Problem 1.

a) (3 marks) An investment of $10,000 now will return an actual income of $1000 in one year, and actual incomes in later years, forever, that increase at the inflation rate of 3% per year. Calculate the real interest rate that is earned.

Given a geometric gradient $A_A = \$1000$ with $g = 3\%$, convert it into real dollar; i.e., $A_R = \frac{A_A}{(1+f)^N}$ for year N. Thus, $A_R = \frac{1000}{1.03} = \$970.873$

This $A_R = \$970.873$ is an annuity that lasts forever; hence

$$PW_R = -10000 + 970.873(P/A, i_R^*, N \to \infty) = 0$$

$$0 = -10000 + \frac{970.873}{i_R^*} \Rightarrow i_R^* = \frac{970.873}{10,000} = 9.71\%$$

b) Joe's savings account pays an actual interest rate of 4% per year. Joe pays tax on the interest at the rate of 25%. The inflation rate is 2.0%

b-i) (1 mark) What is Joe's actual, after-tax interest rate on his savings?

$$i_{A,\,after\text{-}tax} = (1-t)\, i_{A,\,bef\text{-}tax} = (1-0.25)\,4\% = 3\%$$

b-ii) (1 mark) What is Joe's real, after-tax interest rate on his savings?

$$i_{R,\,after\text{-}tax} = \frac{1 + i_{A,\,after\text{-}tax}}{1+f} - 1 = \frac{1.03}{1.02} - 1 = 0.98\%$$

b-iii) (2 marks) What actual interest rate on the savings account would make Joe's real, after-tax interest rate equal to zero? The inflation rate is still 2% and the tax rate is 25%.

Find $i_{A,\,bef\text{-}tax}$ such that $i_{R,\,after\text{-}tax} = 0$

$$i_{R,\,after\text{-}tax} = \frac{1 + i_{A,\,after\text{-}tax}}{1+f} - 1 = 0 \Rightarrow i_{A,\,after\text{-}tax} = f = 2\%$$

$$i_{A,\,bef\text{-}tax}(1-t) = i_{A,\,after\text{-}tax} \Rightarrow i_{A,\,bef\text{-}tax} = \frac{i_{A,\,after\text{-}tax}}{1-t} = \frac{2\%}{1-0.25} = 2.666\%$$

Problem 2.

a) (4 marks)Production of a new consumer electronics device is expected to start at 100,000 units next year, and increase by 5% per year for the following 9 years (i.e. 10 years of production altogether). The before-tax present worth of all costs (first costs, less PW of real salvage values, plus real labour, materials, etc. costs) has been estimated to be \$ 12 million. The real, before-tax MARR is 10%. Write an expression for the real-dollar, before-tax levelized cost of the device. *You should use interest factor symbols whenever possible; be sure to show all known parameter values.*

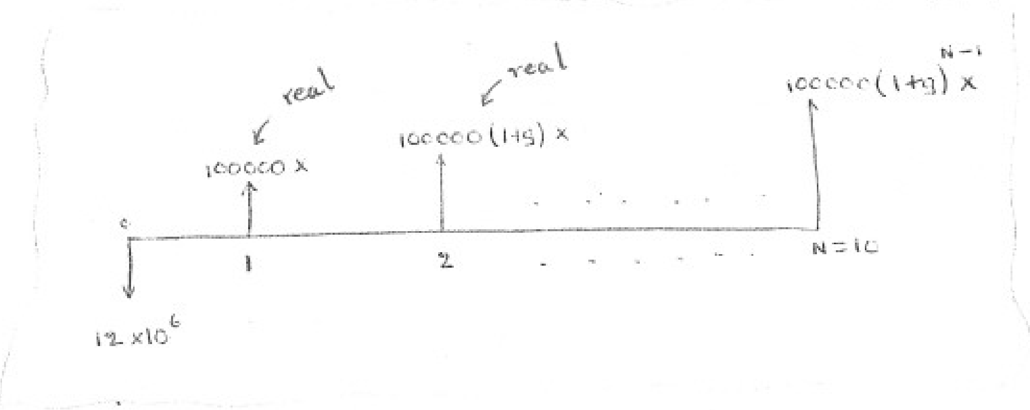


Diagram is not required !!

Data:

$i_{R,\,bef\text{-}tax} = 10\%$

$g = 5\%$

x: selling price / levelized cost

Annual revenue $= 100000(1+g)^{n-1}x$ at year n : Geometric gradient !!

PW (Revenues) = PW (Costs)

$$\sum_{t=1}^{10} \frac{100\,000(1+g)^{t-1}x}{(1+i_{R,\,bef\text{-}tax})^t} = 12 \times 10^6 \Rightarrow x = \boxed{\frac{12 \times 10^6}{\sum_{t=1}^{10} \frac{100\,000(1+g)^{t-1}}{(1+i_{R,\,bef\text{-}tax})^t}}}$$

Second solution:

PW (Costs) = PW (Revenues)

12×10^6 = PW (Revenues as a gradient)

$12 \times 10^6 = 100\,000\,(P/A,\, g = 5\%,\, i_R = 10\%,\, 10)\, X$

$$x = \frac{12 \times 10^6}{100\,000\,(P/A,\, g = 5\%,\, i_R = 10\%,\, 10)}$$

2 b) (4 marks) Machine 1 costs \$20 000 and will have a salvage value of \$10 000 in one year. Machine 2 costs \$30 000 and will have a salvage value of \$25 000 in one year. You must buy one of the two machines. The MARR is 15%. Calculate any relevant IRR and recommend which machine you should buy.

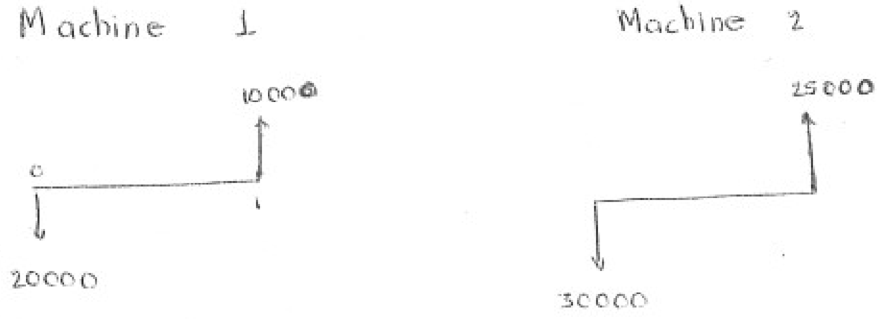


Diagram is not required

We need to find the IRR for the incremental investment

Current defender: Machine 1

Challenger: Machine 2

Incremental investment: Machine 2 - Machine 1

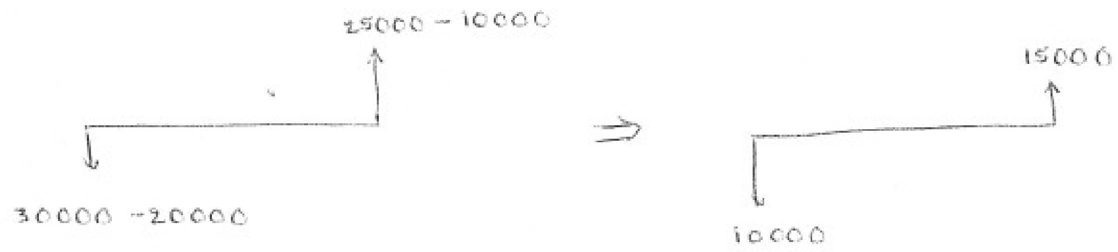


Diagram is not required

$$PW_{2-1} = -10000 + 15000\,(P/F, i^*, 1) = 0$$

$$0 = -10000 + \frac{15000}{1+i^*} \Rightarrow i^* = \frac{15000}{10000} - 1 = 50\%$$

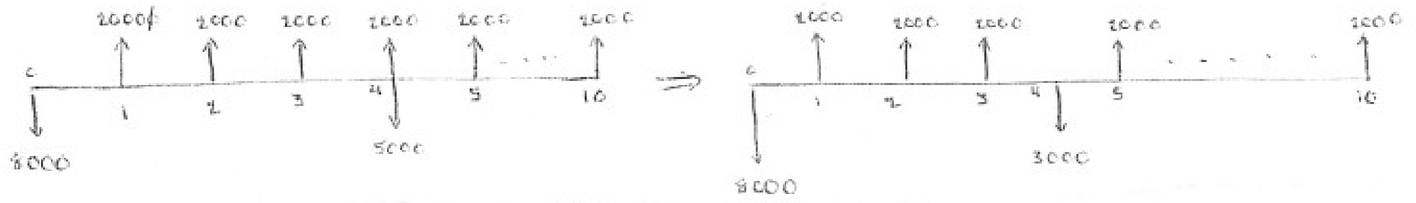
Criterion: $IRR_{2-1} = 50\% > MARR = 15\%$

Thus, the incremental investment is acceptable.

Buy Machine 2.

Problem 3.

(a) (4 marks) A proposed investment has a first cost of $8,000, followed by a cash inflow of $2,000 per year for 10 years, and a special one-time expense of $5,000 at the end of the fourth year. *Without actually solving for all internal rates of return,* state an upper limit on the number of positive internal rates of return, and give a justification for your answer.



Diagrams are not required!!

Signs: $\underbrace{- \;\; +}_{\text{change 1}} \;\; + \;\; \underbrace{+ \;\; -}_{\text{change 2}} \;\; \underbrace{+}_{\text{change 3}} \;\; + \cdots\cdots +$

There are 3 changes of sign

By Descartes rule, there can be up to 3 positive roots

(b) (4 marks) Use the information from part (a) to write out an equation, which, if solved, would give the approximate external rate of return, *i*, for this investment. The MARR is 10%. *Do not solve the equation. Use interest factor symbols whenever possible; show all known parameter values.*

1) Solution 1:

Using FW (receipts at MARR) = FW (disbursments at unknown i_E^*)

$$FW = \underbrace{2000(F/A, 10\%, 10)}_{\text{receipts}} - 8000(F/P, i_E^*, 10) - 5000(F/P, i_E^*, 6) = 0$$

disbursments

2) Solution 2: NOTE: only solution 2 is acceptable this year (not solution 1).

Using FW(net positive cash flows at MARR) = FW(net negative cash flows at i_E^*)

$$FW = \underbrace{2000(F/A, 10\%, 10) - 2000(F/P, 10\%, 6)}_{+\text{ cash flow}} - \underbrace{8000(F/P, i_E^*, 10) - 3000(F/P, i_E^*, 6)}_{\text{negative cash flow}} = 0$$

NOTE: The positive cash flows may also be treated with 2 equivalences:

i) one for cash 1–3 ; and ii) another for cash 5–10

3 (c) (2 marks) Use the information from part (a) to *calculate the payback period of the proposed investment.* Assume that the $2000 per year amounts and the $5000 amount actually occur continuously throughout each year and not at the ends of years. If the company has a policy requiring a six year payback, then *state whether the investment is acceptable by this criterion.*

Because of cash flows in year 4, the annual inflows are not constant; hence, we should use a kind of accumulated benefits until $5000 are recovered.

Year	Annual inflow	Accumulated Benefit	Cost to be recovered
1	2000	2000	6000
2	2000	4000	4000
3	2000	6000	2000
4	-3000	3000	5000
5	2000	5000	3000
6	2000	7000	1000
7	2000	9000	—

} * Recovered

After year 6, only $1000 are to be recovered. Since cash flows occur continuously, a half year is required to pay back. Thus, the payback period is **6.5** years. Since the company requires a six-year payback, the investment is not acceptable.

Students do not need to state this table to find the payback period.

Problem 4. (4 marks) A and B are two mutually exclusive projects, and "do nothing" is a possibility. A has the lower first cost. Each row of the following table gives a different relationship among MARR, IRR(A) and Incremental IRR(B-A).

In each row, write "*A*","*B*", "*none*", *or "cannot decide,"* where "cannot decide" means that there is insufficient information in the "Relationship" column.

Relationship	Decision
IRR(B-A) < IRR(A) < MARR	None
IRR(A) < MARR < IRR(B-A)	cannot decide
IRR(B-A) < MARR < IRR(A)	A
MARR < IRR(B-A) < IRR(A)	B

Explanations are not required

Problem 5. Environmetrics is considering a possible investment in a $20,000 soil testing device. Environmetrics pays taxes at a 40% rate and the CCA rate for the soil testing device is 30%. Environmetrics will use the device for only one year, when it will be sold for a salvage value of $15,000. The difference between revenue and costs during the year of use (i.e., net revenue) will be $8000, before tax.

a) (3 marks) Calculate the ***before-tax*** IRR.

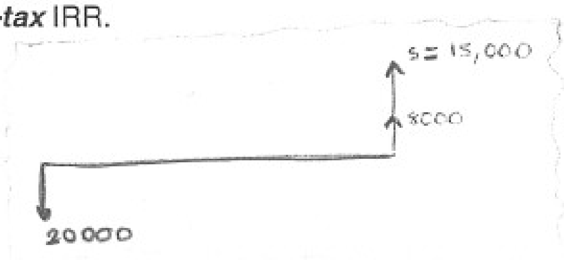


Diagram is not required

$$PW_{bef\text{-}tax} = -20000 + (8000+15000)(P/F,\ i^*_{bef\text{-}tax},\ 1) = 0$$

$$-20000 + \frac{23000}{1+i^*_{bef\text{-}tax}} = 0$$

$$i^*_{bef\text{-}tax} = \frac{23000}{20000} - 1 = 15\%$$

b) (6 marks) Write an equation for the ***exact after-tax*** IRR. *Do not solve it ! Be sure to display the dependence of all terms and factors on the unknown IRR.*

it means "function of"; it's not a factor

$$PW_{after\text{-}tax} = -20000\ CCTF_{new}(i^*_{after\text{-}tax}) + 15000\ CCTF_{old}(i^*_{after\text{-}tax})\,(P/F,\ i^*_{after\text{-}tax},\ 1) + \dots$$

$$\dots + 8000\,(1-t)\,(P/F,\ i^*_{after\text{-}tax},\ 1) = 0$$

CCTF is from the old text.
CCTF-old = CSF.
CCTF-new=CTF .

$$-20000\left[1 - \frac{td\,(1+i^*/2)}{(i^*+d)(1+i^*)}\right] + 15000\left[1 - \frac{td}{i^*+d}\right](P/F, i^*, 1) + 8000\,(1-t)\,(P/F, i^*, 1) = 0$$

Solve for $i^*_{after\text{-}tax}$ with $t = 0.4$, $d = 0.3$

The computation $i_{after\text{-}tax} = (1-t)\, i_{bef\text{-}tax}$ is just an approximation, and hence, not acceptable.

Problem 6.

(a) (5 marks) Suppose that the government has just announced that the CCA rate for computing equipment will increase from 30% to 40%. Do you think that the managers of businesses that buy a lot of computers will be pleased or displeased with this announcement? Explain.

The managers will be pleased because a larger CCA rate means earlier tax savings, which is an improvement for the company because of the time value of money.

[Alternative, math-oriented answer:]

For an investment of \$P in equipment with CCA rate d, the P.W. of tax savings $= \frac{t \cdot d}{i+d} \cdot \left(\frac{1+i/2}{1+i}\right) = \frac{t}{\left(\frac{i}{d}+1\right)} \cdot \left(\frac{1+i/2}{1+i}\right)$

Therefore, if d increases, then the PW of tax savings also increases, a good thing for the company.

(b) (5 marks) A company uses the IRR method to screen proposed investments, with an MARR that is set equal to the actual, after-tax weighted average cost of capital. Is it possible that a proposed investment would be accepted, if it has an actual, after-tax IRR less than the owners' minimum expectation of return on equity? Why or why not?

after-tax

$$MARR = w_d \cdot i_d + w_e \cdot i_e, \quad \text{with} \quad w_d + w_e = 1,$$

and $i_d < i_e$. $\therefore$ $i_d < MARR < i_e$.

A proposed investment is acceptable if its IRR>MARR.

$\therefore$ it is possible that

$$MARR < IRR < i_e.$$

Problem 7. (9 marks) Sam is considering buying a new lawnmower. He has a choice between a "Lawn Guy" mower or a Bargain Joe's "Clip Job" mower. Sam has a before-tax, real MARR of 10%. The mowers' salvage values at the end of their respective service lives are both zero. Other data is as follows (all cash flows are before-tax and real):

	Lawn Guy	Clip Job
First Cost	\$350	\$120
Life	12 years	4 years
Annual gas costs	\$60	\$40
Annual maintenance costs:	\$30	\$60

Determine which alternative is preferable using a before-tax **present worth** comparison. State a key assumption that you must make to solve this problem.

To make the comparison with PW, the options must have the same lives. Therefore, the assumption to be made is that the lawnmower is needed indefinitely, and then we can use repeated lives.

More precisely, assume that a lawnmower is needed for at least 12 years, and the second and third copies of the Clip Job will have the same cash flows as the first one.

1) Lawn Guy

$PW = 350 + (60+30)(P/A, 10\%, 12)$

$= 350 + 90(6.8137) = \$963,233$

2) Clip job (repeat cash flows 3 times)

$PW = 120 + 120(P/F, 10\%, 4) + 120(P/F, 10\%, 8) + (60+40)(P/A, 10\%, 12)$

$= \underbrace{120}_{\text{first cost first life}} + \underbrace{120(0.68301)}_{\text{first cost second life}} + \underbrace{120(0.46651)}_{\text{first cost third life}} + \underbrace{100(6.8137)}_{\text{net annual cost through 12 years}} = \$939.312$

Criterion: Choose the least costly; hence, choose the Clip job

Students must use PW; If somebody uses AW, then penalize by 5 marks

If somebody uses AW, then repeated lives are not needed explicitly in the computation

Problem 8.

(a) (7 marks) All cash flows in this problem are real and before-tax. A new delivery vehicle costs \$30 000. Its maintenance costs are mostly covered under guarantee for the first 5 years, but there are required inspections costing \$500 per year for these 5 years. After the first 5 years, maintenance costs are expected to be \$2500 in the sixth year, and increasing by 10% per year thereafter. The vehicle's real salvage value can be estimated from declining balance depreciation at a rate of 30% per year. Using a before-tax, real MARR of 10%, write two expressions for the real, before-tax EAC of the vehicle, as a function of *n*, the unknown number of years of ownership; one expression is to be for *n* in the range 1 to 5, and the other for *n*>5. State what you would do with your expressions to determine the economic life of the vehicle. *Your expressions should include interest factor symbols whenever possible; be sure to display all known parameter values.*

they can also use $P(A/P,i,N) - S(A/F,i,N)$

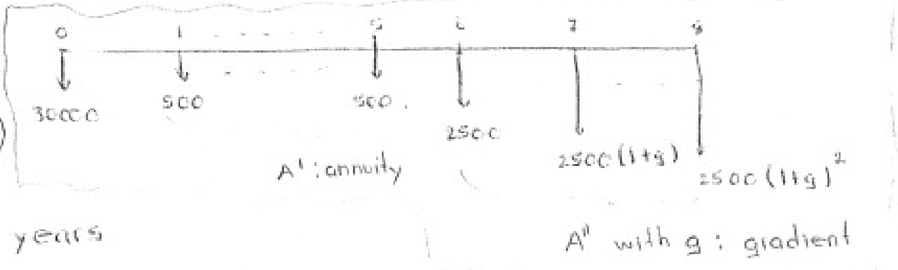


Diagram is not required

For $n = 1, \ldots, 5$ years

$$EAC = (P-S)(A/P, 10\%, n) + Si + A'$$

$$= \left[30000 - 30000(1-d)^n\right](A/P, 10\%, n) + 30000(1-d)^n (0.1) + 500$$

For $n = 6, 7, 8$ years

$$EAC = (P-S)(A/P, 10\%, n) + Si + A'(P/A, 10\%, 5)(A/P, 10\%, n) + A''(P/A, g=10\%, 10\%, n-5)(P/F, 10\%, 5)(A/P, 10\%, n)$$

$$EAC = \left[30000 - 30000(1-d)^n\right](A/P, 10\%, n) + 30000(1-d)^n (0.1) + 500(P/A, 10\%, 5)(A/P, 10\%, n) + \ldots$$

$$\ldots + 2500(P/A, g=10\%, i, n-5)(P/F, 10\%, 5)(A/P, 10\%, n)$$

- Find the n that provides the least cost

(2)

(b) (3 marks) For an equipment replacement decision, the EACs of keeping the Defender for several different numbers of years have been estimated to be as shown in the following table:

Number of years to keep Defender	1	2	3	4
EAC, in \$ per yr.	3,225	2,892	3,154	3,479

The EAC of the Challenger, at its economic life, is \$3,100 per year. Determine whether or not to replace the Defender now. Explain your reasoning.

The EAC for challenger is \$3,100.

The " " defender is \$2,892 if it is kept 2 years.

Therefore, the defender should not be replaced now.

Problem 9. Westmount Waxworks (WW) is considering buying a van to make deliveries of their product. The van has a before-tax first cost of $25 000 and will be used for an 8 year period before being salvaged. WW estimates that the operating costs will be $5000 per year, based on current prices. The actual operating costs, however, are expected to rise at the inflation rate of 5% per year.

Real before-tax savings due to the delivery van are $20 per delivery (which will remain constant over 8 years), with 500 deliveries in the first year, increasing by 100 deliveries per year thereafter. WW a tax rate of 35% and uses an after-tax actual MARR of 15.5%. The CCA rate for vans is 30%. WW uses a declining balance method with a depreciation rate of 25% for estimating real, before-tax salvage values for the van.

a) (1 mark) Calculate the real, after-tax MARR.

$$MARR_{R,\ after\text{-}tax} = \frac{1 + MARR_{A,\ after\text{-}tax}}{1+f} - 1 = \frac{1+0.155}{1+0.05} - 1 = 10\%$$

CCTF is from the old text.
CCTF-old = CSF.
CCTF-new=CTF .

b) (2 marks) Calculate the new and old CCTFs.

$$CCTF_{new} = \left[1 - \frac{td(1+i_A/2)}{(i_A+d)(1+i_A)}\right] = \left[1 - \frac{(.35)(.3)(1.0775)}{(0.155+.3)(1+.155)}\right] = 0.7847$$

$MARR_A = i_A = 15.5\%$ is to be used for the CCTF's factors

$$CCTF_{old} = \left[1 - \frac{td}{i_A+d}\right] = \left[1 - \frac{(.35)(.3)}{(0.155+0.3)}\right] = 0.7692$$

c) (5 marks) Calculate the after-tax present worth of the savings minus operating costs (i.e., leave out first cost and salvage value in this part).

$$PW_{R,after\text{-}tax} = -(1-t)5000(P/A,10\%,8) + (1-t)\Big[\underbrace{10000}_{\$20 \times 500} + \underbrace{2000}_{\$20\times 100}(A/G,10\%,8)\Big](P/A,10\%,8)$$

$$PW_{R,after\text{-}tax} = -(0.65)5000(5.3349) + (0.65)\left[10000+2000(3.0045)\right](5.3349)$$

$$= \$38175.749$$

d) (2 marks) Calculate the after-tax present worth of the first cost.

$$PW_{R,\ after\text{-}taxes} = -25000\ CCTF_{new} = -25000\ (0.7847) = -\$19617.5$$

e) (2 marks) Calculate the after-tax present worth of the salvage value.

$$PW_{R,after\text{-}taxes} = 25000(1-0.25)^8\ CCTF_{old}\ (P/F,10\%,8) = 25000(0.1)(0.7692)(0.46651)$$
$$= \$898.085$$

f) (3 marks) Use your answers above to calculate the total after-tax present worth of the investment in the van, and make a recommendation.

$$PW_{R,after\text{-}tax} = 38175.749 - 19617.5 + 898.055 = \$19456.304$$

Since PW > 0, WW should purchase the van!!

NOTE: For the CCTF's factors d = 0.3; for salvage value, d = 0.25

Problem 10: A company needs to install electric generators – either coal-steam generators, or wind-turbine generators (with batteries to smooth out the fluctuations due to changing wind speeds). The following table gives the expected (most likely) estimates of the costs of the two two types of generation, and some other information. Salvage values for both types are zero, at their service lives. The real, before-tax MARR is 10% per year.

Real Costs of Generation, Before-tax, and other information

Generation Type	Size (MW)	Capital (2005$) $/MW	Fixed Op. (2005$) ($/MW)/yr.	Fuel & Mainten. (2005$) $/MWh	Service Life (Years)
Coal-steam	100	1,500,000	40,000	60	30
Wind-turbine	5	3,800,000	12,000	0	15

(a) (8 marks) Let x = the number of hours per year of operation of a generator. Write two expressions, each as a function of x, for the real, before-tax equivalent annual costs *per MW* of the coal-steam generator and the wind-turbine. Determine the breakeven number of hours per year between the two types of generation, and state which type is better for base load (steady demand of 8760 hours per year).

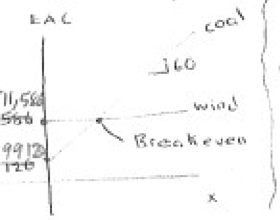
Coal:
- First cost = $\$1.5 \times 10^6$
- Annual fixed cost = $40000 /MW per year
- Production cost = $\dfrac{\$60}{MWh} \cdot \dfrac{x \text{ hours}}{1 \text{ year}} = \$60x$ /MW per year

$$EAC_R = 1.5\times10^6 (A/P, 10\%, 30) + 40000 + 60x$$
$$= 1.5\times10^6 (0.10608) + 40000 + 60x = 199120 + 60x$$

Wind:
- First cost = $\$3.8 \times 10^6$
- Annual fixed cost = $12,000 /MW
- Production cost = $0

$$EAC_R = 3.8\times10^6 (A/P, 10\%, 15) + 12000 = 3.8\times10^6 (0.13147) + 12000 = \$511,586$$

Breakeven point (Graph is not required)



$EAC_{coal} = EAC_{wind}$

$199120 + 60x = 511,586$

$x = \dfrac{511586 - 199120}{60} = 5207.76$ hours

For $[0, 5207.766)$ hours, use coal

For $[5207.766, 8760]$ hours, use wind

So, for base load use wind!

(b) (4 marks) Suppose that the fuel price is uncertain – it could be larger or smaller than shown in the table, by 25%. The real, before-tax MARR is also uncertain – it could be as low as 8% or as high as 12%. Write parameter values in the following table, to define three scenarios. ("Pessimistic" means "makes the EAC per MW of coal greater;" "Optimistic" means the opposite.)

Parameter	Pessimistic	Expected	Optimistic
Fuel Price	75	60	45
real, before-tax MARR	12%	10%	8%

Explanations / Computation for those numbers are not required

Problem 11: Examine the income statement and balance sheet for Waterloo Widget Inc. below, with several entries blanked out, and answer the questions which follow.

Income Statement for Waterloo Widget Inc.	**(in 1000s of $)**
Revenues	
Sales	2010.0
-Cost of Goods Sold	1304.0
Net Revenue from Sales	706.0
Expenses	
Operating Expenses	\$590 ?
Depreciation Expenses	30.0
Interest Expenses	11.0
Total Expenses	\$631 ?
Profit Before Taxes	75.0
-Income tax @ 30%	22.5
Profit After Taxes	52.5

Balance Sheet for Waterloo Widget Inc.	**(in 1000s of $)**
Assets	
Current Assets	
Cash	153.5
Inventory	48.0
Accounts Receivable	\$56 ?
Long-Term Assets	
Equipment	150.0
-accumulated depreciation	30.0
Total long-term assets	120.0
Total Assets	\$377.5 ?
Liabilities and Owners' Equity	
Current Liabilities	
Accounts Payable	25.0
Long-Term Liabilities	
Loan	100.0
Owners' Equity	
Shares	200.0
Retained Earnings	52.5
Total Liabilities & Owners' Equity	377.5

Students must provide support for those numbers - see next description.

(a) (4 marks) Calculate the blanked out values for operating expenses, total expenses, accounts receivable, and total assets.

- Total Expenses:

Profit before taxes = Net Revenues − Total Expenses

\$75 = 706 − Total Expenses

Total expenses = \$706 − \$75 = \$631

- Operating Expenses:

Total Expenses = Operating Expenses + Depreciation Expenses + Interest Expenses

\$631 = Operating Expenses + \$30 + 11

Operating Expenses = \$631 − \$30 − \$11 = \$590

Total Assets

Total Assets = Total Liabilities & Owner's Equity

Total Assets = \$377.5

Accounts Receivable:

Total Assets = Current Assets + Long-term Assets

\$377.5 = Current Assets + 120

Current Assets = \$377.5 − 120 = \$257.5

Current Assets = Cash + Inventory + Accounts Receivable

\$257.5 = 153.5 + 48 + Accounts Receivable ⟹ 257.5 − 153.5 − 48 = \$56

(b) (2 marks) If the owners' minimum expectation for after-tax return on equity is 15%, calculate the after-tax, weighted average cost of capital for Waterloo Widget.

$i_{e,\text{after-tax}} = 15\%$

$t = 30\%$

$$WCC_{\text{after-tax}} = W_d\, i_{d\,\text{after-tax}} + W_e\, i_{e\,\text{after-tax}}$$

$$= W_d (1-t)\, i_d + W_e\, i_e$$

$$W_d = \frac{\text{Debt}}{\text{Debt} + \text{Equity}} = \frac{125}{377.5} = 0.3311$$

Debt = Accounts Payable + Loan

25 + 100

$$W_e = \frac{\text{Equity}}{\text{Debt} + \text{Equity}} = \frac{252.5}{377.5} = 0.6688$$

$$i_d = \frac{\text{Interest}}{\text{Debt}} = \frac{11}{125} = 8.8\%$$

acceptable alternative:

$i_d = \frac{11}{100} = 11\%$

$w_d = \frac{100}{100 + 252.5}$, $w_e = \frac{252.5}{352.5}$

$$WCC_{\text{after-tax}} = (0.3311)(1-.3)(8.8\%) + (0.6688)(15\%) = 12.071\%$$