SAMPLE MIDTERM TEST

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Time Allowed: 2 hours
Closed Book Test
Nonprogrammable Calculators Permitted
ANSWER ALL QUESTIONS

Please follow the instructions printed on the back of your examination booklets! All computations must be explained in the answer books.

QUESTION 1 (3 POINTS)

In a ring network consisting of eight links, as shown in Figure 1, there are two paths connecting any two terminals. Assume that links fail independently with probability q, where $0 < q < 1$.

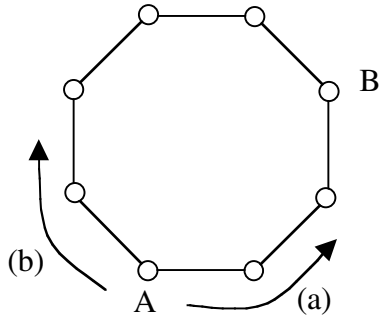

Figure 1

Find the probability of successful transmission of a packet from terminal A to terminal B. (Note: Terminal A transmits the packet in both directions on the ring. Successful transmission means that terminal B received the packet from either direction.)

QUESTION 2 (3 POINTS)

Assume there exists a test to detect a disease whose prevalence is 0.001, that is, the probability that a person picked at random is suffering from the disease is 0.001. The test for the disease has a false positive rate of 0.005. The probability that if you have the disease, the test will indicate that you have it is 1. Given that you test positive for positive for the disease, what is the probability that you actually have it?

QUESTION 3 (7 POINTS)

(a) State the Theorem of Total Probability.

(b) State the definition of the concept that events $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are independent.

(c) State the definition of the concept of a real random variable $\mathbf{X}$ and its cumulative distribution function $F_{\mathbf{X}}(x)$.

(d) If $\mathbf{X}$ has mean 3 and variance 5, what is the mean and variance of $\mathbf{Y} = 3\mathbf{X} - 2$?

(e) If $\mathbf{X} \sim N(\mu, \sigma^2)$, where $\mu = 1$ and $\sigma^2 = 4$, find numerically the value of (i) $\mathcal{P}(\mathbf{X} \geq 1)$, (ii) $\mathcal{P}(\mathbf{X} < 0)$, and (iii) $\mathcal{P}(|\mathbf{X}| > 4)$.

QUESTION 4 (5 POINTS)

Suppose $\mathbf{X}$ is a random variable with probability density function

$$f_{\mathbf{X}}(x) = \begin{cases} ce^{-3x}, & \text{if } x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of c.

(b) If $a > 0$ and $x > 0$, find the value of $\mathcal{P}(\mathbf{X} \geq x + a)$.

(c) If $a > 0$ and $x > 0$, find the value of $\mathcal{P}(\mathbf{X} \geq x + a \mid \mathbf{X} \geq a)$.

QUESTION 5 (5 POINTS)

An integrated circuit chip measuring 1 cm by 1 cm is manufactured on a fabrication line where defects occur on wafers at the average rate of 1 defect per square centimetre.

(a) What is the probability that a randomly selected chip from the fabrication line chip will have no flaw on it? If chips cost \$10 each to make, but only chips without a flaw can be sold, what is the average cost of producing marketable chips?

(b) Suppose now the chip has designed into it a built-in redundancy so that any single flaw on the chip can be effectively repaired, but with this facility, the chip area is increased by 10%, and the cost of producing the new chip is now \$12. What is the probability that a randomly selected chip from the fabrication facility would function? What would the average cost of manufacture of functioning ICs now be?

Hint: For $\mathbf{X} \sim P(\mu)$, $\mathcal{P}(\mathbf{X} = k) = e^{-\mu}\mu^k/k!$.

QUESTION 6 (5 POINTS)

Suppose $\mathbf{X}$ is a continuous random variable with probability density function

$$f_{\mathbf{X}}(x) = \begin{cases} \frac{3}{7}x^2, & \text{if } 1 < x < 2; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find (i) the cumulative distribution function $F_{\mathbf{X}}(x)$, (ii) the mean of $\mathbf{X}$, and (iii) the variance of $\mathbf{X}$.

(b) If $\mathbf{Y} = \mathbf{X}^3 + 1$, find the probability density function of $\mathbf{Y}$.

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.009903	0.009642	0.009387	0.009137	0.008894	0.008656	0.008424
2.4	0.008198	0.007976	0.007760	0.007549	0.007344	0.007143	0.006947	0.006756	0.006569	0.006387
2.5	0.006210	0.006037	0.005868	0.005703	0.005543	0.005386	0.005234	0.005085	0.004940	0.004799
2.6	0.004661	0.004527	0.004396	0.004269	0.004145	0.004025	0.003907	0.003793	0.003681	0.003573
2.7	0.003467	0.003364	0.003264	0.003167	0.003072	0.002980	0.002890	0.002803	0.002718	0.002635
2.8	0.002555	0.002477	0.002401	0.002327	0.002256	0.002186	0.002118	0.002052	0.001988	0.001926
2.9	0.001866	0.001807	0.001750	0.001695	0.001641	0.001589	0.001538	0.001489	0.001441	0.001395
3.0	0.001350	0.001306	0.001264	0.001223	0.001183	0.001144	0.001107	0.001070	0.001035	0.001001
3.1	$9.676\,10^{-4}$	$9.354\,10^{-4}$	$9.043\,10^{-4}$	$8.740\,10^{-4}$	$8.447\,10^{-4}$	$8.164\,10^{-4}$	$7.888\,10^{-4}$	$7.622\,10^{-4}$	$7.364\,10^{-4}$	$7.114\,10^{-4}$
3.2	$6.871\,10^{-4}$	$6.637\,10^{-4}$	$6.410\,10^{-4}$	$6.190\,10^{-4}$	$5.976\,10^{-4}$	$5.770\,10^{-4}$	$5.571\,10^{-4}$	$5.377\,10^{-4}$	$5.190\,10^{-4}$	$5.009\,10^{-4}$
3.3	$4.834\,10^{-4}$	$4.665\,10^{-4}$	$4.501\,10^{-4}$	$4.342\,10^{-4}$	$4.189\,10^{-4}$	$4.041\,10^{-4}$	$3.897\,10^{-4}$	$3.758\,10^{-4}$	$3.624\,10^{-4}$	$3.495\,10^{-4}$
3.4	$3.369\,10^{-4}$	$3.248\,10^{-4}$	$3.131\,10^{-4}$	$3.018\,10^{-4}$	$2.909\,10^{-4}$	$2.803\,10^{-4}$	$2.701\,10^{-4}$	$2.602\,10^{-4}$	$2.507\,10^{-4}$	$2.415\,10^{-4}$
3.5	$2.326\,10^{-4}$	$2.241\,10^{-4}$	$2.158\,10^{-4}$	$2.078\,10^{-4}$	$2.001\,10^{-4}$	$1.926\,10^{-4}$	$1.854\,10^{-4}$	$1.785\,10^{-4}$	$1.718\,10^{-4}$	$1.653\,10^{-4}$
3.6	$1.591\,10^{-4}$	$1.531\,10^{-4}$	$1.473\,10^{-4}$	$1.417\,10^{-4}$	$1.363\,10^{-4}$	$1.311\,10^{-4}$	$1.261\,10^{-4}$	$1.213\,10^{-4}$	$1.166\,10^{-4}$	$1.121\,10^{-4}$
3.7	$1.078\,10^{-4}$	$1.036\,10^{-4}$	$9.961\,10^{-5}$	$9.574\,10^{-5}$	$9.201\,10^{-5}$	$8.842\,10^{-5}$	$8.496\,10^{-5}$	$8.162\,10^{-5}$	$7.841\,10^{-5}$	$7.532\,10^{-5}$
3.8	$7.235\,10^{-5}$	$6.948\,10^{-5}$	$6.673\,10^{-5}$	$6.407\,10^{-5}$	$6.152\,10^{-5}$	$5.906\,10^{-5}$	$5.669\,10^{-5}$	$5.442\,10^{-5}$	$5.223\,10^{-5}$	$5.012\,10^{-5}$
3.9	$4.810\,10^{-5}$	$4.615\,10^{-5}$	$4.427\,10^{-5}$	$4.247\,10^{-5}$	$4.074\,10^{-5}$	$3.908\,10^{-5}$	$3.747\,10^{-5}$	$3.594\,10^{-5}$	$3.446\,10^{-5}$	$3.304\,10^{-5}$
4.0	$3.167\,10^{-5}$	$3.036\,10^{-5}$	$2.910\,10^{-5}$	$2.789\,10^{-5}$	$2.673\,10^{-5}$	$2.561\,10^{-5}$	$2.454\,10^{-5}$	$2.351\,10^{-5}$	$2.252\,10^{-5}$	$2.157\,10^{-5}$
4.1	$2.066\,10^{-5}$	$1.978\,10^{-5}$	$1.894\,10^{-5}$	$1.814\,10^{-5}$	$1.737\,10^{-5}$	$1.662\,10^{-5}$	$1.591\,10^{-5}$	$1.523\,10^{-5}$	$1.458\,10^{-5}$	$1.395\,10^{-5}$
4.2	$1.335\,10^{-5}$	$1.277\,10^{-5}$	$1.222\,10^{-5}$	$1.168\,10^{-5}$	$1.118\,10^{-5}$	$1.069\,10^{-5}$	$1.022\,10^{-5}$	$9.774\,10^{-6}$	$9.345\,10^{-6}$	$8.934\,10^{-6}$
4.3	$8.540\,10^{-6}$	$8.163\,10^{-6}$	$7.801\,10^{-6}$	$7.455\,10^{-6}$	$7.124\,10^{-6}$	$6.807\,10^{-6}$	$6.503\,10^{-6}$	$6.212\,10^{-6}$	$5.934\,10^{-6}$	$5.668\,10^{-6}$
4.4	$5.413\,10^{-6}$	$5.169\,10^{-6}$	$4.935\,10^{-6}$	$4.712\,10^{-6}$	$4.498\,10^{-6}$	$4.294\,10^{-6}$	$4.098\,10^{-6}$	$3.911\,10^{-6}$	$3.732\,10^{-6}$	$3.561\,10^{-6}$
4.5	$3.398\,10^{-6}$	$3.241\,10^{-6}$	$3.092\,10^{-6}$	$2.949\,10^{-6}$	$2.813\,10^{-6}$	$2.682\,10^{-6}$	$2.558\,10^{-6}$	$2.439\,10^{-6}$	$2.325\,10^{-6}$	$2.216\,10^{-6}$
4.6	$2.112\,10^{-6}$	$2.013\,10^{-6}$	$1.919\,10^{-6}$	$1.828\,10^{-6}$	$1.742\,10^{-6}$	$1.660\,10^{-6}$	$1.581\,10^{-6}$	$1.506\,10^{-6}$	$1.434\,10^{-6}$	$1.366\,10^{-6}$
4.7	$1.301\,10^{-6}$	$1.239\,10^{-6}$	$1.179\,10^{-6}$	$1.123\,10^{-6}$	$1.069\,10^{-6}$	$1.017\,10^{-6}$	$9.680\,10^{-7}$	$9.211\,10^{-7}$	$8.765\,10^{-7}$	$8.339\,10^{-7}$
4.8	$7.933\,10^{-7}$	$7.547\,10^{-7}$	$7.178\,10^{-7}$	$6.827\,10^{-7}$	$6.492\,10^{-7}$	$6.173\,10^{-7}$	$5.869\,10^{-7}$	$5.580\,10^{-7}$	$5.304\,10^{-7}$	$5.042\,10^{-7}$
4.9	$4.792\,10^{-7}$	$4.554\,10^{-7}$	$4.327\,10^{-7}$	$4.111\,10^{-7}$	$3.906\,10^{-7}$	$3.711\,10^{-7}$	$3.525\,10^{-7}$	$3.348\,10^{-7}$	$3.179\,10^{-7}$	$3.019\,10^{-7}$
5.0	$2.867\,10^{-7}$	$2.722\,10^{-7}$	$2.584\,10^{-7}$	$2.452\,10^{-7}$	$2.328\,10^{-7}$	$2.209\,10^{-7}$	$2.096\,10^{-7}$	$1.989\,10^{-7}$	$1.887\,10^{-7}$	$1.790\,10^{-7}$
5.1	$1.698\,10^{-7}$	$1.611\,10^{-7}$	$1.528\,10^{-7}$	$1.449\,10^{-7}$	$1.374\,10^{-7}$	$1.302\,10^{-7}$	$1.235\,10^{-7}$	$1.170\,10^{-7}$	$1.109\,10^{-7}$	$1.051\,10^{-7}$
5.2	$9.964\,10^{-8}$	$9.442\,10^{-8}$	$8.946\,10^{-8}$	$8.476\,10^{-8}$	$8.029\,10^{-8}$	$7.605\,10^{-8}$	$7.203\,10^{-8}$	$6.821\,10^{-8}$	$6.459\,10^{-8}$	$6.116\,10^{-8}$
5.3	$5.790\,10^{-8}$	$5.481\,10^{-8}$	$5.188\,10^{-8}$	$4.911\,10^{-8}$	$4.647\,10^{-8}$	$4.398\,10^{-8}$	$4.161\,10^{-8}$	$3.937\,10^{-8}$	$3.724\,10^{-8}$	$3.523\,10^{-8}$
5.4	$3.332\,10^{-8}$	$3.151\,10^{-8}$	$2.980\,10^{-8}$	$2.818\,10^{-8}$	$2.664\,10^{-8}$	$2.518\,10^{-8}$	$2.381\,10^{-8}$	$2.250\,10^{-8}$	$2.127\,10^{-8}$	$2.010\,10^{-8}$
5.5	$1.899\,10^{-8}$	$1.794\,10^{-8}$	$1.695\,10^{-8}$	$1.601\,10^{-8}$	$1.512\,10^{-8}$	$1.428\,10^{-8}$	$1.349\,10^{-8}$	$1.274\,10^{-8}$	$1.203\,10^{-8}$	$1.135\,10^{-8}$
5.6	$1.072\,10^{-8}$	$1.012\,10^{-8}$	$9.548\,10^{-9}$	$9.010\,10^{-9}$	$8.503\,10^{-9}$	$8.022\,10^{-9}$	$7.569\,10^{-9}$	$7.140\,10^{-9}$	$6.735\,10^{-9}$	$6.352\,10^{-9}$
5.7	$5.990\,10^{-9}$	$5.649\,10^{-9}$	$5.326\,10^{-9}$	$5.022\,10^{-9}$	$4.734\,10^{-9}$	$4.462\,10^{-9}$	$4.206\,10^{-9}$	$3.964\,10^{-9}$	$3.735\,10^{-9}$	$3.519\,10^{-9}$
5.8	$3.316\,10^{-9}$	$3.124\,10^{-9}$	$2.942\,10^{-9}$	$2.771\,10^{-9}$	$2.610\,10^{-9}$	$2.458\,10^{-9}$	$2.314\,10^{-9}$	$2.179\,10^{-9}$	$2.051\,10^{-9}$	$1.931\,10^{-9}$
5.9	$1.818\,10^{-9}$	$1.711\,10^{-9}$	$1.610\,10^{-9}$	$1.515\,10^{-9}$	$1.425\,10^{-9}$	$1.341\,10^{-9}$	$1.261\,10^{-9}$	$1.186\,10^{-9}$	$1.116\,10^{-9}$	$1.049\,10^{-9}$
6.0	$9.866\,10^{-10}$	$9.276\,10^{-10}$	$8.721\,10^{-10}$	$8.198\,10^{-10}$	$7.706\,10^{-10}$	$7.242\,10^{-10}$	$6.806\,10^{-10}$	$6.396\,10^{-10}$	$6.009\,10^{-10}$	$5.646\,10^{-10}$
6.1	$5.303\,10^{-10}$	$4.982\,10^{-10}$	$4.679\,10^{-10}$	$4.394\,10^{-10}$	$4.126\,10^{-10}$	$3.874\,10^{-10}$	$3.637\,10^{-10}$	$3.414\,10^{-10}$	$3.205\,10^{-10}$	$3.008\,10^{-10}$
6.2	$2.823\,10^{-10}$	$2.649\,10^{-10}$	$2.486\,10^{-10}$	$2.332\,10^{-10}$	$2.188\,10^{-10}$	$2.052\,10^{-10}$	$1.925\,10^{-10}$	$1.805\,10^{-10}$	$1.693\,10^{-10}$	$1.587\,10^{-10}$
6.3	$1.488\,10^{-10}$	$1.395\,10^{-10}$	$1.308\,10^{-10}$	$1.226\,10^{-10}$	$1.149\,10^{-10}$	$1.077\,10^{-10}$	$1.009\,10^{-10}$	$9.451\,10^{-11}$	$8.854\,10^{-11}$	$8.294\,10^{-11}$
6.4	$7.769\,10^{-11}$	$7.276\,10^{-11}$	$6.814\,10^{-11}$	$6.380\,10^{-11}$	$5.974\,10^{-11}$	$5.593\,10^{-11}$	$5.235\,10^{-11}$	$4.900\,10^{-11}$	$4.586\,10^{-11}$	$4.292\,10^{-11}$
6.5	$4.016\,10^{-11}$	$3.758\,10^{-11}$	$3.515\,10^{-11}$	$3.288\,10^{-11}$	$3.076\,10^{-11}$	$2.877\,10^{-11}$	$2.690\,10^{-11}$	$2.516\,10^{-11}$	$2.352\,10^{-11}$	$2.199\,10^{-11}$
6.6	$2.056\,10^{-11}$	$1.922\,10^{-11}$	$1.796\,10^{-11}$	$1.678\,10^{-11}$	$1.568\,10^{-11}$	$1.465\,10^{-11}$	$1.369\,10^{-11}$	$1.279\,10^{-11}$	$1.195\,10^{-11}$	$1.116\,10^{-11}$
6.7	$1.042\,10^{-11}$	$9.731\,10^{-12}$	$9.086\,10^{-12}$	$8.483\,10^{-12}$	$7.919\,10^{-12}$	$7.392\,10^{-12}$	$6.900\,10^{-12}$	$6.439\,10^{-12}$	$6.009\,10^{-12}$	$5.607\,10^{-12}$
6.8	$5.231\,10^{-12}$	$4.880\,10^{-12}$	$4.552\,10^{-12}$	$4.246\,10^{-12}$	$3.960\,10^{-12}$	$3.692\,10^{-12}$	$3.443\,10^{-12}$	$3.210\,10^{-12}$	$2.993\,10^{-12}$	$2.790\,10^{-12}$
6.9	$2.600\,10^{-12}$	$2.423\,10^{-12}$	$2.258\,10^{-12}$	$2.104\,10^{-12}$	$1.960\,10^{-12}$	$1.826\,10^{-12}$	$1.701\,10^{-12}$	$1.585\,10^{-12}$	$1.476\,10^{-12}$	$1.374\,10^{-12}$