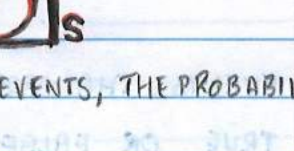
$= 0.90$

c) $P(A^c)$

FORMULA FOR $P(A^c) = 1 - P(A)$

$P(A^c) = 1 - P(A) = 1 - 0.45$

$= 0.55$

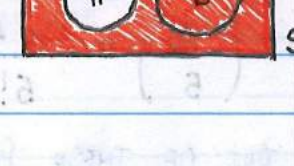


D) $P(B^c)$

FORMULA FOR $P(B^c) = 1 - P(B)$

$P(B^c) = 1 - P(B) = 1 - 0.45$

$= 0.55$



E) $P(A \cup B)^c$

ANOTHER WAY OF SAYING $P(A^c \cup B^c)$, FORMULA $P(A^c \cup B^c) = 1 - P(A \cup B)$

$$P(A \cup B)^c = 1 - P(A \cup B) = 1 - 0.90$$

$$= 0.10$$

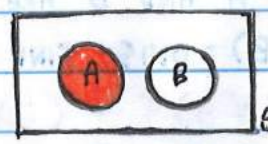
F) $P(A \cap B^c)$

FORMULA: $P(A \cap B^c) = P(A)(1 - P(B))$

$$= P(A) - P(A \cap B)$$

$$= 0.45 - 0$$

$$= 0.45$$



Q11 EVENTS THAT ARE MUTUALLY EXCLUSIVE MUST ALSO BE COMPLEMENTARY: TRUE OR FALSE?

FALSE - COMPLEMENTARY EVENTS MUST BE MUTUALLY EXCLUSIVE, BUT MUTUALLY EXCLUSIVE EVENTS DON'T HAVE T

Q12 THE LOTTO CONTAINS 42 BALLS NUMBERED 1-42. EACH WEEK 5 ARE DRAWN WITHOUT REPLACEMENT. JACKPOT IS WON BY ANYONE WHO CHOOSES ALL 5 CORRECTLY. A RUNNER UP PRIZE IS WON BY ANYONE WHO CORRECTLY SELECTS 4 OUT OF 5 NUMBERS.

A) WHAT IS THE PROBABILITY THAT YOU WIN THE JACKPOT?

THE TOTAL NUMBER OF WAYS OF SELECTING 5 NUMBERS OUT OF 42:

$$\binom{42}{5} = \frac{42!}{5!\,37!} = 850,668$$

OUT OF THESE POSSIBLE COMBOS, ONLY 1 CORRESPONDS TO THE JACKPOT SO:

$$P(\text{JACKPOT}) = \frac{1}{\binom{42}{5}} = \frac{1}{850,668} = 1.176 \times 10^{-6}$$

B) WHAT IS THE PROBABILITY OF WINNING A RUNNER'S UP PRIZE?

TOTAL NUMBER OF WAYS OF SELECTING 4 NUMBERS OUT OF 5:

$$\binom{5}{4} = \frac{5!}{4!\,1!} = 5$$

THE PROBABILITY OF WINNING A RUNNER'S UP PRIZE:

$$P(\text{RUNNER UP}) = \frac{\binom{5}{4}\binom{42-5}{1}}{\binom{42}{5}} \doteq 2.174 \times 10^{-4}$$

Q13 FRED, BURT, CONRAD, OTTO, AND HUGH RUN A RACE. ALL RUNNERS HAVE THE SAME ABILITY, SO THE POSSIBLE ORDERING OF RUNNERS ARE ALL EQUALLY LIKELY.

A) WHAT IS THE PROBABILITY OF BURT WINNING?

THERE ARE 5 RUNNERS, AND 1 WINNER SO

$$P(\text{BURT}) = 1/5 = 0.20$$

B) WHAT IS THE CHANCE THAT BURT WINS AND OTTO COMES 2^{ND}?

TOTAL NUMBER OF WAYS OF SELECTING 5 RUNNERS OUT OF 5 RUNNERS:

$$\binom{5}{5} = \frac{5!}{5!\,0!} = 1$$

TOTAL NUMBER OF WAYS OF SELECTING 1 WINNER OUT OF 5 RUNNERS:

$$\binom{5}{1} = \frac{5!}{1!\,4!} = 5$$

SO THE PROBABILITY OF WINNING 1^{ST} PLACE IS

$$P(\text{FIRST}) = \frac{\binom{5}{5}}{\binom{5}{1}} = 1/5$$

WITH 4 RUNNERS LEFT COMPETING, PROBABILITY OF 2^{ND} PLACE IS:

$$P(\text{SECOND}) = \frac{\binom{4}{4}}{\binom{4}{1}} = 1/4$$

PROBABILITY OF BART FIRST, OTTO SECOND IS

$$P(\text{BART-OTTO}) = P(\text{BART}) \cdot P(\text{OTTO}) = \left(\tfrac{1}{5}\right)\left(\tfrac{1}{4}\right) = 1/20$$

Q[19] SUPPOSE A DIE IS LOADED SO THAT A 6 IS SCORED 4 TIMES MORE OFTEN THAN ANY OTHER SCORE, WHILE ALL OTHER SCORES ARE EQUALLY LIKELY.

A) WHAT IS THE PROBABILITY OF SCORING A TWO?

LET $P(1) = P(2) = P(3) = P(4) = P(5) = x$ AND

$$P(6) = 4x$$

WE KNOW THAT THE SUM OF ALL SIX SAMPLE POINTS MUST EQUAL TO 1 SO

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$x + x + x + x + x + 4x = 1$$

$$9x = 1$$

$$x = 1/9$$

HENCE

$$P(1) = P(2) = P(3) = P(4) = P(5) = 1/9 \text{ AND}$$

$$P(6) = 4x = 4(1/9)$$

$$= 4/9$$

$\therefore$ THE PROBABILITY OF SCORING A TWO IS $1/9$

B) WHAT IS THE PROBABILITY OF ROLLING A SIX?

LET $P(1) = P(2) = P(3) = P(4) = P(5) = x$

$$P(6) = 4x$$

SUM OF THE PROBABILITIES OF TOTAL NUMBER OF SAMPLE POINTS IS 1

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$x + x + x + x + x + 4x = 1$$

$$x = 1/9$$

HENCE

$$P(1) = P(2) = P(3) = P(4) = P(5) = x = 1/9$$

$$P(6) = 4x = 4/9$$

$\therefore$ PROBABILITY OF ROLLING A 6 IS $4/9$

Q15 ANGUS GOES TO 1 OF 2 COFFEE SHOPS IN TOWN. HE GOES TO STARBUCK'S 67% OF THE TIME, OTHERWISE HE GOES TO COSTA. HE BUYS A LATTE 72%, EITHER WAY.

A) ANGUS WENT INTO TOWN TODAY - WHAT IS PROBABILITY HE HAD A LATTE AT STARBUCKS?

LET S = STARBUCKS $P(S) = 0.67$

C = COSTA $P(C) = 0.33$

L = LATTE $P(L|C) = 0.72$ AND $P(L|S) = 0.72$

$$P(L \cap S) = P(S)P(S|T)$$
$$= (0.67)(0.72)$$
$$= 0.4824$$

B) ARE THE 2 EVENTS THAT GAVE JOINT PROBABILITY IN a) INDEPENDENT?

YES - HE BUYS A LATTE 72% OF THE TIME REGARDLESS OF THE COFFEE SHOP

C) GIVEN THAT ANGUS HAD A LATTE IN TOWN, WHAT IS THE PROBABILITY THAT HE DRANK AT COSTA?

SINCE DRINKING A LATTE IS INDEPENDENT OF COFFEE SHOP:

$$P(C|L) = \frac{P(C)P(L|C)}{P(C)P(L|C) + P(S)P(L|S)} = P(C)$$

$$= 0.33$$

D) WHAT IS THE PROBABILITY HE WENT TO STARBUCKS OR HAD A LATTE OR BOTH?

$$P(S \cup L) = P(S) + P(L) - P(S)P(L|S)$$
$$= 0.67 + 0.72 - (0.67)(0.72)$$
$$= 0.9076$$

Q^{10} BREATHALYSER TESTS ARE NOT ENTIRELY ACCURATE: 8% OF DRIVERS WHO HAVE NOT CONSUMED EXCESS ALCOHOL GIVE A POSITIVE READING; 10% OF DRIVERS WHO HAVE CONSUMED EXCESS ALCOHOL GIVE A NEGATIVE READING. SUPPOSE 18% OF DRIVERS ARE ACTUALLY ABOVE LEGAL LIMIT.

A) PROBABILITY THAT DRIVER IS INCORRECTLY CLASSIFIED AS OVER LIMIT:

DEFINE EVENTS

A: {ABOVE LIMIT} $\quad P(A) = 0.18$

A^c: {BELOW LIMIT} $\quad P(A^c) = 1 - P(A) = 0.82$

T: {TEST POSITIVE}

T^c: {TEST NEGATIVE}

GIVEN:

$P(T \mid A^c) = 0.08$ - BELOW LIMIT, TEST POSITIVE

$P(T^c \mid A) = 0.10$ - ABOVE LIMIT, TEST NEGATIVE

FIND:

$P(T^c \mid A^c) = 1 - P(T \mid A^c) = 1 - 0.08$

$= 0.92$ - BELOW LIMIT, TEST NEGATIVE

$P(T \mid A) = 1 - P(T^c \mid A) = 1 - 0.10$

$= 0.90$ - ABOVE LIMIT, TEST POSITIVE

THE PROBABILITY THAT DRIVER IS INCORRECTLY CLASSIFIED AS OVER LIMIT:

$$P(T \cap A^c) = P(T \mid A^c)P(A^c)$$
$$= (0.08)(0.82)$$
$$= 0.0656$$

B) PROBABILITY THAT DRIVER IS CORRECTLY CLASSIFIED AS OVER LIMIT:

$$P(T \cap A) = P(T \mid A)P(A)$$
$$= (0.90)(0.18)$$
$$= 0.162$$

C) PROBABILITY DRIVER GIVES A POSITIVE READING

$$P(T) = P(A)P(T|A) + P(A^c)P(T|A^c)$$
$$= (0.18)(0.90) + (0.82)(0.08)$$
$$= 0.2276$$

D) PROBABILITY THAT DRIVER IS BELOW LIMIT, GIVEN NEGATIVE READING:

$$P(A^c|T^c) = \frac{P(A^c)P(T^c|A^c)}{P(A^c)P(T^c|A^c) + P(A)P(T^c|A)}$$

$$= \frac{(0.82)(0.92)}{(0.82)(0.92) + (0.18)(0.10)}$$

$$= 0.9767$$

Q17 80% OF SCHOOLS SUSCRIBE TO CHANNEL 1 : 20% OF THE SUSCRIBERS NEVER USE CHANNEL 1, WHILE 40% CLAIM TO USE IT MORE THAN 5 TIMES WEEKLY. FIND PROBABILITY THAT A RANDOMLY SELECTED SCHOOL SUSCRIBES TO CHANNEL 1 AND USES IT MORE THAN 5 TIMES WEEKLY.

GIVEN

S: SUSCRIBE $\quad P(S) = 0.80$

S^c: DON'T SUSCRIBE $\quad P(S^c) = 0.20$

U: USE 5 TIMES WEEKLY

U^c: DON'T USE

$P(U|S) = 0.40$ - SUSCRIBE, USE 5 TIMES WEEKLY

$P(U^c|S) = 0.20$ - SUSCRIBE, NEVER USE

PROBABILITY THAT RANDOM SCHOOL SUSCRIBES AND USES CHANNEL 1:

$$P(U \cap S) = P(S)P(U|S)$$
$$= (0.80)(0.40)$$
$$= 0.32$$

Q[18] FACTORY A PRODUCES 4 TIMES AS MANY COMPUTERS AS FACTORY B. THE PROBABILITY AN ITEM PRODUCED BY FACTORY A IS DEFECTIVE IS 0.01 AND THE PROBABILITY A DEFECTIVE ITEM IS PRODUCED BY B IS 0.049. A COMPUTER RANDOMLY SELECTED IS DEFECTIVE. WHAT IS THE PROBABILITY IT CAME FROM FACTORY A?

DEFINE EVENTS

A : FACTORY A $\quad P(A) = 0.4$

B : FACTORY B $\quad P(B) = 0.1$

W : WORKS

W^c : DOES NOT WORK

GIVEN

$P(W^c|A^c) = 0.0001$ – DEFECTIVE ITEM PRODUCED BY A

$P(W^c|B) = 0.00049$ – DEFECTIVE ITEM PRODUCE BY B

FIND

$$P(A|W^c) = \frac{P(W^c|A)P(A)}{P(W^c|A)P(A) + P(W^c|B)P(B)}$$

$$= \frac{(0.0001)(0.4)}{(0.0001)(0.4) + (0.00049)(0.1)}$$

$$= 0.4494$$

Q^{15} 90% OF CHEQUING ACCOUNTS OPEN FOR AT LEAST A YEAR - REMAINING % OPEN FOR LESS THAN THAT. FOR THOSE OPEN LESS THAN A YEAR, CHEQUES RETURNED DUE TO INSUFFICIENT FUNDS IS 4% WHILE FOR THOSE OPEN FOR OVER A YEAR IT WAS 2%.

A) WHAT IS PROBABILITY CHEQUE WILL BE RETURNED?

Y: OVER A YEAR

Y^c: LESS THAN A YEAR

I: INSUFFICIENT FUNDS

I^c: SUFFICIENT FUNDS

GIVEN:

$P(Y) = 0.90$

$P(Y^c) = 0.10$

$P(I|Y) = 0.02$

$P(I|Y^c) = 0.04$

FIND:

$$P(I) = P(Y)P(I|Y) + P(Y^c)P(I|Y^c)$$

$$= (0.90)(0.02) + (0.10)(0.04)$$

$$= 0.022$$

$\therefore$ 2.2% "CHANCE OF CHEQUE RETURNED DUE TO INSUFFICIENT FUNDS

B) IF CHEQUE RETURNED DUE TO INSUFFICIENT FUNDS, WHAT IS PROBABILITY IT CAME FROM BANK ACCOUNT OPEN FOR OVER A YEAR?

FIND:

$$P(Y|I) = \frac{P(Y)P(I|Y)}{P(Y)P(I|Y) + P(Y^c)P(I|Y^c)}$$

$$= \frac{(0.90)(0.02)}{(0.90)(0.02) + (0.10)(0.04)}$$

$$= 0.8181$$

$\therefore$ 81.81% CHANCE OF RETURNED CHEQUE FOR BANK ACCOUNT OPEN FOR >1 YR

Q^{20} A QUIZ CONSISTS OF 4 MULTIPLE CHOICE QUESTIONS, EACH OF WHICH HAS 6 ANSWERS, ONLY ONE OF WHICH IS CORRECT. IF YOU MAKE RANDOM GUESSES FOR EACH QUESTION:

A) WHAT IS PROBABILITY THAT ALL 4 OF YOUR ANSWERS ARE INCORRECT?

THE PROBABILITY OF GETTING A SINGLE ANSWER CORRECT IS 1/6

C : CORRECT

C^c : INCORRECT

$$\Rightarrow P(C) = 1/6$$

$$P(C^c) = 1 - P(C) = 1 - 1/6$$

$$= 5/6$$

HENCE, THE PROBABILITY OF GETTING ALL 4 INCORRECT IS:

$$P(C^c) = (5/6)^4$$

$$= 0.4823$$

B) WHAT IS PROBABILITY OF GETTING ALL 4 ANSWERS CORRECT?

$$P(C) = (1/6)^4$$

$$= 0.00077$$

Q^{21} 81% OF PEOPLE ARRESTED ARE MALE, 12% UNDER THE AGE OF 18, AND 6% WERE FEMALE AND UNDER 18. LET M REPRESENT "MALE." AND U18 BEIN "UNDER 18".

A)

	M	M^c	ROW PROBABILITIES
U18	0.06	0.06	0.12
$U18^c$	0.75	0.13	0.88
COLUMN PROBABILITY	0.81	0.19	1

$M \cap U18$ - MALE UNDER 18 $\quad M^c \cap U18$ - FEMALE UNDER 18

$M \cap U18^c$ - MALE OVER 18 $\quad M^c \cap U18^c$ - FEMALE OVER 18

B) PERSON ARRESTED IS RANDOMLY CHOSEN. WHAT IS PROBABILITY THAT PERSON IS MALE OR UNDER 18?

FIND:

$$P(M \cup U18) = P(M) + P(U18) - P(M \cap U18)$$
$$= (0.81) + (0.12) - (0.06)$$
$$= 0.87$$

C) PROBABILITY THAT PERSON IN b) IS NEITHER MALE NOR UNDER 18?

FIND:

$$P(M^c \cap U18) = 1 - P(M \cup U18)$$
$$= 1 - 0.87$$
$$= 0.13$$

D) PROBABILITY THAT ALL PEOPLE ARRESTED ARE MALE AND OVER 18?

FIND:

$$P(M \cap U18^c) = 0.75$$

E) ARE M AND U18 MUTUALLY EXCLUSIVE EVENTS?

NO, BECAUSE $P(M \cap U18) \neq 0$

Q22 18% OF PEOPLE CONTRACT DISEASE A, 25% WILL CONTRACT DISEASE B, AND 74% OF PEOPLE WILL NOT CONTRACT EITHER DISEASE IN THEIR LIFETIME.

A)

DISEASE	A	A^c	ROW PROBABILITIES
B	0.17	0.08	0.25
B^c	0.01	0.74	0.75
COLUMN PROBABILITIES	0.18	0.82	1

B) PROBABILITY PERSON CONTRACTS DISEASE A OR DISEASE B?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.18 + 0.25 - 0.17$$
$$= 0.26$$

C) PROBABILITY THAT PERSON CHOSEN CONTRACTS ONLY 1 OF 2 DISEASES

$$P(A^c \cup B^c) = P(A^c \cap B) + P(A \cap B^c)$$
$$= 0.08 + 0.01$$
$$= 0.09$$

D) SUPPOSE PERSON CONTRACTS DISEASE A. WHAT IS PROBABILITY THEY CONTRACT DISEASE B?

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.17}{0.18}$$
$$= 0.9444$$

E) IF PERSON CONTRACTS DISEASE B, WHAT IS THE PROBABILITY THEY WILL ALSO CONTRACT DISEASE A?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.17}{0.25}$$
$$= 0.6800$$

Q23

A) IF THE KNOWLEDGE THAT EVENT A HAS OCCURED IMPLIES THAT A 2ND EVENT B CANNOT OCCUR, THEN EVENTS A AND B ARE:

DISJOINT / MUTUALLY EXCLUSIVE

B) IF EVENT A AND B ARE DISJOINT AND BOTH HAVE A PROBABILITY OF 0.1, THEN THE PROBABILITY THAT A OR B OCCURS IS:

$$P(A \cup B) = P(A) + P(B)$$
$$= 0.1 + 0.1$$
$$= 0.2$$

Q[24] 47% OF PRISONERS WHO TRY TO ESCAPE USE ROAD A, 40% USE ROAD B, AND 13% USE ROAD C. 64% OF THOSE WHO USED ROAD A WERE CAPTURED. 48% USING ROAD B WERE CAPTURED, AND 87% USING ROAD C WERE CAPTURED.

A) WHAT IS PROBABILITY THAT AN ESCAPED PRISONER IS NOT CAPTURED?

DEFINE EVENTS:

A : ROAD A $\Rightarrow P(A) = 0.47$

B : ROAD B $\Rightarrow P(B) = 0.40$

C : ROAD C $\Rightarrow P(C) = 0.13$

K : CAPTURED

K^c : NOT CAPTURED

$P(K|A) = 0.64$

$P(K|B) = 0.48$

$P(K|C) = 0.87$

FIND

$$P(K) = P(A)P(K|A) + P(B)P(K|B) + P(C)P(K|C)$$
$$= (0.47)(0.64) + (0.4)(0.48) + (0.13)(0.87)$$
$$= 0.6059$$

$\therefore$ 60.59% CHANCE OF BEING CAPTURED

$$P(K^c) = 1 - P(K)$$
$$= 1 - 0.6059$$
$$= 0.3941$$

$\therefore$ 39.41% CHANCE OF NOT BEING CAPTURED

B) PROBABILITY A CAPTURED PRISONER USED ROAD A?

$$P(A|K) = \frac{P(A)P(K|A)}{P(K)} = \frac{(0.47)(0.64)}{0.6059}$$
$$= 0.4965$$

C) PROBABILITY THAT AN UNCAPTURED PRISONER USED ROAD C?

$$P(C|K^c) = \frac{P(C)P(K^c|C)}{P(K^c)}$$

$$= \frac{P(C)P(1 - P(K|C))}{P(K^c)} \Rightarrow \text{SINCE } P(K^c|C) = 1 - P(K|C)$$

$$= \frac{(0.13)(1 - 0.87)}{0.3941} = 0.0429$$

∴ 4.29 % CHANCE AN UNCAPTURED PRISONER USED ROAD C

Q25 GAMBLER FACES 3 SLOT MACHINES - A, B, C - WHICH HE RANDOMLY CHOOSES AND PLAYS ONCE. PROBABILITY OF WINNING USING MACHINE A IS 0.84, 0.54 FOR MACHINE B, AND 0.79 FOR MACHINE C.

A) WHAT IS THE PROBABILITY OF THE GAMBLER WINNING?

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$P(A) = P(B) = P(C) = 1/3$ - PROBABILITY OF PICKING A, B, OR C

$P(W|A) = 0.84$ - WINNING USING MACHINE A

$P(W|B) = 0.54$ - WINNING USING MACHINE B

$P(W|C) = 0.79$ - WINNING USING MACHINE C

FIND

$$P(W) = P(A)P(W|A) + P(B)P(W|B) + P(C)P(W|C)$$

$$= (\tfrac{1}{3})(0.84) + (\tfrac{1}{3})(0.54) + (\tfrac{1}{3})(0.79)$$

$$P(W) = 0.7233$$

B) IF HE WINS, WHAT IS THE PROBABILITY HE PLAYED MACHINE C?

$$P(C|W) = \frac{P(C)P(W|C)}{P(W)} = \frac{(1/3)(0.79)}{0.7233}$$

$$P(C|W) = 0.3641$$

C) IF HE DOESN'T WIN, WHAT IS PROBABILITY HE PLAYED MACHINE A?

$$P(A|W^c) = \frac{P(A)P(W^c|A)}{P(W^c)}$$

$$= \frac{P(A)(1-P(W|A))}{(1-P(W))}$$

$\Rightarrow P(W^c|A) = 1 - P(W|A)$

$\Rightarrow P(W^c) = 1 - P(W)$

$$= \frac{(1/3)(1-0.84)}{(1-0.7233)} = 0.1928$$

Q[26] SAND USED FOR CASTINGS IS TOO DRY 5.1% OF THE TIME AND TOO WET 1.5% OF THE TIME. DEFECTIVE CASTINGS OCCUR 0.24% OF THE TIME WHEN THE SAND HAS CORRECT LEVEL OF MOISTURE; 5.2% WHEN SAND IS TOO DRY; 33.2% WHEN IT IS TOO WET.

A) WHAT IS PROBABILITY THAT THE CASTING IS CORRECT?

$P(D) = 0.051$ $\quad P(X|D) = 0.052$

$P(W) = 0.015$ $\quad P(X|W) = 0.332$

$P(X|C) = 0.0024$

C : CORRECT LEVEL OF MOISTURE

X : DEFECTIVE

X^c : NOT DEFECTIVE

$P(X^c|D) = 1 - P(X|D) = 0.948$

$P(X^c|W) = 1 - P(X|W) = 0.668$

$P(X^c|C) = 1 - P(X|C) = 0.9976$

$P(C) = 1 - P(D) - P(W) = 0.934$

FIND :

$P(X^c) = P(C)P(X^c|C) + P(W)P(X^c|W) + P(D)P(X^c|D)$

$= (0.934)(0.9976) + (0.015)(0.668) + (0.051)(0.948)$

$= 0.9901$

B) IF THE CASTING IS GOOD, WHAT IS PROBABILITY SAND IS TOO WET?

$$P(W|X^c) = \frac{P(W)P(X^c|W)}{P(X^c)} = \frac{(0.015)(0.668)}{0.9901}$$

$$= 0.0101$$

C) IF CASTING IS GOOD, WHAT IS PROBABILITY THAT THE MOISTURE IS CORRECT?

$$P(C|X^c) = \frac{P(C)P(X^c|C)}{P(X^c)} = \frac{(0.934)(0.9976)}{0.9901}$$

$$= 0.9410$$