

Heat Transfer

Tutorial # 4

Roberto Ortiz-Fernandez

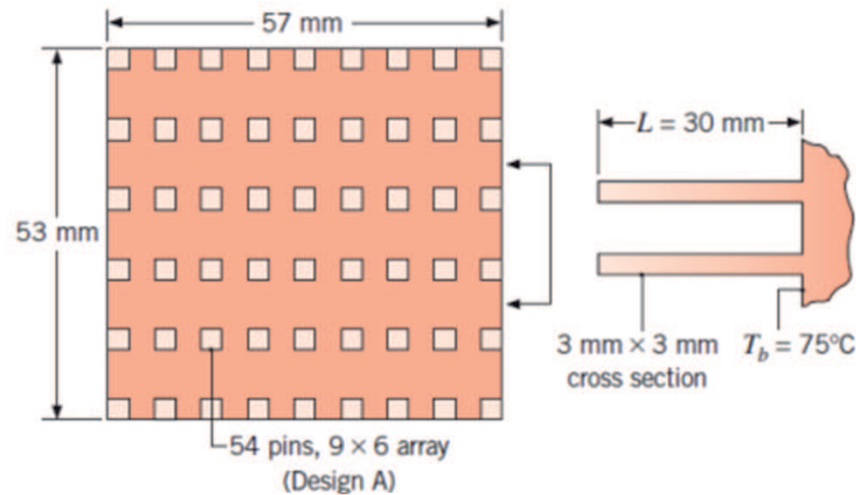
Office hours: Every Tuesday from 1:30pm to
2:30pm (CBY A0505)

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Exercise # 1

Because of the large number of devices in today's PC chips, finned heat sinks are often used to maintain the chip at an acceptable operating temperature. Two fin designs are to be evaluated, both of which have base (unfinned) area dimensions of 53mm x 57mm. The fins are of square cross-section and fabricated from an extruded aluminum alloy with a thermal conductivity of 175W/mK. Cooling air may be supplied at 25°C, and the maximum allowable chip temperature is 75°C. Other features of the design and operating conditions are tabulated.

Design	Fin Dimensions		Number of Fins in Array	Convection Coefficient (W/m ² ·K)
	Cross Section $w \times w$ (mm)	Length L (mm)		
A	3 × 3	30	6 × 9	125
B	1 × 1	7	14 × 17	375



Determine which fin arrangement is superior. In your analysis, calculate the heat rate, efficiency and effectiveness of a single fin, as well as the total heat transfer rate of the array.

Exercise # 1

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx _{x=L}$	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$ (3.75)	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx _{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

$$\theta \equiv T - T_\infty \quad m^2 \equiv hP/kA_c$$

$$\theta_b = \theta(0) = T_b - T_\infty \quad M \equiv \sqrt{hPkA_c} \theta_b$$

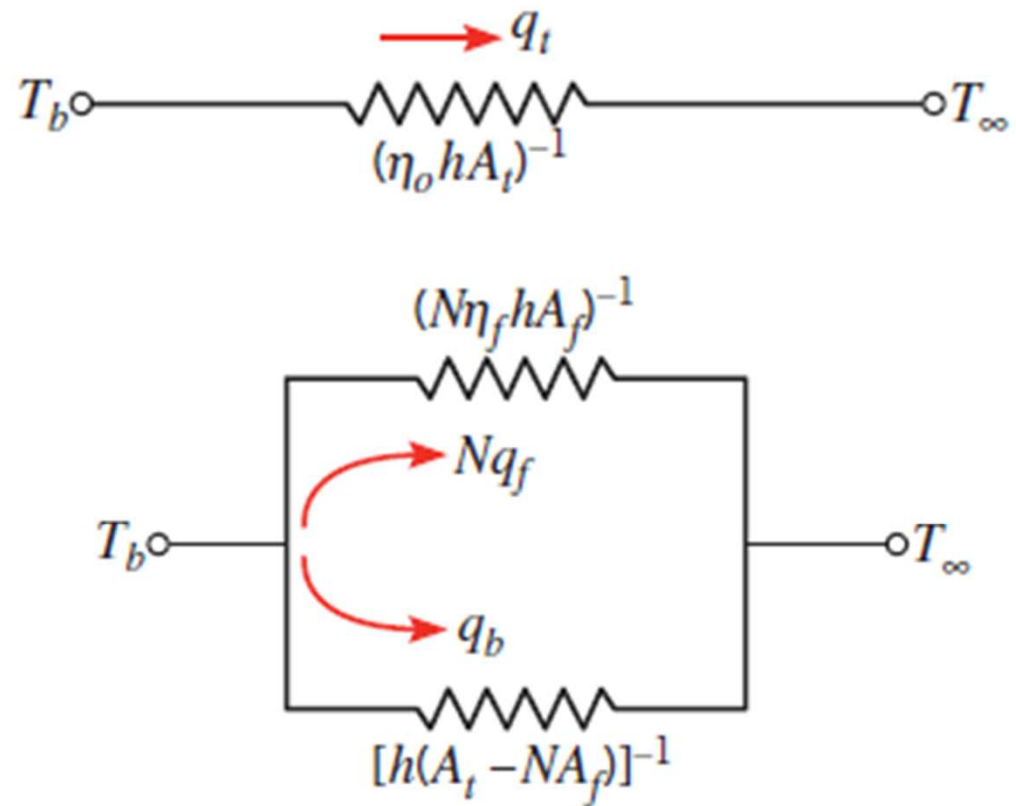
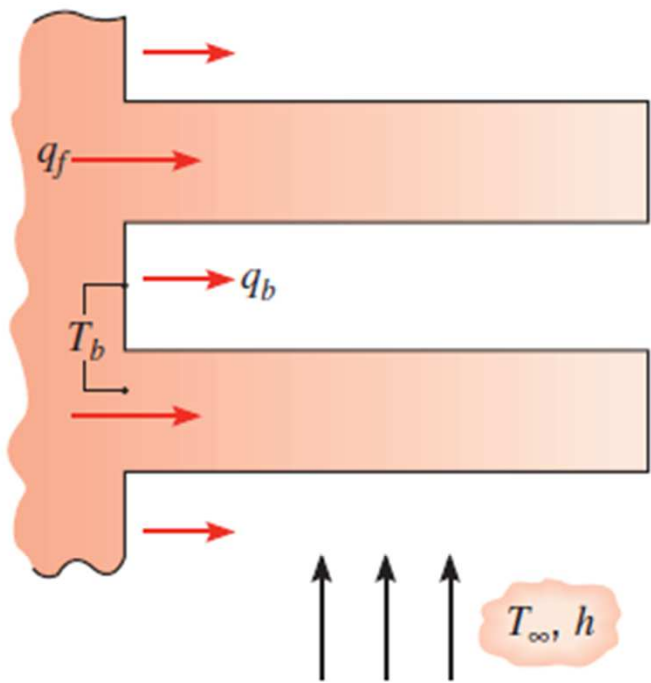
Exercise # 1

Tb	75 C		
Tinf	25 C		
Lb	0.053 m ²		
wb	0.057 m ²		
Ab	0.003021 m ²		
K	175 W/mK		
Design	A	B	
w	0.003	0.001	
Ac	0.000009	0.000001	
L	0.03	0.007	
N	54	238	
h	125	375	
Afin (1 fin)	0.000369	0.000029	
Ab (1 fin)	0.000009	0.000001	
m	30.86067	92.58201	
M	2.430278	0.810093	
q_fin (1 fin)	1.80	0.475	
fin efficiency	0.779	0.873	
fin effectiveness	31.9	25.3	
q_fin total	97.0	112.9	
q_unfin	15.8	52.2	
q_total	112.9	165.1	

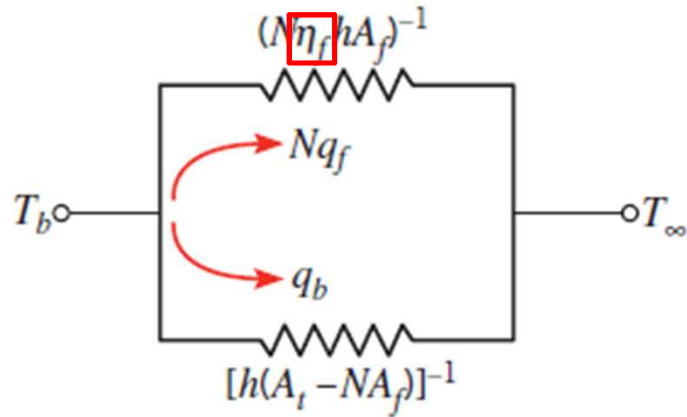
Several important trends may be inferred from this result. Obviously, fin effectiveness is enhanced by the choice of a material of **high thermal conductivity**. Aluminum alloys and copper come to mind. However, although copper is superior from the standpoint of thermal conductivity, aluminum alloys are the more common choice because of additional benefits related to **lower cost and weight**. Fin effectiveness is also enhanced by increasing the ratio of the perimeter to the cross-sectional area. For this reason, **the use of thin, but closely spaced fins, is preferred, with the proviso that the fin gap not be reduced to a value for which flow between the fins is severely impeded, thereby reducing the convection coefficient.**

Exercise # 2

Determine the percentage increase in heat transfer associated with attaching aluminum fins of rectangular profile to a plane wall. The fins are 50mm long, 0.5mm thick and are equally spaced at a distance of 4mm (250 fins/m). The convection coefficient associated with the bare wall is 40 W/m²K, while that resulting from attachment of the fins is 30 W/m²K.



Exercise # 2



$$\eta_f \equiv \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f\theta_b}$$

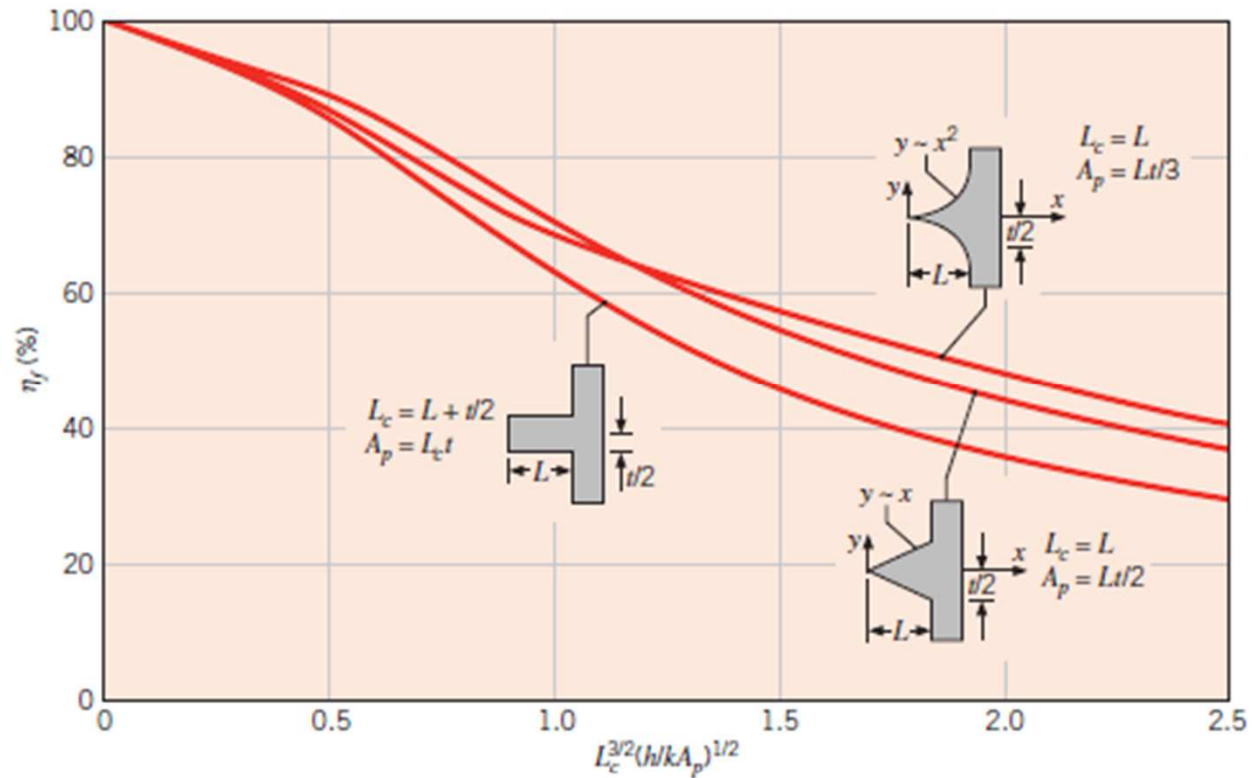
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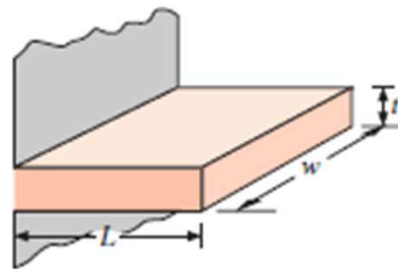
Straight Fins

Rectangular^a

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$



$$\eta_f = \frac{\tanh mL_c}{mL_c}$$