Solution for Assignment on Design of Compression members

Problem 1:

A compression member has an L152x102x16 cross-section and a 300W material. The member span is 3,960mm and its effective length factors are $K_x = K_y = K_z = 1.0$. It is required to calculate the compressive resistance of the member.

Solution

(a) Properties

From the handbook, the relevant section properties are:

$A = 3780 mm^2$

$r'_x = 51.6mm,\ r'_y = 22.0mm,\ x_0 = 34.4mm,\ y_0 = 32.3mm, r_0 = 73.3mm$

$J = 319,000 mm^4, C_w = 0.427 \times 10^9 mm^6$

(b) Class 3 Check

For the long leg

$$\frac{b}{t} = \frac{152}{16} = 9.5 < \left(\frac{b}{t}\right)_{\lim} = \frac{250}{\sqrt{F_y}} = \frac{250}{\sqrt{300}} = 14.4$$

Long leg meets Class 3 requirements

Since Class 3 requirements are met for the long leg, they must be met for the short leg

$$\frac{b}{t} = \frac{102}{16} = 6.4 < \left(\frac{b}{t}\right)_{\lim} = \frac{250}{\sqrt{F_y}} = \frac{250}{\sqrt{300}} = 14.4$$

Thus, the section meets Class 3 requirements

(b) Check Slenderness

$$\left(\frac{K_{x'} L}{r_{x'}}\right) = \frac{1 \times 3,960}{51.6} = 76.7 < 200$$

$$\left(\frac{K_{y'} L}{r_{y'}}\right) = \frac{1 \times 3,960}{22.0} = 180 < 200$$

Slenderness requirements are met

(c) Buckling Stresses

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{K_{x'} L}{r_{x'}}\right)^2} = \frac{\pi^2 (200,000)}{(76.7)^2} = 336 MPa$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{K_{x'} L}{r_{x'}}\right)^2} = \frac{\pi^2 (200,000)}{(180)^2} = 60.9 MPa$$

$$F_{ez} = \left[\frac{\pi^2 E C_w}{(K_z L_z)^2} + GJ\right] \frac{1}{A\overline{r}_0^2}$$

$$\left[\frac{\pi^2 (200,000)(0.427\times 10^9)}{(3,960)^2} + (77,000)(319,000)\right] \frac{1}{3,780 \times 73.3^2} = 1,212 MPa$$

Since the Section is asymmetric, the above stresses are <u>not</u> the buckling stresses. The buckling stresses are obtained by solving the following cubic equation.

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2 (F_e - F_{ey})\left(x_0/\overline{r}_0\right)^2 - F_e^2 (F_e - F_{ex})\left(y_0/\overline{r}_0\right)^2 = 0$$

Substituting

$$(F_e - 336)(F_e - 60.9)(F_e - 1,212)$$
$$-F_e^2 (F_e - 60.9)\left(\frac{34.4}{73.3}\right)^2 - F_e^2 (F_e - 336)\left(\frac{32.3}{73.3}\right)^2 = 0$$

Solving (using the Solver or the Goal Seek utility, both in excel), obtain the three solution of the above cubic equations. The <u>smallest</u> root is the elastic buckling stress sought

$$F_e = 60.3 MPa$$

(d) Factored compressive resistance

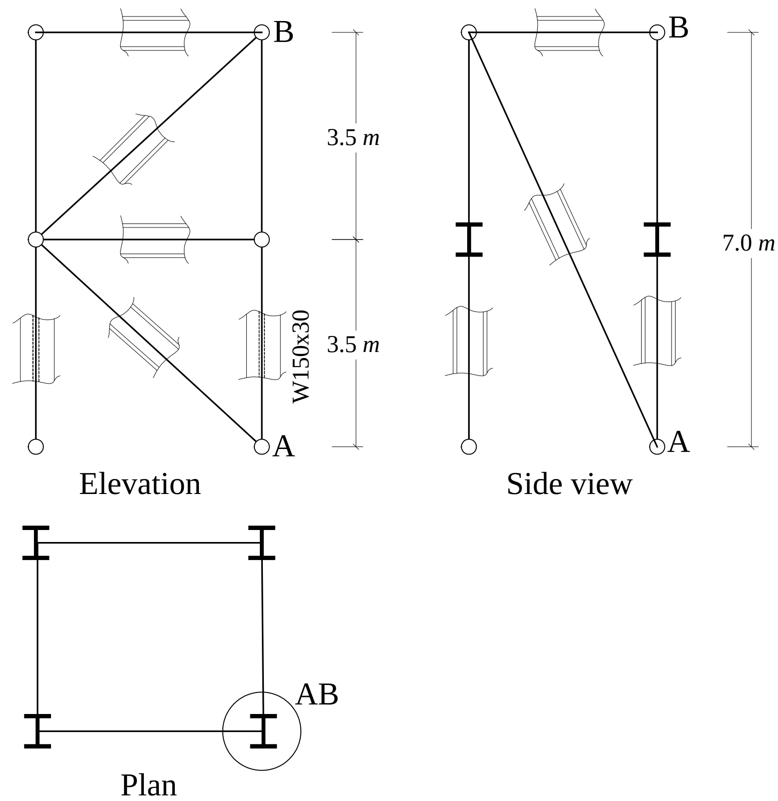
$$\lambda = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{300}{60.3}} = 2.23$$

For most sections (including angles) n=1.34. Thus, the compressive resistance for the compression member is

$$C_r = \phi A F_y \left(1 + \lambda^{2n}\right)^{-1/n} = 0.9 \times 3,780 \times 0.30 \times \left(1 + 2.23^{2\times 1.34}\right)^{-1/1.34} = 189 kN$$

Problem 2:

It is required to calculate the factored resistance for the shown vertical W150x30 member. Material is 350W. Perform all calculations and checks as required in CAN-CSA S16. For torsional buckling, the effective length can be taken as $K_z L$ = 3,500mm



Solution:

(a) Section Properties

From the Handbook, the relevant cross-section properties for a W150x30 are extracted:

r_x = 67.3 mm

r_y = 38.3 mm

A = 3,790 mm^2

C_w = 30.3x10^9 mm^6

J = 100,000 mm^4

I_x = 17.1x10^6 mm^4

I_y = 5.56x10^6 mm^4

(b) Class 3 Check

Flange: $\frac{b}{t} = 8.22 < \left(\frac{b}{t}\right)_{\lim} = \frac{200}{\sqrt{350}} = 10.7$ Flange meets Class 3 requirements

Web: $\frac{b}{t} = 21.0 < \left(\frac{b}{t}\right)_{\lim} = \frac{670}{\sqrt{350}} = 35.8$ Web meets Class 3 requirements

Section meets Class 3 requirements.

(c) Check Slenderness

From the figure, K_xL = 7,000, K_yL = 3,500*mm*

$$\frac{K_xL}{r_x} = 104 < 200$$

$$\frac{K_yL}{r_y} = 91.4 < 200$$

OK

(d) Buckling Stresses

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{K_xL}{r_x}\right)^2} = \frac{\pi^2 (200,000)}{(104)^2} = 182.5MPa$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{K_yL}{r_y}\right)^2} = \frac{\pi^2 (200,000)}{(91.4)^2} = 236.3MPa$$

$$F_{ez} = \left[\frac{\pi^2 EC_w}{(K_zL_z)^2} + GJ\right]\frac{1}{I_x + I_y}$$

$$\left[\frac{\pi^2 (200,000)(30.3\times10^9)}{(3,500)^2} + (77,000)(100,000)\right]\frac{1}{(17.1+5.56)\times10^6}$$

$$= \left[4.884\times10^9 + 7.7\times10^9\right]\frac{1}{(17.1+5.56)\times10^6} = \frac{12.58\times10^9}{(17.1+5.56)\times10^6} = 555MPa$$

Since the column cross-section is doubly symmetric, the buckling stress is the smallest of F_{ex}, F_{ey}, F_{ez}, i.e.,

$$F_e = \min\left(F_{ex}, F_{ey}, F_{ez}\right) = 182.5MPa$$

(e) The factored compressive resistance for the column is

$$\lambda = \left(\frac{K_x L}{r_x}\right)\sqrt{\frac{F_y}{\pi^2 E}} = 104\sqrt{\frac{350}{\pi^2\left(200000\right)}} = 1.385$$

or $$\lambda = \sqrt{\frac{350}{182.5}} = 1.385$$

For rolled wide flange sections n=1.34. Thus, the compressive resistance for the column is

$$C_r = \phi A F_y \left(1+\lambda^{2n}\right)^{-1/n} = 0.9\times 3{,}790\ \times 0.350\times\left(1+1.385^{2\times 1.34}\right)^{-1/1.34}$$
$$= 1194\times 0.4018 = 479kN$$