Professor: Michael Reeks

STUDENT NAME: _____

STUDENT NUMBER: _____

Read carefully:

Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have read and ensured that you are complying with the above statement.

Signature: _____

Directions

Answer each question in the space provided. Please write clearly and legibly. ***Show all of your work—your work must justify your answer, and clearly identify your final answer. No books, notes, or electronic devices of any kind may be used during the exam period. You must simplify results of function evaluations when it is reasonable to do so. For example,*** $\sin(\pi/2)$ ***should be evaluated*** (***replaced by*** 1)***.***

1. Let L_1 and L_2 be lines in 3-space with vector equations:

$$L_1 : \ \langle x, y, z\rangle = \langle 5, 1, 3\rangle + s\langle -4, -2, -14\rangle \qquad L_2 : \ \langle x, y, z\rangle = \langle 2, 3, 9\rangle + t\langle 2, 1, 7\rangle$$

Do L_1 and L_2 intersect each other?

- IF SO, find the (x, y, z)-coordinates of the point where they intersect.
- IF THEY DO NOT INTERSECT, decide whether they are parallel or skew.

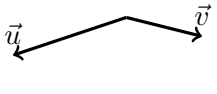
2. Let L be the line represented by the equation

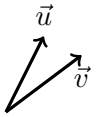
$$2y - 7 = -5x + 1.$$

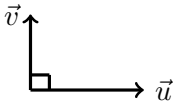
(a) Find a vector equation for L

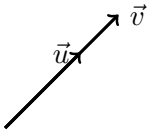
(b) Find parametric equations for L.

3. Suppose that we know $\vec{u} \cdot \vec{v} = 12$, but we don't have coordinates for $\vec{u}$ or $\vec{v}$. Which of the following pictures could be a drawing of $\vec{u}$ and $\vec{v}$? (Circle your answer.)

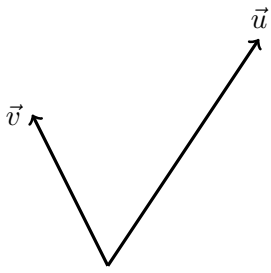




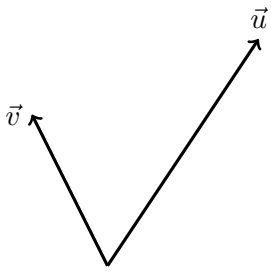




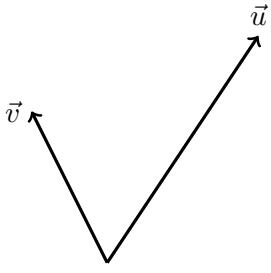
4. In each diagram, draw the vector indicated in bold below the diagram. (If you need more room or make a mistake, replicate the diagram on the back of this page and draw the answer there.)



(a) $\vec{\mathbf{u}} + \vec{\mathbf{v}}$



(a) $\vec{\mathbf{v}} - \vec{\mathbf{u}}$



(a) $\frac{1}{2}\vec{\mathbf{u}} - 2\vec{\mathbf{v}}$

5. Find the derivative of the following functions.

   (a) $f(x) = 2^{-x}\ln(2x+1)$

   (b) $g(s) = \dfrac{\sin(3x+1)}{\cos(\sqrt{x})}$

6. Let $g(x) = \dfrac{1}{(2x+1)^2}$. Find the equation of the line tangent to $g(x)$ at $x = -1$.

7. Suppose $h(x) = e^{f(x)} + 2g(-x)$ for some differentiable functions $f(x)$ and $g(x)$, and suppose that $f(-1) = 0$, $f'(-1) = -2$, $f''(-1) = 5$, $g(1) = \sqrt{2}$, $g'(1) = 2$, and $g''(1) = 3$.

   (a) Find $h'(-1)$.

   (b) Find $h''(-1)$.

8. Suppose that $\vec{v} = \langle -2\sqrt{2}, 2\sqrt{2} \rangle$.

   (a) Find $||\vec{v}||$.

   (b) Find the direction of $\vec{v}$. Your answer should be the POSITIVE angle θ in radians between $\vec{v}$ and the positive x-axis.

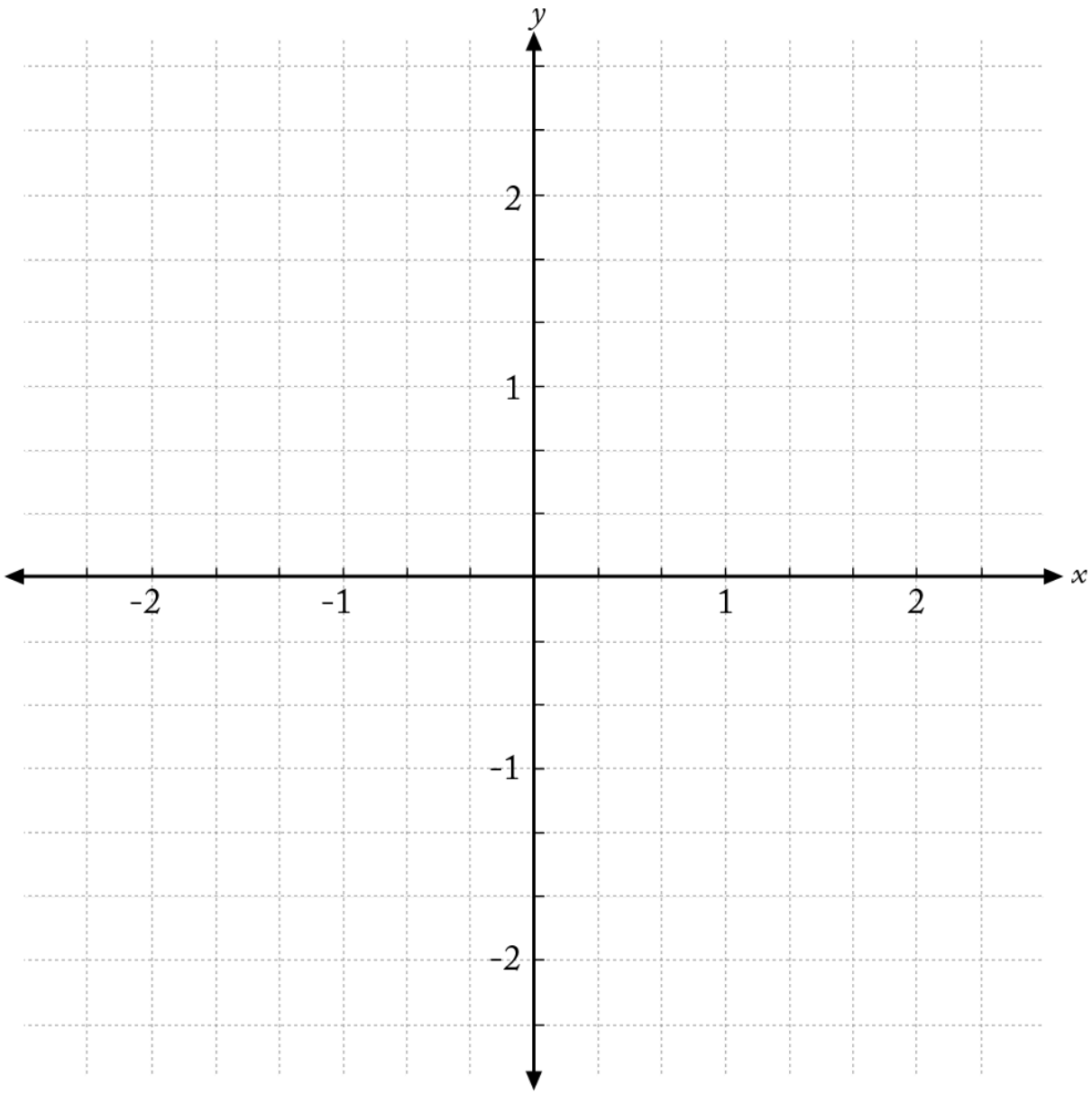
9. Evaluate the following limits. If the limit does not exist, explain why.

   (a) $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^2 - 3x + 4 & x < 1 \\ 0 & x = 1 \\ 2\sin\left(\frac{\pi x}{2}\right) & x > 1 \end{cases}$

   (b) $\lim_{x \to \infty} \dfrac{3x^3 - 14x^5}{1 + \pi x 4x^3}$

10. Sketch a graph of $f(x)$, given the following data:

- $f(x)$ is continuous everywhere except at $x = 1$ and $x = -1$, and its derivative exists everywhere except at $x = \pm 1$ and $x = 0$
- $f(x)$ has vertical asymptotes at $x = \pm 1$.
- $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = 0$
- $f(x)$ has x-intercepts at $(-1/2, 0)$ and $(1/2, 0)$
- $f'(x)$ is positive on $(-\infty, -1)$, $(-1, 0)$, and $(2, \infty)$ and negative on $(0, 1)$ and $(1, 2)$
- $f''(x)$ is positive on $(-\infty, -1)$, $(-1/2, 0)$, $(0, 1/2)$, and $(1, \infty)$; and negative on $(-1, -1, 2)$ and $(1/2, 1)$.



11. Let

$$\vec{u} = \langle 1, 1, -2 \rangle$$
$$\vec{v} = \langle 2, -2, 3 \rangle$$
$$\vec{w} = \langle 1, 2, -3 \rangle.$$

Find $((\text{proj}_{\vec{u}}\, \vec{v}) \times \vec{v}) + \vec{w}$.

12. Suppose $f(x) = \dfrac{4}{1+x^2}$, $f'(x) = \dfrac{-8x}{(1+x^2)^2}$, and $f''(x) = \dfrac{8(3x^2-1)}{(1+x^2)^3}$.

(a) What are the intervals where f is increasing? Decreasing? Your answer may be a sign chart.

(b) Find and classify all relative extrema of f.

(c) On what intervals is f concave up, and on what intervals is it concave down? Your answer may be a sign chart.

13. Let $\vec{v} = \langle 3, 3\sqrt{3} \rangle$ and $\vec{u} = \langle -4, 0 \rangle$.

(a) Find θ_{vu}, the angle ($0 \leq \theta_{vu} \leq \pi$) between $\vec{v}$ and $\vec{u}$.

(b) Find a vector $\vec{w}$ such that $\vec{w}$ is orthogonal to both $\vec{v}$ and $\vec{u}$. Justify your answer by taking dot products. (Hint: you can consider $\vec{u}$ and $\vec{v}$ as three dimensional vectors with z-coordinate 0, i.e. $\vec{v} = \langle 3, 3\sqrt{3}, 0 \rangle$ and $\vec{u} = \langle -4, 0, 0 \rangle$.)