

Department of Economics
Carleton University
ECON 2009A – Managerial Economics
Quiz #4 - November 15, 2018 (45 minutes)

1. (30 points) You are given the following information about the current conditions of a particular perfectly competitive industry with identical firms:

Market demand: $Q^D = 1,410 - 40P$

Market supply: $Q^S = 30P - 60$

Firm total cost function: $TC(q) = 50 + 2q + 2q^2$

- (a) (5 points) What is the current equilibrium total price and quantity in the market?

The equilibrium price and quantity are those where market demand equals market supply ($Q^D = Q^S$):

$$1,410 - 40P^* = 30P^* - 60$$

$$1,470 = 70P^*$$

$$21 = P^*$$

Replacing either supply or demand, the equilibrium quantity is:

$$Q^* = 1,410 - 40 \times 21 = 570$$

- (b) (5 points) What is the supply curve of each firm?

The supply curve of each firm is the marginal cost curve above the average variable cost.

$$MC = 2 + 4q$$

$$P = 2 + 4q \text{ (inverse supply curve)} \rightarrow \boxed{q^S = \frac{P}{4} - \frac{1}{2}} \text{ (supply curve)}$$

Note that this supply curve is always above the AVC , so there are no additional limitations for it.

$$P \geq AVC$$

$$2 + 4q \geq 2 + 2q$$

$$4q \geq 2q \text{ is always true for positive } q \quad (1)$$

Note also, that because the MC (or inverse supply curve) has an intercept of 2, that means there will be no quantity supplied for a price equal to 2 or below.

- (c) (5 points) What is the amount of output supplied by each firm? How many firms are there in the market?

The optimal amount of output supplied by each individual firm will be where profit is maximized, i.e. where $P = MC$. We know that price is \$21 from above. You can start from this condition or you can maximize profit to get to this condition:

$$\begin{aligned}\pi &= Pq - TC(q) \\ \pi &= 21q - (50 + 2q + 2q^2)\end{aligned}$$

π is maximized where:

$$\begin{aligned}\frac{d\pi}{dq} &= 0 \\ 21 - 2 - 4q^* &= 0 \\ \underbrace{21}_P &= \underbrace{2 + 4q^*}_{MC} \rightarrow \boxed{q^* = 4.75}\end{aligned}\tag{2}$$

Because the total quantity traded in the market is $Q^* = 570$ (from part (a)) and each firm produces $q^* = 4.75$, there are $n = 570/4.75 = 120$ firms in this industry.

- (d) (5 points) Are individual firms making a profit?

The profit earned by each firm is then:

$$\pi = 21 \times 4.75 - 50 - 2 \times 4.75 - 2 \times 4.75^2 = -\$4.875$$

- (e) (10 points) Is the equilibrium you just calculated the long-run equilibrium in the economy? If yes, explain why. If not, calculate the long-run equilibrium price and quantity in the market and for each firm. Assume the cost function given applies in the long run.

Because each firm is making negative profits, we would expect firms to exit this industry in the long run. As firms exit the market, the total output will fall and price will increase. In the long run, the market price increases to the point where firm profits are zero, i.e. price is equal to the minimum average cost. This is also the point where marginal cost is equal to average cost.

$$\begin{aligned}AC(q) &= \frac{TC(q)}{q} = \frac{50 + 2q + 2q^2}{q} = \frac{50}{q} + 2 + 2q \\ MC(q) &= \frac{dC(q)}{dq} = 2 + 4q\end{aligned}$$

Set $AC = MC$ to find the level of output for which AC is minimized:

$$\begin{aligned}\frac{50}{q} + 2 + 2q &= 2 + 4q \quad (\text{multiply by } q) \\ 50 + 2q + 2q^2 &= 2q + 4q^2 \\ 50 &= 2q^2 \rightarrow q^2 = 25 \rightarrow \boxed{q_{LR} = 5}\end{aligned}$$

The minimum price is defined by the AC or MC when $q = 5$.

$$AC(q = 5) = \frac{50}{5} + 2 + 2 \times 5 = 22$$

$$MC(q = 5) = 2 + 4 \times 5 = 22$$

At this minimum price and corresponding output, firms make zero profits:

$$\begin{aligned}\pi &= Pq - TC(q) \\ &= 22 \times 5 - 50 - 2 \times 5 - 2 \times 5^2 \\ &= 110 - 50 - 10 - 50 \\ &= 0\end{aligned}$$

At that price also, consumers demand a total of:

$$Q^D = 1,410 - 40 \times 22 = 530.$$

So there 530 units is the equilibrium quantity in the market.

2. (30 points) A monopolist faces the inverse demand for its output: $p = 30 - Q$. The monopolist also has a constant marginal and average cost of \$4/unit.

(a) (10 points) What is the monopolist's profit-maximizing level of output? What is the monopolist's profit at this level of output?

You can start with the monopolist's profit-maximizing condition $MR = MC$ or from the profit function:

$$\begin{aligned}\pi &= p(Q)Q - C(Q) \\ \pi &= (30 - Q)Q - 4Q \quad \text{Note: } TC = AC \times Q = 4Q \\ \pi &= 30Q - Q^2 - 4Q \\ \pi &= -Q^2 + 26Q\end{aligned}$$

The first order condition is:

$$\frac{d\pi}{dQ} = -2Q + 26 = 0 \quad \text{This is identical to setting } MR = MC$$

$$\frac{26}{2} = Q^*$$

$$13 = Q^*, \text{ replacing in the demand function } \rightarrow p^* = \$17$$

Replacing both of these back into the monopolist's profit function:

$$\pi = p(Q^*)Q^* - C(Q^*) = 17 \times 13 - 4 \times 13 = 169$$

- (b) (10 points) Calculate the consumer surplus and show it in a graph along with the monopolist's equilibrium; label the monopolist's equilibrium e^M .

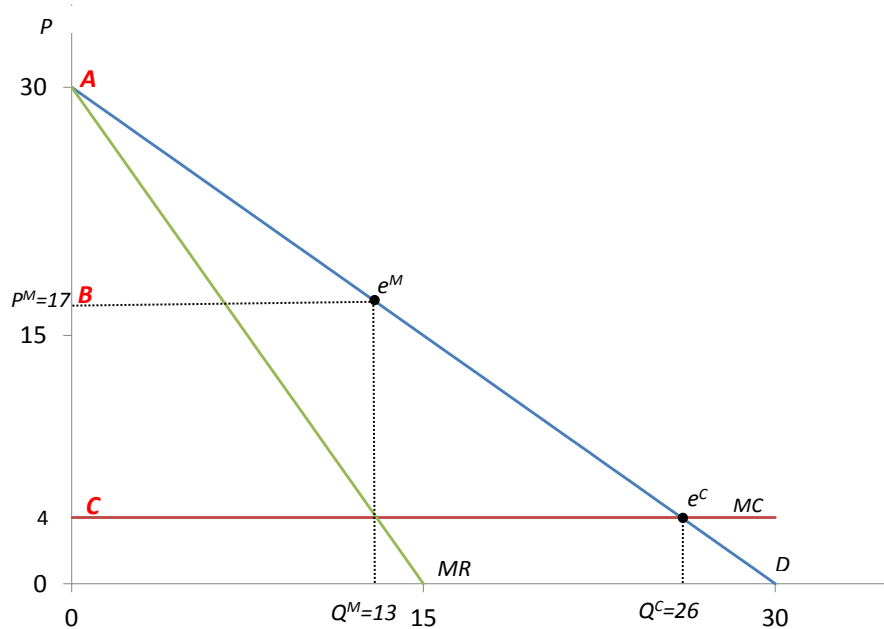
The consumer surplus is the area under the inverse demand curve up to the equilibrium price. Because the demand curve is linear, we know this area is a triangle (ABe^M in the graph). To find the height of the triangle we need to find the intercept of the inverse demand curve, i.e. the constant in $p(Q)$, and the equilibrium price, which we already calculated above. So the height of the triangle is $30 - 17 = 13$.

The base of the triangle is the quantity sold, which we also calculated above: $Q = 13$.

The consumer surplus (or area of the triangle) is:

$$CS^M = \frac{13 \times 13}{2} = \frac{169}{2} = 84.5$$

The monopolist's equilibrium in the graph is e^M , where the equilibrium quantity Q^M is determined by the intersection of the MC and the MR and the equilibrium price, p^M is given by the demand at the equilibrium quantity.



- (c) (5 points) What would the competitive equilibrium be if the demand curve were the same as the one faced by the monopolist and the market supply curve were equal to the monopolist's marginal cost, i.e. $P = MC$? Show this equilibrium in the same graph as above and label it e^C .

The competitive equilibrium is determined by the intersection of supply and demand. Because the monopolist's marginal cost curve is constant $MC = 4$, then the supply curve is horizontal at $P^* = MC = 4$. This will be the competitive price which we replace in the demand curve to get the equilibrium market quantity.

$$P^* = 30 - Q^*$$

$$4 = 30 - Q \rightarrow \boxed{Q^* = 26}$$

The competitive equilibrium in the graph above is given by point e^C .

- (d) (5 points) What is the gain in consumer surplus if this market were perfectly competitive instead of a monopoly?

Now the consumer surplus is triangle ACe^C . The height of the triangle is $(30 - 4)$ and the base is 26. Therefore the consumer surplus in the competitive equilibrium is

$$CS^C = 26^2/2 = 338$$

The increase in consumer surplus is $CS^C - CS^M = 338 - 84.5 = 253.5$.

3. (40 points) Answer the following questions briefly.

- (a) (10 points) What is the difference between the marginal revenue of a competitive firm and the marginal revenue of a monopolist?

The marginal revenue of a competitive firm is equal to the equilibrium competitive price determined by the market supply and demand curve. Hence, it is a fixed number because a single competitive firm cannot change the market price.

The marginal revenue of a monopoly, on the other hand, is decreasing because the monopolist affects the market price when it sells additional units. The incremental revenue from selling an additional unit is composed of two opposing effects: the increased revenue from the extra unit sold and the decreased revenue from the previous units being sold at a lower price.

- (b) (10 points) Explain why having a competitive industry turn into a monopoly would not be desirable.

Total welfare (consumer surplus + producer surplus) is maximized under perfect competition. A monopoly would reduce the quantity sold and increase the market price, reducing total welfare. Even if it increases profits for itself, the increase in producer surplus occurs at the expense of consumers and a deadweight loss is generated.

- (c) (10 points) Explain whether it is possible for a competitive firm to operate and have negative profit.

A competitive firm can operate at a negative profit in the short run if the loss is smaller than the fixed costs it would have to pay if it decided to shut down.

- (d) (10 points) Explain why and how is it that competitive firms make zero profit in the long run.

Competitive firms make zero profit in the long run because, if they didn't, free entry or exit of firms would occur until profits are pushed to zero. If profits are positive, entry would occur, increasing supply and decreasing price until profits are lowered to zero. If they are negative, firms would exit, decreasing supply and pushing price up until profits are zero.