ST 259 Week 4

The Law of Total Probability

- Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for any other event B,

$$P(B) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

Bayes' Theorem

- Let $A_1, \ldots, A_k$ be a collection of mutually exclusive and exhaustive events with $P(A_i) > 0$ for $i = 1, \ldots, k$. Then for any other event B, for which $P(B) > 0$

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}, \text{ for } j = 1,2, \ldots, k$$

Independent

- Two events A and B are independent if $P(A|B) = P(A)$ and are dependent otherwise.

- A and B are independent if and only if
$$P(A \cap B) = P(A)P(B)$$

Mutually Independent

- Events $A_1, \ldots, A_n$ are mutually independent if for every k $(k = 2,3, \ldots, n)$ and every subset of indices $i_1, i_2, \ldots, i_k$,

$$P\left(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}\right) = P\left(A_{i_1}\right)P\left(A_{i_2}\right) \ldots P\left(A_{i_k}\right)$$