Carleton University

ECON 2009 I – Managerial Economics

Midterm Exam

February 26, 2019

1. (35 points) Short questions:

   (a) (6 points) The monthly demand for breakfast cereal was estimated to be the following

   $$Q^D = 500 - 55P - 5P_{milk} + 7P_{eggs} + 0.3I$$

   where

   - Q^D is the quantity demanded of cereal (boxes),
   - P is the price of a box of cereal,
   - P_{milk} is the price of a gallon of milk,
   - P_{eggs} is the price of a dozen eggs and
   - I is average income (in thousands of dollars).

   Currently, $P = \$7$, $P_{milk} = \$5$, $P_{eggs} = \$4$ and the average income is \$50,000.
   Note: For your calculations below, provide the formulas for the elasticities requested in addition to your calculation.

   i. (2 points) What is the price elasticity of demand for breakfast cereal? What does this number mean exactly? Is it elastic or inelastic?

   $$\eta = \frac{dQ}{dP}\frac{P}{Q}$$
   $$\eta = -55\frac{7}{Q}$$

   We calculate Q by replacing the data given into the demand function:

   $$Q = 500 - 55 \times 7 - 5 \times 5 + 7 \times 4 + 0.3 \times 50 = 133$$

   Replacing back into the elasticity formula:

   $$\eta = -55\frac{7}{133} = -2.89$$

   This figure means that an increase in price of 1% will lead to decrease in quantity of 2.89%. Because the decrease in quantity is more than proportional to the increase in price, the demand for breakfast cereal is elastic.

ii. (2 points) What is the cross-price elasticity of demand between cereal and milk? What does this number mean? What is the relationship between these two goods?

$$\eta_{Pmilk} = \frac{dQ}{dP_{milk}} \frac{P_{milk}}{Q}$$
$$\eta_{Pmilk} = -5\frac{5}{133} = -0.19$$

A cross-price elasticity of -0.19 means that a 1% increase in the price of milk will lead to a decrease of 0.19% in the quantity of cereal demanded. This means the two goods are complements.

iii. (2 points) What is the cross-price elasticity of demand between cereal and eggs? What does this number mean? What is the relationship between these two goods?

$$\eta_{Peggs} = \frac{dQ}{dP_{eggs}} \frac{P_{eggs}}{Q}$$
$$\eta_{Peggs} = 7\frac{4}{133} = 0.21$$

A cross-price elasticity of 0.21 means that a 1% increase in the price of eggs will lead to an increase of 0.21% in the quantity of cereal demanded. This means the two goods are substitutes.

(b) (10 points) For each case below describing changes affecting the **market for bread**, note whether the statement is true, false, or uncertain, and explain your answer. (You may draw a graph but full marks require an explanation of how each event affects the market for bread).

i. Dietitians of Canada publish a report stating a new finding showing that eating bread increases the mortality rate by 10% and, at the same time, the cost of flour increases; then the equilibrium quantity of bread will fall, and the price of bread will fall.
The report shifts demand to the left because consumers will like bread less
An increase in the cost of flour shifts supply to the left due to higher input costs
Equilibrium quantity will definitely fall, but whether price will rise, fall, or remain constant depends on the relative sizes of the supply and demand shifts.

ii. If the price of crackers (a substitute for bread) increases and the price of flour (used to make bread) falls, the quantity and price of bread will both increase.
The increase in the price of the substitute increases demand for bread (shift to the right)
The fall in the price of the input increases supply (shift to the right).
Equilibrium quantity increases, but price may rise, fall, or remain unchanged depending on the relative magnitude of the shifts.

(c) (10 points) If a firm's product has a current price elasticity of -2. How should the firm change its price to increase revenue? How will this affect profit? Explain.
Because demand is elastic, dropping price will increase revenue.
This will not necessarily increase profit. A drop in price will increase quantity produced and will therefore increase total cost will increase. Total profit goes up due to higher revenue but drops due to higher cost. The final effect will depend on which effect is larger.

(d) (9 points) What is the difference between the average product of labor (AP_L) and the marginal product of labor (MP_L)? What do these two measures have in common?
AP_L measures the average output per worker, while MP_L measures the output of the last worker hired.
Both are measures of productivity.

2. (25 points) Maria derives utility from swimming and yoga in the following way:

$$U(x, y) = x^{0.5}y^{0.5}$$

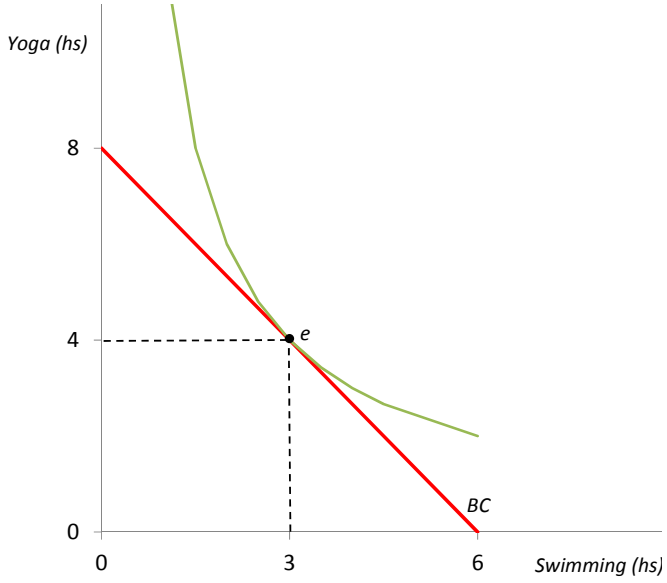
where x is the number of hours of swimming and y is the number of hours of yoga per week. Maria has budgeted $240 per week for her swimming and yoga time. If the price of swimming is $40 per hour and the price of yoga is $30 per hour, answer the following questions.

(a) (10 points) Write the equation for Maria's budget constraint and draw it on a graph showing swimming hours on the x-axis and yoga hours on the y-axis. Label the axes and show and explain the values of the intercepts.

Maria's budget constraint is:

$$I = p_x x + p_y y$$
$$240 = 40x + 30y$$

The intercepts show the number of units of each good that Maria can buy if she spends all of her income on that good.



(b) (3 points) What is Maria's marginal utility of swimming and what is her marginal utility of yoga?

$$\begin{aligned} MU_x &= \frac{\partial U}{\partial x} \\ &= 0.5x^{-0.5}y^{0.5} \\ MU_y &= \frac{\partial U}{\partial y} \\ &= 0.5x^{0.5}y^{-0.5} \end{aligned}$$

(c) (2 points) What is Maria's marginal rate of substitution between the two goods?

Since the question does not specify which MRS, either answer is correct.

The MRS of swimming for yoga:

$$MRS = \frac{MU_x}{MU_y} = \frac{y}{x} \tag{1}$$

The MRS of yoga for swimming is:

$$MRS = \frac{MU_y}{MU_x} = \frac{x}{y} \tag{2}$$

(d) (10 points) What is the amount of swimming and yoga hours that maximize Maria's utility given the current prices and her budget? Show this optimal bundle and the indifference curve that goes through it on the graph in part (a) (The indifference curve does not have to be drawn to scale). Label the optimal bundle e.

Maria maximizes her utility if her budget constraint is satisfied and the tangency condition between an indifference curve and her budget constraint is satisfied.

Tangency condition:

$$\begin{aligned} MRS &= MRT \\ \frac{MU_x}{MU_y} &= \frac{p_x}{p_y} \\ \frac{y}{x} = \frac{40}{30} &\rightarrow \boxed{y = \frac{4}{3}x} \end{aligned} \tag{3}$$

We replace equation 3 in Maria's budget constraint:

$$\begin{aligned} 240 &= 40x + 30y \\ 240 &= 40x + 30\frac{4}{3}x \\ 240 = 80x &\rightarrow \boxed{x = 3} \end{aligned} \tag{4}$$

Replacing this result back into equation 3, we can obtain the result for $y = 4$. So the optimal bundle for Maria is 3 hours of swimming and 4 hours of yoga.

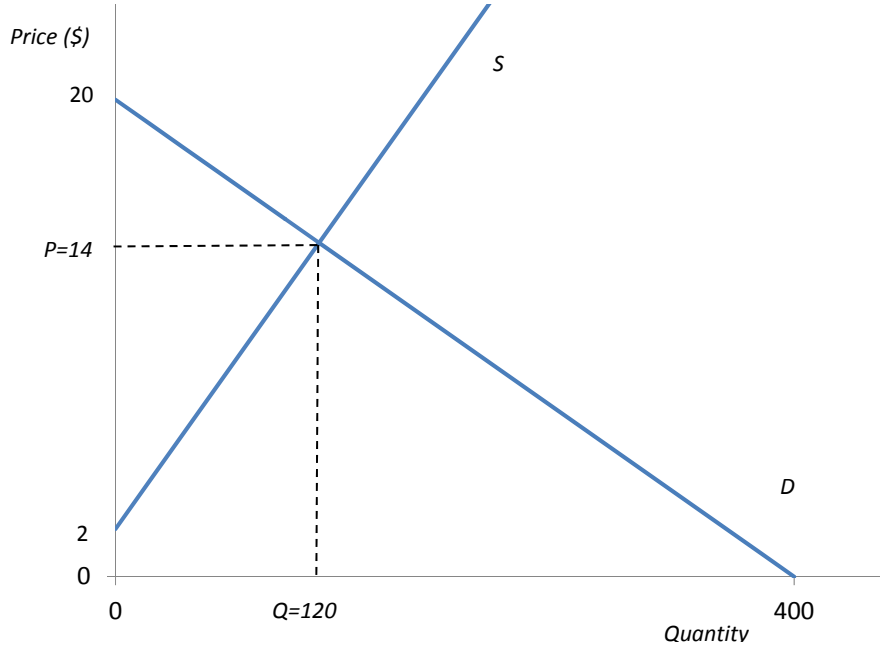
3. (20 points) The market demand for chairs is given by $Q^D = 400 - 20p$ and the market supply is $Q^S = -20 + 10p$.

   (a) (10 points) Draw the market demand and supply curves on a graph with quantities in the x-axis and price in the y-axis. What is the market equilibrium price and quantity?

   In equilibrium:

$$\begin{aligned} Q^D &= Q^S \\ 400 - 20p &= -20 + 10p \\ 420 &= 30p \rightarrow \boxed{p = 14} \end{aligned} \tag{5}$$

   At this price, the equilibrium quantity is $Q = Q^D = Q^S = -20 + 10p = 400 - 20p \rightarrow \boxed{Q = 120}$



   (b) (5 points) Now suppose a new competitor enters the market and is able to supply 30 additional chairs. Calculate and explain how the market equilibrium changes.

   An additional competitor will shift the supply curve to the right, which will lead to a larger quantity and lower price in equilibrium.

   Mathematically, the new supply curve will shift to the right and is now

$$\begin{aligned} Q^S_{new} &= Q^S + 30 \\ Q^S_{new} &= -20 + 10p + 30 \\ Q^S_{new} &= 10p + 10 \end{aligned} \tag{6}$$

With this new supply curve we recalculate the equilibrium:

$$\begin{aligned} Q^D &= Q^S_{new} \\ 400 - 20p &= 10p + 10 \\ 390 &= 30p \rightarrow \boxed{p = 13} \end{aligned} \tag{7}$$

At this price, the equilibrium quantity is $Q = Q^D = Q^S = 10 + 10p = 400 - 20p \rightarrow \boxed{Q = 140}$.

(c) (5 points) Now suppose that, in addition to the new supplier there in part (b) there is also an increase in demand because of a new tax rebate the government is granting all consumers. This rebate means that consumers will be purchasing 30 more units. Calculate the change in the market equilibrium.

$$\begin{aligned} Q^D_{new} &= Q^S_{new} \\ 400 - 20p + 30 &= 10p + 10 \\ 420 &= 30p \rightarrow \boxed{p = 14} \end{aligned} \tag{8}$$

At this price, the equilibrium quantity is $Q = Q^D = Q^S = 10 + 10p = 430 - 20p \rightarrow \boxed{Q = 150}$

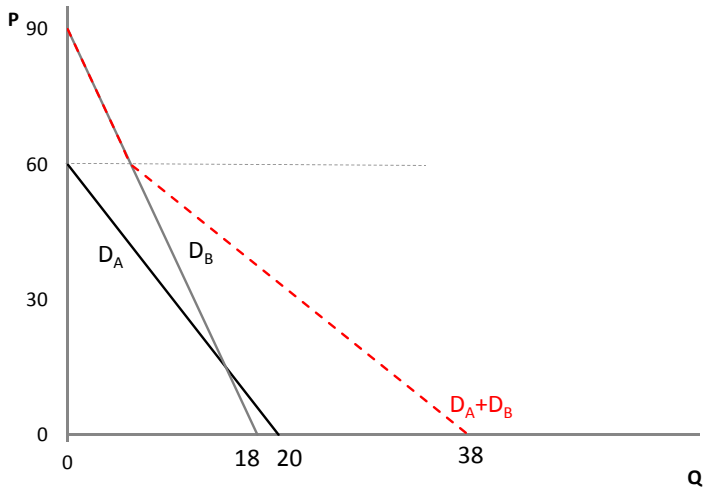
4. (20 points) Consumers A and B have the following inverse demand curves:

$$p = 60 - 3Q_A$$

$$p = 90 - 5Q_B$$

(a) (3 points) Draw both demand curves. Label them D_A and D_B. Make sure you also find the values for the intercepts.

Figure 1. Demand aggregation



(b) (7 points) On the same graph, draw the aggregate demand curve. Label it $D_A + D_B$.

(c) (10 points) Calculate the aggregate demand for both consumers mathematically.
Because we were given inverse demands, we need to rewrite them as demands to add up the quantities:

$$p = 60 - 3Q_A \quad \rightarrow \quad Q_A = 20 - \frac{1}{3}p$$
$$p = 90 - 5Q_B \quad \rightarrow \quad Q_B = 18 - \frac{1}{5}p$$

Now we can add up the demand curves:

$$Q_A + Q_B = 20 - \frac{1}{3}p + 18 - \frac{1}{5}p$$
$$Q_A + Q_B = 38 - \left(\frac{1}{3} + \frac{1}{5}\right)p$$
$$Q_A + Q_B = 38 - \frac{8}{15}p$$

This is the lower portion of the aggregate demand, i.e. for prices at or below 60. The aggregate demand equation for prices between 60 and 90 corresponds to that of consumer B.