CARLETON UNIVERSITY

FINAL EXAMINATION
April 2018

DURATION: 3 HOURS **SCANTRON FORMS REQUIRED**

Department Name and Course Number: School of Mathematics and Statistics, MATH 1005A,B,C,D,E,F
Course Instructor(s): Dr. S. Melkonian (Section A), Dr. B. Fodden (Section B), Dr. E. Hua (Sections C and D), Dr. G. Li (Section E), Dr. S. Desjardins (Section F).

AUTHORIZED MEMORANDA
Non-programmable, non-graphic calculators

1. If y is the solution of the initial-value problem $\dfrac{dy}{dx} = 6xy^{2/3}$, $y(0) = 1$, then $y(1) =$

   (a) 1 (b) 2 (c) 0 (d) 8

2. The orthogonal trajectories of the general solution of the equation $y' = -\dfrac{x}{y}$ satisfy the equation

   (a) $y' = \dfrac{x}{y}$ (b) $y' = -\dfrac{y}{x}$ (c) $y' = \dfrac{y}{x}$ (d) $y' = -\dfrac{x}{y}$

3. The general solution of the differential equation $xy' = y + \dfrac{y^2}{x}$ is $y =$

   (a) $\dfrac{-x}{\ln|x| + c}$ (b) $\dfrac{x}{\ln|x| + c}$ (c) $\dfrac{1}{\ln|x| + c}$ (d) $\dfrac{-1}{\ln|x| + c}$

4. If y is the solution of the initial-value problem $x^2y' + xy = 1$, $y(1) = 0$, then $y(e) =$

   (a) 1 (b) e (c) $\dfrac{1}{e}$ (d) $-\dfrac{1}{e}$

5. The general solution of the equation $2x + y + (x + 3y^2)\dfrac{dy}{dx} = 0$ is

   (a) $x^2 + 3y^2 = c$ (b) $x^2 + xy + y^3 = c$ (c) $x^2 + y^3 = c$ (d) $x^2 - y^3 = c$

6. An integrating factor which makes the equation $x + y^3 + 3y^2\dfrac{dy}{dx} = 0$ exact is $I(x) =$

   (a) 1 (b) e^{-x} (c) x (d) e^x

7. The general solution of the equation $y'' + 4y' + 4y = 0$ is $y =$

   (a) $e^{-2x}(c_1 + c_2 x)$ (b) $e^{-2x}(c_1 + c_2 \ln|x|)$ (c) $x^{-2}(c_1 + c_2 \ln|x|)$ (d) $c_1 e^{-2x} + c_2 e^{-2x}$

8. The solution of the initial-value problem $y'' + 9y = 0$, $y(0) = 0$, $y'(0) = -6$, is $y =$

   (a) $-2\sin(3x)$ (b) $e^{-3x} - e^{3x}$ (c) $-6\sin(3x)$ (d) $2\cos(3x)$

9. The general solution of the equation $x^2 y'' + 3xy' + 2y = 0$ for $x \neq 0$ is $y =$

   (a) $c_1 x^{-1} + c_2 x^{-2}$ (b) $c_1 e^{-x} + c_2 e^{-2x}$ (c) $e^{-x}[c_1\cos(x) + c_2\sin(x)]$
   (d) $x^{-1}[c_1\cos(\ln|x|) + c_2\sin(\ln|x|)]$

10. The general solution of the equation $y'' - y' = 6e^{-x}$ is $y =$

   (a) $xe^{-x} + c_1 e^{x} + c_2 e^{-x}$ (b) $6e^{-x} + c_1 + c_2 x$ (c) $3e^{-x} + c_1 e^{x} + c_2 e^{-x}$
   (d) $3e^{-x} + c_1 + c_2 e^{x}$

11. The general solution of the system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{pmatrix} 4 & 5 \\ -2 & -3 \end{pmatrix}$, is $\mathbf{x} =$

   (a) $c_1 e^{2t}\begin{pmatrix} 5 \\ 2 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (b) $c_1 e^{-2t}\begin{pmatrix} 5 \\ 2 \end{pmatrix} + c_2 e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
   (c) $c_1 e^{2t}\begin{pmatrix} 5 \\ -2 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (d) $c_1 e^{-2t}\begin{pmatrix} 5 \\ -2 \end{pmatrix} + c_2 e^{t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

12. Given that the matrix $A = \begin{pmatrix} -2 & -1 \\ 2 & 0 \end{pmatrix}$ has the complex eigenvalue $\lambda = -1 + i$ and the corresponding eigenvector $\mathbf{v} = \begin{pmatrix} -1 \\ 1+i \end{pmatrix}$, a fundamental matrix for the system $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{X} =$

   (a) $e^{-t}\begin{pmatrix} -\cos(t) & -\sin(t) \\ \cos(t)+\sin(t) & \sin(t)+\cos(t) \end{pmatrix}$ (b) $e^{-t}\begin{pmatrix} -\cos(t) & -\sin(t) \\ \cos(t)-\sin(t) & \sin(t)+\cos(t) \end{pmatrix}$
   (c) $e^{-t}\begin{pmatrix} -\cos(t) & 0 \\ \cos(t) & \sin(t) \end{pmatrix}$ (d) $e^{-t}\begin{pmatrix} -\sin(t) & 0 \\ \sin(t) & \cos(t) \end{pmatrix}$

13. The sum of the series $\displaystyle\sum_{n=1}^{\infty} 3 \cdot 2^{1-2n} 3^n$ is

   (a) 6 (b) 9 (c) 24 (d) 18

14. The series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^3+1}}$

   (a) converges (b) diverges

15. The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n[\ln(n)]^2}$

    (a) converges (b) diverges

16. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

    (a) converges absolutely (b) converges conditionally (c) diverges

17. The series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^2-1}$

    (a) converges absolutely (b) converges conditionally (c) diverges

18. The series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

    (a) converges absolutely (b) converges conditionally (c) diverges

19. The radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n2^n}$ is $R =$

    (a) $\frac{1}{2}$ (b) 3 (c) 2 (d) ∞

20. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{n}$ is $I =$

    (a) $[-2, 0)$ (b) $[-2, 0]$ (c) $[-1, 1)$ (d) $(-2, 0]$

21. The Maclaurin series (Taylor series about 0) of $f(x) = e^{-x}$ is

    (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{-1}{n!} x^n$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} x^{-n}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} (x+1)^n$

22. The coefficient of $(x-2)^3$ in the Taylor series of $f(x) = \ln(x)$ about (centred at) 2 is

    (a) $\frac{1}{24}$ (b) $\frac{1}{12}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{24}$

23. The coefficient of x^3 in the Maclaurin series of $f(x) = \sqrt{1+x}$ is

    (a) $-\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{5}{32}$

24. Let $f(x) = x^2$ for $0 \le x < 3$, and $f(x+3) = f(x)$ for all x. At $x = 17$, the Fourier series of f converges to

    (a) 1 (b) 9 (c) $\frac{9}{2}$ (d) 4

25. Let $f(x) = \left\{ \begin{array}{ll} 1, & 0 \leq x < 1 \\ 0, & 1 \leq x \leq 2 \end{array} \right\}$. The Fourier sine series of f on $[0, 2]$ is

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2[1-(-1)^n]}{n\pi} \sin(n\pi x)$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \sin(n\pi x)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n\pi} \left[1 - \cos\left(\frac{n\pi}{2}\right)\right] \sin\left(\frac{n\pi x}{2}\right)$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\pi} \left[1 - \cos\left(\frac{n\pi}{2}\right)\right] \sin\left(\frac{n\pi x}{2}\right)$

Answers

1. d
2. c
3. a
4. c
5. b
6. d
7. a
8. a
9. d
10. d
11. c
12. b
13. d
14. a
15. a
16. a
17. c
18. b
19. c
20. a
21. a
22. a
23. c
24. d
25. c