Carleton University

ECON 2009 A – Managerial Economics

Midterm Exam

October 18, 2018

1. (35 points) Short questions:

   (a) (6 points) Paper producers estimate that the monthly market demand for printing paper is

$$Q = 1,000 - 10P - 5P_t + 0.05I$$

where Q is the quantity demanded of paper (reams), P is the price of a ream of printing paper, P_t is the price of printing toner, and I is income. Currently, $P = \$10$, $P_t = \$200$, and $I = \$40,000$.
Note: For your calculations below, provide the formulas for the elasticities requested in addition to your calculation.

   i. (2 points) What is the price elasticity of demand for printing paper? What does this number mean exactly? Is it elastic or inelastic?

$$\eta = \frac{dQ}{dP}\frac{P}{Q}$$
$$\eta = -10\frac{10}{Q}$$

We calculate Q by replacing the data given into the demand function:

$$Q = 1,000 - 10 \times 10 - 5 \times 200 + 0.05 \times 40,000 = 1,900$$

Replacing back into the elasticity formula:

$$\eta = -10\frac{10}{1,900} = -0.0526$$

This figure means that an increase in price of 1% will lead to decrease in quantity of 0.05%. Because the decrease in quantity is less than proportional to the increase in price, the demand for printing paper is inelastic.

ii. (2 points) What is the cross-price elasticity of demand between printing paper and toner? What does this number mean? What is the relationship between these two goods?

$$\eta_{Pt} = \frac{dQ}{dP_t}\frac{P_t}{Q}$$
$$\eta_{Pt} = -5\frac{200}{1,900} = -0.5263$$

A cross-price elasticity of -0.5263 means that a 1% increase in the price of toner will lead to a decrease of 0.5263% in the quantity of printing paper demanded. This means the two goods are complements.

iii. (2 points) What is the income elasticity of printing paper? Is printing paper a normal or inferior good? If normal, is it a luxury or a necessity?

$$\eta_I = \frac{\partial Q}{\partial I}\frac{I}{Q}$$
$$\eta_I = 0.05\frac{40,000}{1,900} = 1.05$$

Because the income elasticity is positive, printing paper is a normal good. This means that an increase in income will increase the quantity demanded. In addition, because the elasticity is larger than one, printing paper is a luxury. An increase in income leads to a more than proportional increase in quantity demanded.

(b) (10 points) For each case below describing changes affecting the **market for bread**, note whether the statement is true, false, or uncertain, and explain your answer. (You may draw a graph but full marks require an explanation of how each event affects the market for bread).

i. If scientists find that bread is much healthier than previously thought and, at the same time, the number of bakeries increases by 30%; then the equilibrium quantity of bread will rise, and the price of bread will fall.

Uncertain. The scientific discovery shifts demand to the right, increasing quantity demanded and increasing price. An increase in the number of suppliers shifts supply also to the right, which increases quantity but decreases price. In this case, both the supply and the demand curves shift to the right. Quantity will definitely increase, but whether price will rise, fall, or remain constant depends on the relative sizes of the supply and demand shifts.

ii. If the price of crackers (a substitute for bread) falls and the price of flour (used to make bread) increases, the quantity and price of bread will both fall.

Uncertain. In this case, the fall in the price of the substitute decreases demand for bread (shift to the left) and the increase in the price of the input decreases supply (shift to the left), so the demand and supply curves both shift to the left. Quantity decreases, but price may rise, fall, or remain unchanged depending on the relative magnitude of the shifts.

(c) (10 points) If a firm's product has a current price elasticity of -0.5. How should the firm change its price to increase revenue? How will this affect profit? Explain.

If demand is inelastic as here, then increasing price will increase revenue. This is because if price increases by 1%, quantity will fall by less than that; in this case 0.5%.

This will definitely increase profit because, in addition to revenue increasing, total cost will likely decrease. Since the firm will increase its price, total quantity sold will fall, which will lead total cost to fall as well.

(d) (9 points) What is the difference between the average product of labor (AP_L) and the marginal product of labor (MP_L)? What do these two measures have in common?

Both are measures of productivity but the AP_L measures the average output per worker, while MP_L measures the output of the last worker hired.

2. (25 points) Maria derives utility from swimming and yoga in the following way:

$$U(x, y) = x^{0.5}y^{0.5}$$

where x is the number of hours of swimming and y is the number of hours of yoga per week. Maria has budgeted \$120 per week for her swimming and yoga time. If the price of swimming is \$20 per hour and the price of yoga is \$12 per hour, answer the following questions.

(a) (10 points) Write the equation for Maria's budget constraint and draw it on a graph showing swimming hours on the y-axis and yoga hours on the x-axis. Label the axes and show and explain the values of the intercepts.

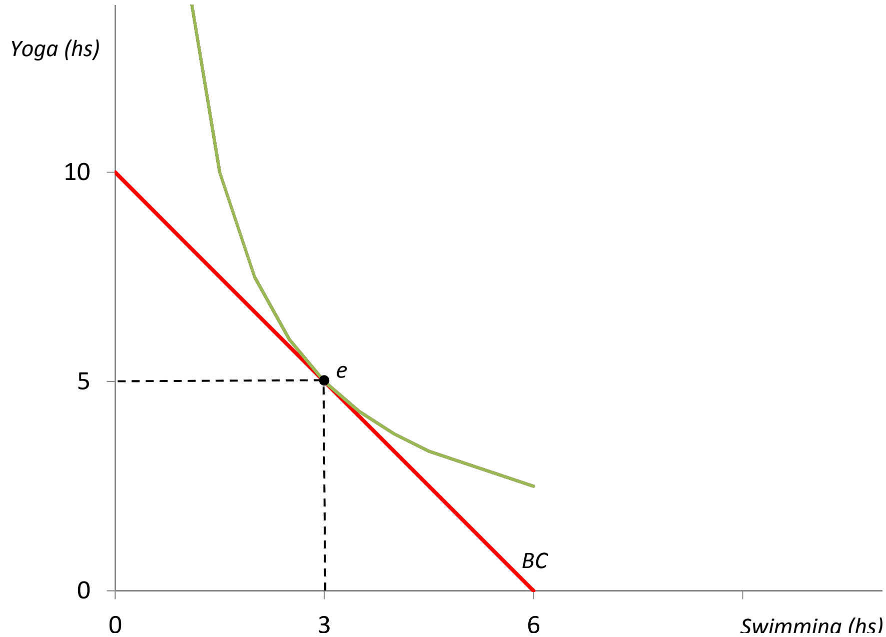
Maria's budget constraint is:

$$I = p_x x + p_y y$$
$$120 = 20x + 12y$$

Because we draw yoga hours on the y-axis we can solve for y in the budget constraint to find the intercept and slope:

$$y = 120/12 - 20/12x = 10 - 5/3x$$

The intercepts show the number of units of each good that Maria can buy if she spends all of her income on that good. If $x = 0$, $y = 10$ because Maria spends all her income on yoga and so the intercept of the y-axis is 10 yoga hours. If Maria spends all her income on swimming, then she can buy $x = 120/20 = 6$ swimming hours.



(b) (3 points) What is Maria's marginal utility of swimming and what is her marginal utility of yoga?

$$MU_x = \frac{\partial U}{\partial x} = 0.5x^{-0.5}y^{0.5}$$
$$MU_y = \frac{\partial U}{\partial y} = 0.5x^{0.5}y^{-0.5}$$

(c) (2 points) What is Maria's marginal rate of substitution between the two goods?

Since the question does not specify which MRS, either answer is correct.

The MRS of swimming for yoga:

$$MRS = \frac{MU_x}{MU_y} = \frac{y}{x} \tag{1}$$

The MRS of yoga for swimming is:

$$MRS = \frac{MU_y}{MU_x} = \frac{x}{y} \tag{2}$$

(d) (10 points) What is the amount of swimming and yoga hours that maximize Maria's utility given the current prices and her budget? How much utility does this optimal bundle give Maria? Show this optimal bundle and the indifference curve that goes through it on the graph in part (a) (The indifference curve does not have to be drawn to scale). Label the optimal bundle e.

Maria maximizes her utility if her budget constraint is satisfied and the tangency condition between an indifference curve and her budget constraint is satisfied.

Tangency condition:

$$MRS = MRT$$
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$
$$\frac{y}{x} = \frac{20}{12} \rightarrow \boxed{y = \frac{5}{3}x} \tag{3}$$

We replace equation 3 in Maria's budget constraint:

$$120 = 20x + 12y$$
$$120 = 20x + 12\frac{5}{3}x$$
$$120 = 40x \rightarrow \boxed{x = 3} \tag{4}$$

Replacing this result back into equation 3, we can obtain the result for $y = 5$. So the optimal bundle for Maria is 3 hours of swimming and 5 hours of yoga. We replace these values in the utility function to derive the level of utility for this bundle: $U = 3^{0.5}5^{0.5} = 3.87$

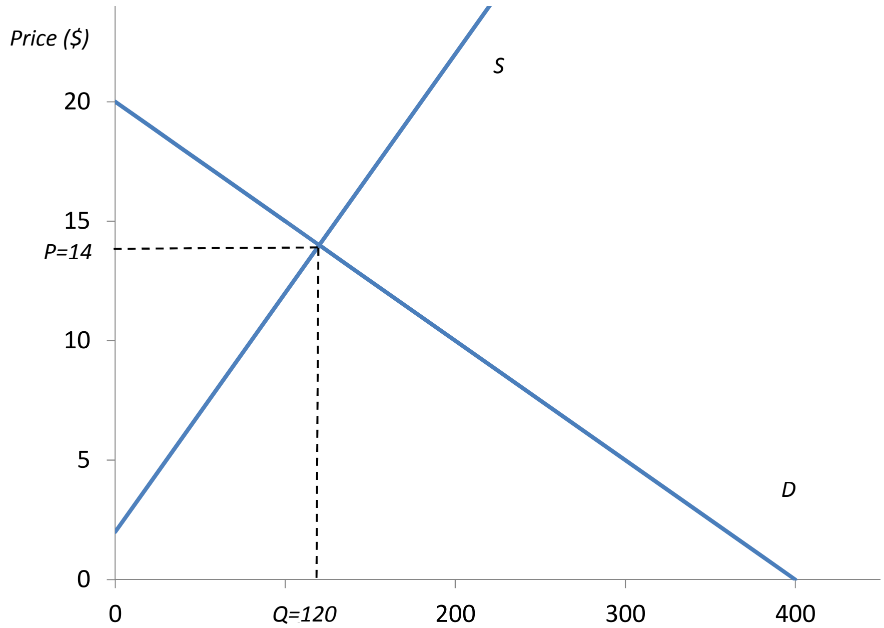
3. (20 points) The market demand for chairs is given by $Q^D = 400 - 20p$ and the market supply is $Q^S = -20 + 10p$.

(a) (10 points) Draw the market demand and supply curves on a graph with quantities in the x-axis and price in the y-axis. What is the market equilibrium price and quantity?

The equilibrium price and quantity are determined by the intersection of market supply and demand. Making both quantities demanded and supply equal we get:

$$Q^D = Q^S$$
$$400 - 20p = -20 + 10p$$
$$420 = 30p \rightarrow \boxed{p = 14} \tag{5}$$

At this price, the equilibrium quantity is $Q = Q^D = Q^S = -20 + 10p = 400 - 20p \rightarrow$ $\boxed{Q = 120}$



(b) (5 points) Now suppose a new competitor enters the market and is able to supply 30 additional chairs at every price. Explain how the market equilibrium changes.

This questionn does not require calculations. It is sufficient to say that an additional competitor will shift the supply curve to the right, which will lead to a larger quantity and lower price in equilibrium.

Mathematically, the new supply curve will shift to the right and is now

$$
\begin{aligned}
Q^S_{new} &= Q^S + 30 \\
Q^S_{new} &= -20 + 10p + 30 \\
Q^S_{new} &= 10p + 10
\end{aligned} \tag{6}
$$

With this new supply curve we recalculate the equilibrium:

$$
\begin{aligned}
Q^D &= Q^S \\
400 - 20p &= 10p + 10 \\
390 &= 30p \rightarrow \boxed{p = 13}
\end{aligned} \tag{7}
$$

At this price, the equilibrium quantity is $Q = Q^D = Q^S = 10 + 10p = 400 - 20p \rightarrow$ $\boxed{Q = 140}$ So the new equilibrium leads to a lower price and a higher quantity.

(c) (5 points) Now suppose that, in addition to the new supplier there in part (b) there is also an increase in demand because of a new tax rebate the government is granting all consumers. This rebate means that consumers will be purchasing 30 more units at every price. How does the market equilibrium change?

Here also you do not need to do the calculations, although you can. The increase in demand means it shifts to the right and that pushes the equilibrium price and quantity up. Mathematically,

$$Q^D = Q^S$$
$$400 - 20p + 30 = 10p + 10$$
$$420 = 30p \rightarrow \boxed{p = 14} \quad (8)$$

At this price, the equilibrium quantity is $Q = Q^D = Q^S = 10 + 10p = 430 - 20p \rightarrow \boxed{Q = 150}$ So the new equilibrium leads to a higher price than in (b) and same as original price in (a) and a higher quantity.

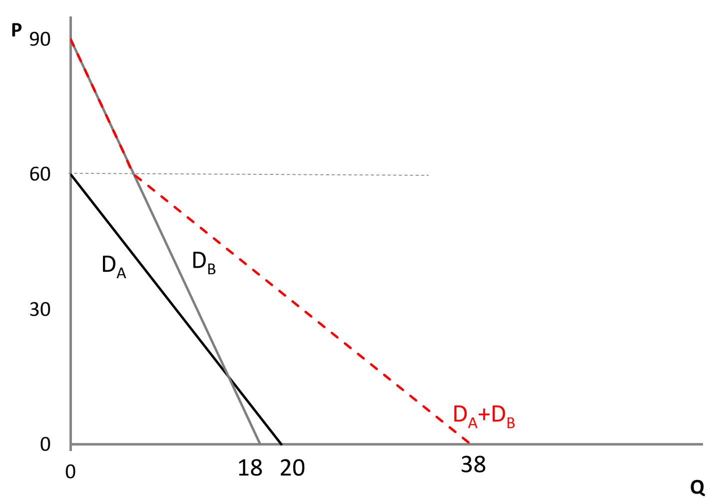
4. (20 points) Consumers A and B have the following inverse demand curves:

$$p = 60 - 3Q_A$$

$$p = 90 - 5Q_B$$

(a) (3 points) Draw both demand curves. Label them D_A and D_B. Make sure you also find the values for the intercepts.

Figure 1. Demand aggregation



(b) (7 points) On the same graph, draw the aggregate demand curve. Label it $D_A + D_B$.

(c) (10 points) Calculate the aggregate demand for both consumers mathematically.

Because we were given inverse demands, we need to rewrite them as demands to add up the quantities:

$$p = 60 - 3Q_A \qquad \rightarrow \qquad Q_A = 20 - \frac{1}{3}p$$
$$p = 90 - 5Q_B \qquad \rightarrow \qquad Q_B = 18 - \frac{1}{5}p$$

Now we can add up the demand curves:

$$Q_A + Q_B = 20 - \frac{1}{3}p + 18 - \frac{1}{5}p$$
$$Q_A + Q_B = 38 - \left(\frac{1}{3} + \frac{1}{5}\right)p$$
$$Q_A + Q_B = 38 - \frac{8}{15}p$$