Elements of Solution Assignment # 1

Problem 1

A country ("Home") is populated with 300 workers who produce either food (F) and/or clothing (C). Each food worker produces 6 units of food and each clothing worker produces 3 units of clothing. The preferences of the consumers over food and clothing are represented by the utility function:

$$\overline{L} = 300 \qquad a_{L_F} = 1/6 \qquad a_{L_C} = 1/3$$

$$u(D_F, D_C) = (D_F)^{\frac{2}{3}}(D_C)^{\frac{1}{3}}$$

1) *Find the relationship between relative demand, D_F/D_C , and the relative price, P_F/P_C.*

$$\begin{aligned} MRS_{D_F,D_C} &\equiv \frac{\partial U/\partial D_F}{\partial U/\partial D_C} = \frac{P_F}{P_C} \\ \frac{\frac{2}{3}\frac{U}{D_F}}{\frac{1}{3}\frac{U}{D_C}} &= \frac{P_F}{P_C} \Longrightarrow \\ \frac{D_F}{D_C} &= \frac{2}{\frac{P_F}{P_C}} \end{aligned}$$

2) *Autarky price in Home (the relative price of food and clothing)*

In autarky the relative price is equal to the opportunity cost in Home.

$$\begin{aligned} \frac{P_F}{P_C} &= \frac{a_{L_F}}{a_{L_C}} = \frac{1/6}{1/3} \\ \frac{P_F}{P_C} &= \frac{1}{2} \end{aligned}$$

3) *Equilibrium relative produced quantity (Q_F/Q_C) in autarky in Home?*

$$\begin{aligned}
\frac{Q_F}{Q_C} &= \frac{D_F}{D_C} = \frac{2}{\frac{P_F}{P_C}} \\
\frac{Q_F}{Q_C} &= \frac{2}{1/2} \\
\frac{Q_F}{Q_C} &= 4
\end{aligned}$$

See Figure 1 for a graphical representation.

4) *Expression of the production possibility frontier*

$$\begin{aligned}
\overline{L} &= a_{L_F} Q_F + a_{L_C} Q_C \\
300 &= \frac{1}{6} Q_F + \frac{1}{3} Q_C \\
1800 &= Q_F + 2Q_C
\end{aligned}$$

Equilibrium quantities of food (Q_F) and clothing (Q_C) in autarky

The equilibrium quantities are found at the intersection of the relative quantity produced and the production possibility frontiers. We have the following system of two equations:

$$\begin{aligned}
\frac{Q_F}{Q_C} &= 4 \\
1800 &= Q_F + 2Q_C
\end{aligned}$$

The solution to this system is $Q_F = 1200$ and $Q_C = 300$.

Numbers of workers (L_F, and L_C) employed in each of the two industries in autarky.

$L_F = a_{L_F} Q_F \Longrightarrow L_F = \frac{1}{6} 1200 = 200$

$L_C = a_{L_C} Q_C \Longrightarrow L_C = \frac{1}{3} 300 = 100$

One can easily check that $L_F + L_C = 300$

Foreign country

$\overline{L}^* = 900$ $\qquad a^*_{L_F} = 1/1$ $\qquad a^*_{L_C} = 1/2$

5) Answer questions (2) and (3) for the Foreign economy in autarky.

Question 2 for Foreign

In autarky the relative price is equal to the opportunity cost in Foreign.

$$\begin{aligned} \frac{P^*_F}{P^*_C} &= \frac{a^*_{L_F}}{a^*_{L_C}} = \frac{1}{1/2} \\ \frac{P^*_F}{P^*_C} &= 2 \end{aligned}$$

Question 3 for Foreign

Same utility function as in Home

Equilibrium relative produced quantity (Q^*_F/Q^*_C) in autarky

$$\begin{aligned} \frac{Q^*_F}{Q^*_C} &= \frac{2}{\frac{P^*_F}{P^*_C}} \\ \frac{Q^*_F}{Q^*_C} &= \frac{2}{2} \\ \frac{Q^*_F}{Q^*_C} &= 1 \end{aligned}$$

6) Now assume that these two countries open to trade with one-another.

(a) *World relative supply curve* : See Figure 2

(b) The world equilibrium relative price will be at the intersection of the relative demand and relative supply curves

The two curves cross at the vertical portion of the relative supply curve where

$$\left(\frac{Q_F + Q_F^*}{Q_C + Q_C^*}\right)^{eq} = 1$$
$$\left(\frac{P_F}{P_C}\right)^{eq} = 2$$

(c) As the equilibrium price is larger than the opportunity cost of food in Home, Home will specialize in the production of food

But the equilibrium price is exactly equal to the opportunity cost in Foreign. The latter will produce both goods at that price. It will not specialize.

Home will benefit from trading at the equilibrium price by exporting food to Foreign and importing cloth.

Foreign will not have any benefit from trading with Home as the former is offered the same price as its autarky price. Foreign does not have any incentive to engage in trade with Home.

Problem 2

i. $K^A/L^A = 15/45 = 1/3$ $\qquad$ $K^B/L^B = 10/20 = 1/2$

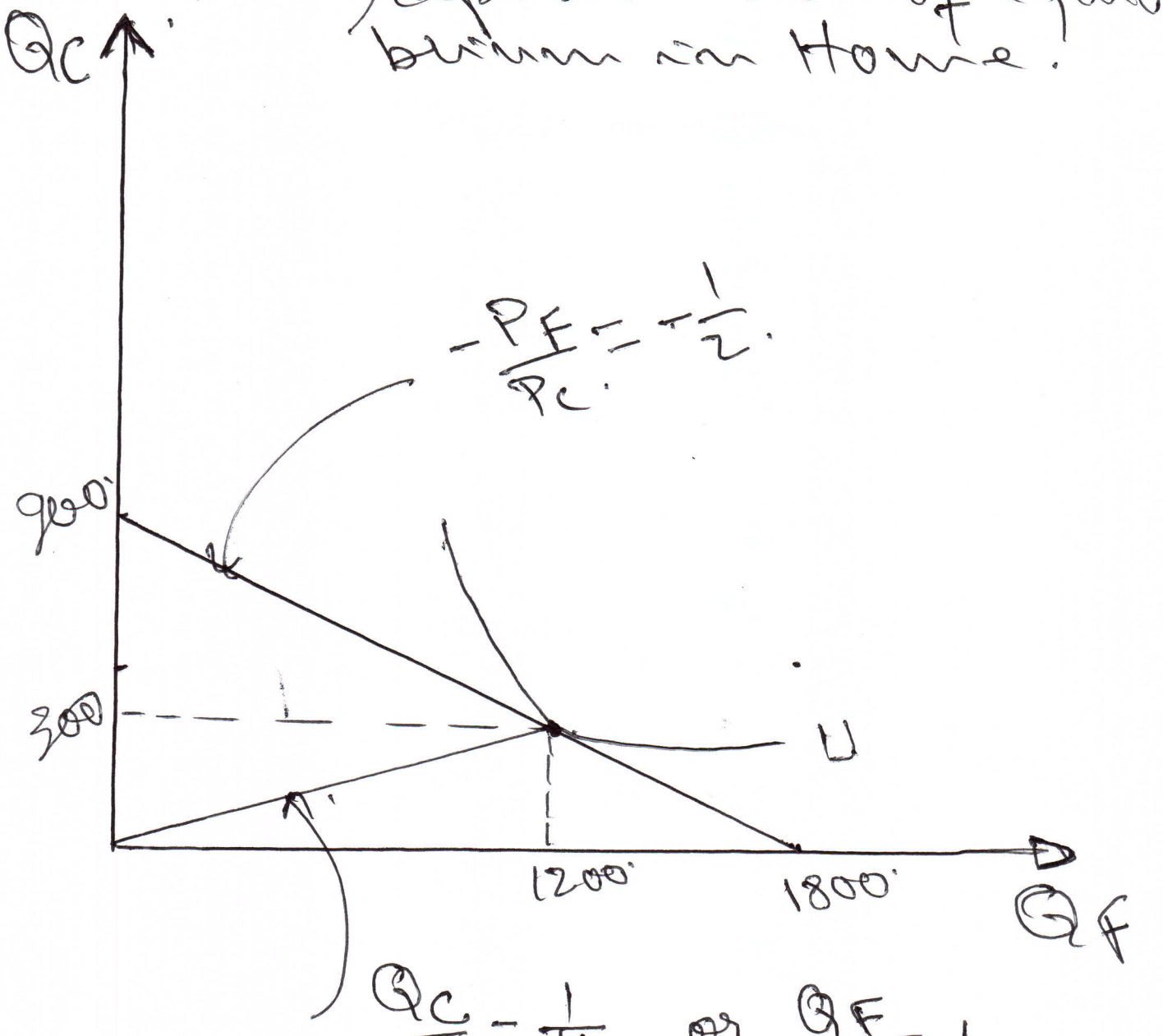
Country B is capital abundant and Country A is labour abundant. According to Heckscher-Ohlin Theorem a country will export the good that is intensive in the factor in which it is abundant and import the good that is intensive in its scarce factor of production. Given that good 1 is capital intensive and country B is abundant in capital it will export good 1. Conversely, as country A is abundant in labour, which is used intensively in the production of good 2, Country A will export good 2.

ii. Country A that is labour abundant will still export good 2 which is labor intensive. Based on Heckscher-Ohlin theory, the pattern of trade depends only on factor proportions. The assumption that A is much richer than B does not affect the relative factor endowments. So A will continue to export the labor intensive good, 2, and import the capital intensive good, 1,

from Country B.

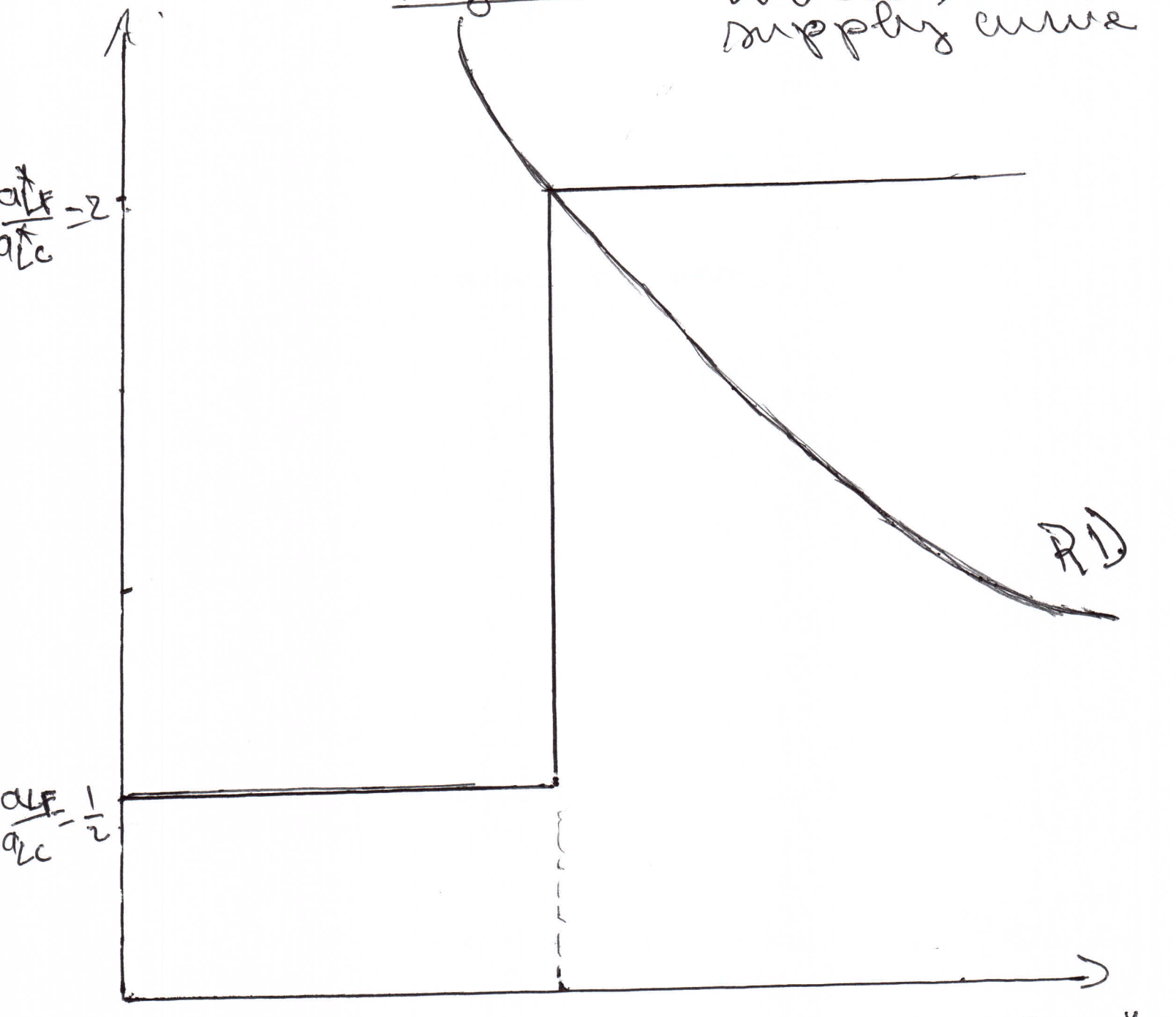
iii. If both labour and capital stock triple in country B, $K^B/L^B = 30/60 = 1/2$ the relative factor endowment does not change in that country. It follows that according to Heckscher-Ohlin theory, the trade pattern would not change.

Figure 1: Graphical representation of equilibrium in Home.



$\frac{Q_C}{Q_F} = \frac{1}{4}$ or $\frac{Q_F}{Q_C} = 4$.

Figure 2



$$\frac{\bar{L}/a_{LF}}{\bar{L}^*/a_{Lc}^*} = \frac{300/1/6}{900/1/2} = 1$$