3.84

A 10-m high open cylinder, $A_{cyl} = 0.1\ m^2$, contains 20°C water above and 2 kg of 20°C water below a 198.5-kg thin insulated floating piston, shown in Fig. P3.84. Assume standard *g*, *Po*. Now heat is added to the water below the piston so that it expands, pushing the piston up, causing the water on top to spill over the edge. This process continues until the piston reaches the top of the cylinder. Find the final state of the water below the piston (*T*, *P*, *v*) and the heat added during the process.

Solution:

Piston force balance at initial state: $F\uparrow = F\downarrow = P_A A = m_p g + m_B g + P_0 A$

State $1_{A,B}$: Comp. Liq. $\Rightarrow$ $v \cong v_f = 0.001002\ m^3/kg$; $u_{1A} = 83.95$ kJ/kg

$V_{A1} = m_A v_{A1} = 0.002\ m^3$; $m_{tot} = V_{tot}/v = 1/0.001002 = 998$ kg

mass above the piston $m_{B1} = m_{tot} - m_A = \mathbf{996\ kg}$

$$P_{A1} = P_0 + (m_p + m_B)g/A = 101.325 + \frac{(198.5 + 996) \times 9.807}{0.1 \times 1000} = \mathbf{218.5\ kPa}$$

State 2_A: $P_{A2} = P_0 + \frac{m_p g}{A} = \mathbf{120.8\ kPa}$; $v_{A2} = V_{tot}/m_A = \mathbf{0.5\ m^3/kg}$

$x_{A2} = (0.5 - 0.001047)/1.4183 = 0.352$; $T_2 = \mathbf{105°C}$

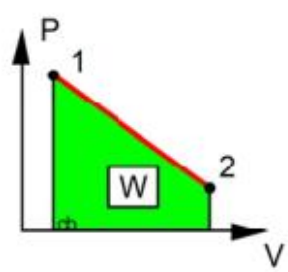
$u_{A2} = 440.0 + 0.352 \times 2072.34 = 1169.5$ kJ/kg

Continuity eq. in A: $m_{A2} = m_{A1}$

Energy: $m_A(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P linear in V as m_B is linear with V

$${}_1W_2 = \int PdV = \frac{1}{2}(218.5 + 120.82)\ \text{kPa}\ (1 - 0.002)\ m^3 = \mathbf{169.32\ kJ}$$



$${}_1Q_2 = m_A(u_2 - u_1) + {}_1W_2 = 2170.1 + 169.3 = \mathbf{2340.4\ kJ}$$

Assumptions 0.5: Point.

1st law written in full: 0.5 point.

Reduction of first law: 1 point

P-V diagram: 1 Point

Work Eqn: 1 Point

Q=1 point

3.114

A spring loaded piston/cylinder contains 1.5 kg of air at 27C and 160 kPa. It is now heated to 900 K in a process where the pressure is linear in volume to a final volume of twice the initial volume. Plot the process in a P-v diagram and find the work and heat transfer.

Take CV as the air.

$m_2 = m_1 = m$; $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$ => ${}_1W_2 = \int P\, dV = \text{area} = 0.5(P_1 + P_2)(V_2 - V_1)$

State 1: Ideal gas. $V_1 = mRT_1/P_1 = 1.5\times 0.287 \times 300/160 = 0.8072\ m^3$

Table A.7 $u_1 = u(300) = 214.36$ kJ/kg

State 2: $P_2V_2 = mRT_2$ so ratio it to the initial state properties

$P_2V_2 /P_1V_1 = P_2 2 /P_1 = mRT_2 /mRT_1 = T_2 /T_1$ =>

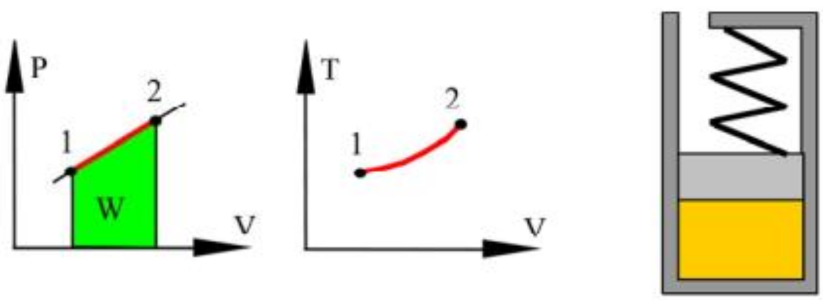
$P_2 = P_1 (T_2 /T_1)(1/2) = 160 \times (900/300) \times (1/2) = 240$ kPa

Work is done while piston moves at linearly varying pressure, so we get

${}_1W_2 = 0.5(P_1 + P_2)(V_2 - V_1) = 0.5\times(160 + 240)\ \text{kPa} \times 0.8072\ m^3 =$ **161.4 kJ**

Heat transfer is found from energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.5\times(674.824 - 214.36) + 161.4 =$ **852.1 kJ**



Assumptions 0.5: Point.

1st law written in full: 0.5 point.

Reduction of first law: 1 point

P-V diagram: 1 Point

Work Eqn: 1 Point

Q=1 point