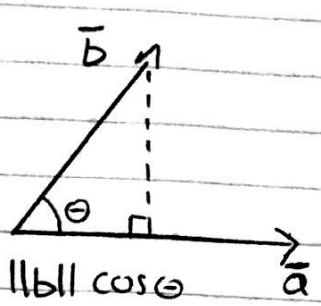
Dot Product

$$\bar{a} \cdot \bar{b} = \|\bar{a}\| \, \|\bar{b}\| \cos\theta$$



$$\text{comp}_{\bar{a}} \bar{b} = \|\bar{b}\| \cos\theta = \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\|}$$

$$\text{comp}_{\bar{b}} \bar{a} = \|\bar{a}\| \cos\theta = \frac{\bar{a} \cdot \bar{b}}{\|\bar{b}\|}$$

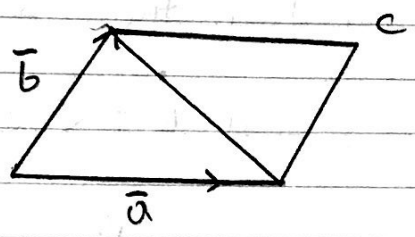
$$\text{Proj}_{\bar{a}} \bar{b} = \text{comp}_{\bar{a}} \bar{b} \ (\text{unit vector } \bar{a})$$

$$\left\{ = \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\|} \cdot \frac{\bar{a}}{\|\bar{a}\|} \right\} \Leftarrow \text{Vector}$$

Cross Product

$$\|\bar{a} \times \bar{b}\| = \|\bar{a}\| \, \|\bar{b}\| \sin\theta$$

Area of Parallelogram



$$A_{\square} = \|\bar{a} \times \bar{b}\|$$

$$A_{\triangle} = \frac{1}{2} \|\bar{a} \times \bar{b}\| = \frac{1}{2} \|\bar{a}\| \, \|\bar{b}\|$$

$$\text{Volume} = \|\bar{a} \times \bar{b}\| \cdot \bar{c}$$

* Is the line passing through point $(2, 0, 9)$ & $(-4, 1, -5)$ $\parallel$ or $\perp$ or neither to the vector $\langle 0, -9, -4 \rangle$:

$\bar{a} = \langle -6, 1, -14 \rangle$

$\bar{b} = \langle 0, -9, -4 \rangle$

$\bar{a} \cdot \bar{b} = (0 - 9 + 56) = 47 \neq 0$

not $\parallel$
$\bar{a} = c\bar{b}$

$\bar{a}$ not $\perp$ $\bar{b}$

* Find vector $\bar{b}$ is $\parallel$ to $\bar{a} = \langle 3, 7 \rangle$ & $\|\bar{b}\| = 2$

$\bar{b} = \|\bar{b}\| \cdot \frac{\bar{a}}{\|\bar{a}\|} = \frac{2(3, 7)}{\sqrt{3^2 + 7^2}} = \frac{\langle 6, 14 \rangle}{\sqrt{58}}$

$\bar{b} = \left\langle \frac{6}{\sqrt{58}}, \frac{14}{\sqrt{58}} \right\rangle$

~~* Find vector b is $\parallel$ to $\bar{a} = \langle 3, 7 \rangle$ & $\|\bar{b}\| = 2$~~

$\vec{b}$

Find the $\measuredangle$ b/w $\langle -2, 1 \rangle$, & $\langle 3, 4 \rangle$:

$\bar{a} \cdot \bar{b} = \langle -2, 1 \rangle \cdot \langle 3, 4 \rangle = -6 + 4 = -2$

$\|\bar{a}\| = \sqrt{4 + 1} = \sqrt{5}$

$\|\bar{b}\| = \sqrt{9 + 16} = 5$

$\cos\theta = \frac{-2}{5\sqrt{5}} \Rightarrow \cos^{-1}\left(\frac{-2}{5\sqrt{5}}\right)$

$\theta = 100.5^\circ$

Find $\measuredangle$ using cross product: $\bar{a} \langle -2, 1 \rangle$
$\bar{b} \langle 3, 4 \rangle$

$\| \bar{a} \times \bar{b} \| = -8\hat{k} - 3\hat{k} = \| -11\hat{k} \|$

$= 11\hat{k}$

$\bar{a} \times \bar{b} = 11\hat{k}$

$\sin\theta = \dfrac{\|a \times b\|}{\|\bar{a}\| \|\bar{b}\|} = \dfrac{11}{\sqrt{5}\sqrt{5}} \Rightarrow \theta = \sin^{-1} \dfrac{11}{5\sqrt{5}}$

$= 79.7°$