Carleton University

ECON 2009 A – Managerial Economics

Midterm Exam #2

November 22, 2018

1. (20 points) Short questions:

   (a) (5 points) A firm has a production function given by $Q = 3L^2 + 2K$. What is this firm's average product of labor and marginal product of labor if $K = 10$?

$$AP_L = \frac{Q}{L} = \frac{3L^2 + 2 \times 10}{L} = 3L + 20/L$$
$$MP_L = \frac{dQ}{dL} = 6L$$

   (b) (5 points) What does it mean for a firm to experience scope economies in the production of two products? When can this occur?

   Experiencing scope economies means that the joint production of the two products is cheaper than producing each product independently. This can occur for several reasons (only ONE is required and any example that implicitly refers to one of these two is also sufficient):

   - Sharing of inputs: maybe the same equipment is required or maybe the same labor skills are required, or administrative departments can be shared (HR, payroll, etc.)
   - Cost complementarities: when part of the production process for one good makes the production cost cheaper for the other. E.g. crude oil and natural gas, both require the drilling of a well and then they are produced jointly from the same well

   (c) (5 points) What is the profit-maximizing two-part tariff for a firm that faces identical consumers with demand curves given by $q = 10 - P$ if the marginal cost of the firm is $MC = AVC = 2$?

   The profit-maximizing two-part tariff is composed of a per-unit price equal to marginal cost and a fixed fee equal to the implied consumer surplus of that per-unit price. Hence:

$$P^* = MC = 2$$

$$FF^* = CS = (10 - 2)8/2 = 32$$

   (d) (5 points) Explain which of the following firms would be most able to perfectly price discriminate: a supermarket, a gas station, a doctor, an art auction house.

An art auction house is most able to exercise perfect price discrimination because the auction mechanism can induce bidders to bid a value equal to their willingness to pay or reservation price.

2. (20 points) Steel baskets can be produced using steel and labor. The number of baskets that can be produced, given the technology available follows a production function given by $Q = \frac{1}{2}L^{0.4}S^{0.6}$, where $Q =$ number of baskets, $L =$ units of labor, and $S =$ kilos of steel used.

   (a) (5 points) Does the steel basked production function exhibit constant, decreasing or increasing returns to scale? Explain

   The above is a Cobb-Douglas production function where the exponents on the inputs add up to 1, which means it exhibits <u>constant returns to scale</u>.

   You can also show this mathematically. If output is doubled when inputs are doubled, then there are constant returns to scale. (you can increase inputs by any other proportion)

$$
\begin{aligned}
Q(2L, 2S) &= 2Q(L, S) \\
\frac{1}{2}(2L)^{0.4}(2S)^{0.6} &= 2\frac{1}{2}L^{0.4}S^{0.6} \\
\frac{1}{2}2^{0.4}L^{0.4}2^{0.6}S^{0.6} &= L^{0.4}S^{0.6} \\
\frac{1}{2}2^{0.4+0.6}L^{0.4}S^{0.6} &= L^{0.4}S^{0.6} \\
L^{0.4}S^{0.6} &= L^{0.4}S^{0.6} \rightarrow \text{Constant Returns to Scale}
\end{aligned}
$$

   (b) (15 points) The price of a unit of labor is $p_L = \$10$ and the price of steel is $p_S = \$60$ per kilo. If the company has $M = \$10,000$ to spend on labor and steel, what is the maximum amount of baskets it can produce and how much labor and steel would it use?

   The maximization of output subject to the cost constraint requires that the marginal rate of technical substitution is equal to the ratio of prices (tangency condition) and that the constraint holds.

Tangency condition:

$$\begin{aligned} MRTS &= \frac{p_L}{p_S} \\ \frac{MP_L}{MP_S} &= \frac{p_L}{p_S} \\ \frac{\frac{0.4}{2}L^{-0.6}S^{0.6}}{\frac{0.6}{2}L^{0.6}S^{-0.4}} &= \frac{10}{60} \\ \frac{0.4}{0.6}L^{-1}S^{1} &= \frac{1}{6} \\ S &= \frac{1}{6}\frac{0.6}{0.4}L \rightarrow \boxed{S = 0.25L} \end{aligned} \tag{1}$$

Replacing in the cost constraint:

$$\begin{aligned} M &= p_L L + p_S S \\ 10,000 &= 10L + 60S \\ 10,000 &= 10L + 60 \times 0.25L \\ 10,000 &= 10L + 15L \rightarrow \boxed{L = 400} \end{aligned}$$

Replacing back into 1, $\boxed{S = 100}$. Total production with these inputs is

$$Q = \frac{1}{2}L^{0.4}S^{0.6} = \frac{1}{2}400^{0.4}100^{0.6} = \boxed{87}$$

3. (35 points) A competitive firm has a short-run and long-run total cost function given by $TC = 2 + 2q + 2q^2$. All firms in this market are identical and the short-run market demand and supply curves are given by:

$$Q^D = 400 - 20P$$

$$Q^S = 25P - 50$$

(a) (5 points) What is the competitive equilibrium in this market?

$$\begin{aligned} Q^D &= Q^S \\ 400 - 20P &= 25P - 50 \\ 450 &= 45P \rightarrow \boxed{P^* = 10} \end{aligned} \tag{2}$$

Replacing this result back into the supply or demand curve: $\boxed{Q^* = 200}$

(b) (10 points) What is the supply curve for each competitive firm? What is the number of units supplied by each firm? How many firms are there in the market?

The inverse supply curve is given by the MC curve.

$$P = MC$$
$$P = 2 + 4q \quad \text{is the inverse supply curve of an individual firm} \tag{3}$$

Because $MC > AVC$ for any positive q, then all of the MC curve will be the supply curve of the competitive firm.

Since, the market price is \$10, the quantity supplied given by the supply curve is $\boxed{q^* = 2}$.

The number of firms is given by the total quantity traded divided by the quantity supplied by each firm: $n = \frac{Q^*}{q^*} = \frac{200}{2} = 100$.

(c) (10 points) What is each firm's profit in the short run? Is this also a long-run equilibrium? If not, what would it be? Explain.

Profit:

$$\pi = TR - TC = P^*q^* - TC(q^*) = 10 \times 2 - (2 + 2 \times 2 + 2 \times 2^2) = \$6$$

This is not a long-run equilibrium because profit is positive and it will cause firms to enter and push price down until profit is zero.

Long-run equilibrium is achieved when $P = LRMC = minLRAC$. This can be calculated by either minimizing $LRAC$ or setting $LRMC = LRAC$.

$LRAC = \frac{TC}{q} = \frac{2+2q+2q^2}{q} = \frac{2}{q} + 2 + 2q$

$LRMC = \frac{dTC}{dq} = 2 + 4q$

$$LRMC = LRAC$$
$$\frac{2}{q} + 2 + 2q = 2 + 4q$$
$$\frac{2}{q} = 2q$$
$$1 = q^2 \rightarrow \boxed{q^{LR} = 1}$$

Replacing back into either cost function $\boxed{P^{LR} = LRMC = LRAC = 6}$.

(d) (10 points) Now suppose the industry turns into a monopoly. This monopoly has a marginal cost function given by $MC = \frac{Q}{25} + 2$ and faces the same market demand as above, $Q^D = 400 - 20P$. Find the monopolist's profit-maximizing price and quantity.

For the monopolist $MR = MC$. Because demand is linear, MR has the same intercept as the inverse demand and double the slope.

$$Q^D = 400 - 20P \quad \rightarrow \text{Inverse demand: } P = 400/20 - Q/20 \rightarrow \boxed{MR = 20 - Q/10}$$

$$
\begin{aligned}
MR &= MC \\
20 - \frac{Q}{10} &= \frac{Q}{25} + 2 \\
18 &= \frac{Q}{25} + \frac{Q}{10} \\
18 = 0.14Q &\rightarrow \boxed{Q^M = 128.6}
\end{aligned}
$$

Replacing this back in the inverse demand curve $P^M = 20 - Q^M/20 = \boxed{13.6}$.

4. (25 points) Figaro is the only barber in in a rural town. He is trying to determine how much to charge adults for a haircut and how much to charge kids or whether to charge the same price, regardless of the customer. Figaro's total cost is given by $TC = 100 + 5Q$. Demands for haircuts from adults and kids are given by: $Q_A = 70 - 2P_A$ and $Q_K = 55 - P_K$.

   (a) (11 points) What price should Figaro charge adults and kids if he wanted to price-discriminate? What is his profit? In order to maximize profit, Figaro must set $MR_A = MC$ and $MR_K = MC$. First derive the inverse demand curves. MR curves will then have the same intercept and double the slope as the inverse demand curves.

$$
\begin{aligned}
Q_A = 70 - 2P_A \quad &\rightarrow P_A = 35 - 0.5Q_A \rightarrow \boxed{MR_A = 35 - Q_A} \\
Q_K = 55 - P_K \quad &\rightarrow P_K = 55 - Q_K \rightarrow \boxed{MR_K = 55 - 2Q_K}
\end{aligned}
$$

Setting $MR = MC$ for each case:

$$
\begin{aligned}
MR_A &= MC \\
35 - Q_A = 5 &\rightarrow \boxed{Q_A = 30} \quad (4) \\
MR_K &= MC \\
55 - 2Q_K = 5 &\rightarrow \boxed{Q_K = 25} \quad (5)
\end{aligned}
$$

Replacing both quantities in the inverse demands leads to $\boxed{P_A = \$20}$ and $\boxed{P_K = \$30}$. His profit is then:

$$
\begin{aligned}
\pi_{disc} &= TR - TC \\
\pi_{disc} &= P_A Q_A + P_K Q_K - TC(Q_A + Q_K) \\
\pi_{disc} &= 20 \times 30 + 30 \times 25 - 100 - 5 \times 55 \\
\pi_{disc} &= \boxed{\$975}. \quad (6)
\end{aligned}
$$

   (b) (11 points) Suppose that the town council prohibits Figaro from charging different prices for the same service. What is the single price he would have to charge for haircuts if he wants to continue to serve both adults and kids? What is his profit?

Now Figaro needs to set MR from the total demand (adults + kids) equal to MC. Aggregate demand:

$$Q = Q_A + Q_K$$
$$Q = 70 - 2P + 55 - P$$
$$Q = 125 - 3P \quad \rightarrow P = 125/3 - Q/3 \rightarrow \boxed{MR = 125/3 - 2/3Q} \tag{7}$$
$$MR = MC$$
$$125/3 - 2Q/3 = 5 \rightarrow \boxed{Q = 55} \tag{8}$$

Replacing this price back in the aggregate inverse demand yields $\boxed{P = 23.33}$.

Profit is then:

$$\pi = TR - TC = 55 \times 23.33 - 100 - 5 \times 55 = \boxed{\$908.33}$$

(c) (3 points) Explain whether Figaro prefers to charge a single price or whether he prefers to price discriminate.

Because his profit with price discrimination ($975) is higher than his profit with single-pricing ($908.33), he will prefer to price-discriminate.