University of Ottawa
School of Electrical Engineering and Computer Science
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Assignment #2 Solution

16. **a)** This is correct, using universal instantiation and modus tollens.

b) This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.

c) After applying universal instantiation, it contains the fallacy of affirming the conclusion.

d) This is correct, using universal instantiation and modus ponens.

28. We want to show that the conditional statement $\neg R(a) \to P(a)$ is true for all a in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if $\neg R(a)$ is true for a particular a, then $P(a)$ is also true. For such an a, universal modus tollens applied to the second premise gives us $\neg(\neg P(a) \wedge Q(a))$. By rules from propositional logic, this gives us $P(a) \vee \neg Q(a)$. By universal generalization from the first premise, we have $P(a) \vee Q(a)$. Now by resolution we can conclude $P(a) \vee P(a)$, which is logically equivalent to $P(a)$, as desired.

16. We give a proof by contraposition. If it is not true than m is even or n is even, then m and n are both odd. By Exercise 6, this tells us that mn is odd, and our proof is complete.

30. We write these in symbols: $a < b$, $(a+b)/2 > a$, and $(a+b)/2 < b$. The latter two are equivalent to $a+b > 2a$ and $a+b < 2b$, respectively, and these are in turn equivalent to $b > a$ and $a < b$, respectively. It is now clear that all three statements are equivalent.

26. If we were to end up with nine 0's, then in the step before this we must have had either nine 0's or nine 1's, since each adjacent pair of bits must have been equal and therefore all the bits must have been the same. Thus if we are to start with something other than nine 0's and yet end up with nine 0's, we must have had nine 1's at some point. But in the step before that each adjacent pair of bits must have been different; in other words, they must have alternated 0, 1, 0, 1, and so on. This is impossible with an odd number of bits. This contradiction shows that we can never get nine 0's.

36. The average of two different numbers is certainly always between the two numbers. Furthermore, the average a of rational number x and irrational number y must be irrational, because the equation $a = (x+y)/2$ leads to $y = 2a - x$, which would be rational if a were rational.

36. From $a \equiv b \pmod{m}$ we know that $b = a + sm$ for some integer s. Multiplying by c we have $bc = ac + s(mc)$, which means that $ac \equiv bc \pmod{mc}$.

40. Write $n = 2k+1$ for some integer k. Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$. Since either k or $k+1$ is even, $4k(k+1)$ is a multiple of 8. Therefore $n^2 - 1$ is a multiple of 8, so $n^2 \equiv 1 \pmod{8}$.