Department of Economics

Carleton University

ECON 2009A – Managerial Economics

Quiz #3 - November 1, 2018 (45 minutes)

1. (20 points) Explain whether you agree or disagree with each statement below based on cost concepts discussed in class.

   (a) (6 points) A few years after graduating you have the opportunity of recording a single and must decide if you want to continue working as an accountant (your college major) or if you want to make a career change and become a singer. The cost of your education will matter for your decision.

   No, the cost of your education is a sunk cost and should not matter for your decisions.

   (b) (7 points) If the marginal cost of producing a good is increasing as a firm produces more of the good, then the marginal product of labor (MP_L) is decreasing.

   This is correct. The marginal cost and marginal product have an inverse relationship, i.e. when one increases the other decreases. Intuitively, when the cost of producing one more unit is higher than the cost of producing the previous unit, then it has to be the case that the last employee hired is less productive than the previous one — all else constant.

   (c) (7 points) The New Developments construction company has the following short-run total cost function: $TC = q^3 - 10q^2 + 36q$. The average cost (AC) of this company is minimized when $q = 6$ and $AC = 12$.

   You can find the minimum of the AC by either using that $\frac{dAC}{dq} = 0$ or you can use the fact that $MC = AC$ at the minimum of the average cost. $AC = \frac{TC}{q} = \frac{q^3-10q^2+36q}{q} = q^2-10q+36$

$$\frac{dAC}{dq} = 0$$

$$2q - 10 = 0 \rightarrow \boxed{q = 5}$$

   So AC is minimized when $q = 5$ and $AC = 11$.

   The alternative way is setting $AC = MC$.

$$MC = \frac{dTC}{dq} = 3q^2 - 20q + 36$$

$$
\begin{aligned}
AC &= MC \\
q^2 - 10q + 36 &= 3q^2 - 20q + 36 \\
0 &= 2q^2 - 10q \\
0 &= q(2q - 10) \rightarrow \boxed{q = 5}
\end{aligned}
$$

And we get the same answer as before. Note that $q = 0$ is also an answer but it is not relevant as it would mean that the firm is not producing any output.

2. (20 points) Show and explain whether the following production functions exhibit constant, increasing or decreasing returns to scale:

   For each of the functions below we compare whether output scales by more than, less than or the same proportion as inputs. I use doubling of inputs and compare with double the output, but you can multiply inputs by any integer.

   (a) (5 points) $F(L, K) = L + K$

   $$
   \begin{aligned}
   \text{Double output} &\gtreqless \text{Output with double inputs} \\
   2F(L, K) &\gtreqless F(2L, 2K) \\
   2(L + K) &\gtreqless 2L + 2K \\
   2(L + K) &= 2(L + K)
   \end{aligned}
   $$

   Because when inputs double, output also doubles, this production function exhibits constant returns to scale.

   (b) (5 points) $F(L, K) = LK$

   $$
   \begin{aligned}
   2F(L, K) &\gtreqless F(2L, 2K) \\
   2(LK) &\gtreqless 2L2K \\
   2LK &< 4LK
   \end{aligned}
   $$

   Because when inputs double, output more than doubles, this production function exhibits increasing returns to scale.

   (c) (5 points) $F(L, K) = L^{0.25}K^{0.75}$

$$
\begin{aligned}
2F(L,K) &\gtreqless F(2L,2K)\\
2L^{0.25}K^{0.75} &\gtreqless (2L)^{0.25}(2K)^{0.75}\\
2L^{0.25}K^{0.75} &\gtreqless 2^{0.25}L^{0.25}2^{0.75}K^{0.75}\\
2L^{0.25}K^{0.75} &\gtreqless 2^{0.25+0.75}L^{0.25}K^{0.75}\\
2L^{0.25}K^{0.75} &= 2L^{0.25}K^{0.75}
\end{aligned}
$$

Because when inputs double, output also doubles, this production function exhibits constant returns to scale.

(d) (5 points) $F(L,K) = 10L + 10K$

$$
\begin{aligned}
2F(L,K) &\gtreqless F(2L,2K)\\
2(10L+10K) &\gtreqless 10(2L) + 10(2K)\\
20L + 20K &= 20L + 20K
\end{aligned}
$$

Because when inputs double, output also doubles, this production function exhibits constant returns to scale.

3. (20 points) What is the average product of labor for each of the production functions below when $K = 16$.

(a) (5 points) $F(L,K) = L + K$

$$
\begin{aligned}
Q &= L + 16\\
AP_L &= \frac{Q}{L}\\
AP_L &= \frac{L+16}{L}
\end{aligned}
$$

(b) (5 points) $F(L,K) = LK$

$$
\begin{aligned}
Q &= 16L\\
AP_L &= \frac{Q}{L}\\
AP_L &= \frac{16L}{L} = 16
\end{aligned}
$$

(c) (5 points) $F(L,K) = L^{0.25}K^{0.75}$

$$Q = L^{0.25}16^{.75} = 8L^{.25}$$
$$AP_L = \frac{Q}{L}$$
$$AP_L = \frac{8L^{.25}}{L} = 8L^{-.75}$$

(d) (5 points) $F(L,K) = 10L + 10K$

$$Q = 10L + 160$$
$$AP_L = \frac{Q}{L}$$
$$AP_L = \frac{10L + 160}{L}$$

4. (20 points) Use the cost function definitions discussed in class to fill in the data in the table below. Make a list of the steps you follow to do so.

Q	Fixed Cost	Variable Cost	Total Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost	Marginal Cost
0					-		
1					50		
2				50	40		
10					10		

The complete table is:

Q	Fixed Cost	Variable Cost	Total Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost	Marginal Cost
0	100	0	100	-	-	-	
1	100	50	150	100	50	150	50
2	100	80	180	50	40	90	30
10	100	100	200	10	10	20	2.5

We can obtain those values following these steps:

(a) Fixed Cost (FC) $= AFC \times Q = 50 \times 2 = 100$, and since FC don't change with output, this is the full column

(b) Variable Cost (VC) $= AVC \times Q$, for each level of output

(c) Total Cost is just the sum of the Fixed and Variable Costs

(d) Average Total Cost is the sum of the Avg. Fixed and Avg. Variable Costs

(e) Marginal cost $= \frac{\Delta TC}{\Delta Q}$, i.e. the discrete way of calculating the marginal cost because we do not have an equation for the cost function, just the table. The last value is the only slightly different because the output jumps from 2 to 10, so $MC = \frac{200-180}{10-2} = \frac{20}{8} = 2.5$.

5. (20 points) A firm's production function is

$$Q = 5LK$$

where:
$Q =$ units of output
$K =$ units of capital
$L =$ units of labor
and the price per unit of labor is $p_L = \$10$ and the price per unit of capital is $p_K = \$5$.

(a) (5 points) If the firm has only \$100 to spend on capital and labor, write down the firm's cost constraint or isocost line corresponding to that budget using the data provided.
The isocost line for the firm is like the consumer's budget constraint. Total spending on inputs has to be equal to the budget \$100:

$$p_K K + p_L L = \$100$$

$$5K + 10L = \$100$$

(b) (15 points) Calculate the maximum amount of output the firm is able to produce and the optimal amounts of labor and capital used to produce that level of output.
In order to maximize output subject to the isocost line, we need to satisfy the tangency condition (between the isocost line and the isoquant) and the isocost equation from (a)
Tangency condition:

$$\begin{aligned} \text{Slope of isoquant} &= \text{Slope of isocost} \\ MRTS &= \frac{p_L}{p_K} \\ \frac{MP_L}{MP_K} &= \frac{p_L}{p_K} \\ \frac{5K}{5L} &= \frac{10}{5} \\ K &= 2L \end{aligned} \tag{1}$$

Replacing this back into the isocost condition:

$$5K + 10L = 100$$
$$5(2L) + 10L = 100$$
$$10L + 10L = 100 \rightarrow \boxed{L = 5}$$

Replacing this result back into (1), $K = 10$. The total amount of output with these quantities of inputs is:

$$Q = 5LK$$

$$250 = 5 \times 5 \times 10$$