Solution for the assignment on the Design of Flexural Members

Question 1:

A simply supported beam has an 8m span. The beam is laterally and torsionally restrained at both ends, and non-retrained laterally and torsionally between both ends. The beam is subject to three concentrated factored forces, each having a magnitude of 100 kN, acting at the quarter span, the mid-span, and the three quarter span points. The beam is bent about its minor axis. It is required to find an economic section to carry the given loads. (Fy=350MPa). Perform all the necessary checks to carry the applied loads. You can neglect the self weight of the beam.

Question 2:

A simply supported beam has a 6.5m span. The beam is laterally and torsionally unsupported between both ends. The beam is subject to a transverse distributed factored load of 10 kNm (including a self-weight allowance). The beam is bent about its major axis. It is required to find an economic section to carry the given loads (Fy=350MPa)

Question 3:

A simply supported beam has a 7.5m span. Cross-section is W310x67 and steel grade is 350W. The beam is laterally and torsionally unsupported at both ends and non-restrained laterally and torsionally in between supports. The beam has a span of and is simply supported at both ends. The beam is subject to two concentrated loads P (factored) acting on the third span points

a) Determine the maximum loads *P* that the beam can withstand based on its flexural resistance according to CAN/CSA S16.

b) Given that the bottom flange of the beam is bearing against a base plate of width *N*=200 *mm* at both ends of the beam, determine whether bearing stiffeners are required at the ends of the beam.

c) Check whether the shear capacity of the beam is adequate

Solution for Problem 1:

The reaction at each support reaction is $(100+100+100)/2 = 150kN$. The bending moment at mid-span is $150\times 4-100\times 2 = 400kNm$. Thus the section be selected must withstand a 400kN moment according to CAN/CSA S16. Since the section is bent about the minor axis, lateral-torsional buckling is not a possible mode of failure. Assume Section is class 2. This assumption must be verified after selection of a section.

Required plastic modulus:

$$M_{ry} = \phi Z_y F_y \Rightarrow Z_{yreq} > \frac{M_{ry}}{F_y} = \frac{400\times 10^6}{0.9\times 350} = 1,270\times 10^3\,mm^3$$

By referring to the tables provided in Section 6 the handbook, and looking for the lightest W section with a plastic section modulus Z_y higher higher than the required value, we select a $W310\times 179$. This section has the following properties

$$d = 333mm,\quad b = 313mm,\quad t = 28.1mm,\quad w = 18.0mm$$
$$I_y = 144\times 10^6\,mm^4,\quad S_y = 919\times 10^3\,mm^3,\quad Z_y = 1400\times 10^3\,mm^3$$

Proceeding with a $W310\times 179$ Section

Classification:

Class 2 requirements:
For the flange:

$$\frac{b_{el}}{t} = \frac{313}{2\times 28.1} = 5.6 \le \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{350}} = 9.1$$

Flanges meet Class 2 requirement

For the web:

$$\frac{h}{t} = \frac{333-28.1\times 2}{18.0} = 15.4 \le \frac{1,700}{\sqrt{F_y}} = 90.9$$

Web meets Class 2 requirements. The section then meets Class 2 requirements.

Factored resistance:

$$M_{ry} = \phi Z_y F_y = 0.9\times 1400\times 10^3\times 0.350\times 10^{-3} = 441 > 400\,kNm \qquad OK.$$

Check the shear capacity for the section:

In this case, shear must be carried by two flanges. Using the mechanics of material equation, the shear stress distribution is

$$\tau = \frac{Q_y V}{I_y t},$$

where Q_y is the moment of area at the shear plane of interest, and V is the maximum shear force given by

$$V = \frac{1}{2}150 = 75kN \qquad \text{For each flange}$$

I_y is the minor moment of inertia, and t is the thickness at the section of interest. Based on on the above equation, we have a parabolic stress distribution. The maximum value of τ takes place in the middle of the flange. In this case, we have

$$Q_y = \left(\frac{1}{2}bt\right)\left(\frac{1}{4}b\right) = \frac{1}{8}\times 313^2 \times 28.1 = 344\times 10^3 mm^3$$

$$\tau = \frac{75\times 10^3 \times 344\times 10^3}{144\times 10^6 \times 28.1} = 6.4 < 0.66F_y = 231kPa \qquad OK$$

If one is seeking a square HSS section for this section, the most economical section for the this problem would be an $HSS305\times 305\times 13$ with a mass of $113kg/m$. It is class 2 section and passes all other checks.

Solution for Problem 2:

1. Determine maximum factored moment

The distributed factored load of 10 *kNm* induces a bending moment with a peak bending moment at mid-span of $10\times(6.5)^2/8 = 52.8\,kNm$. The chosen section must withstand this resistance.

2. Required section modulus

Assume the section is Class 2. This assumption must be verified after selecting a section. We have:

$$M_r = \phi Z_x F_y \Rightarrow Z_{x\min} = \frac{M_{ry}}{F_y} = \frac{52.8\times10^6}{0.9\times350} = 167.6\times10^3 mm^3$$

3. Cross-section Selection

If the beam was laterally supported, one would try a section with Z_x slightly larger than $167.6\times10^3 mm^3$. In that case, a $W\ 200\times19$ with $Z_x = 187\times10^3 mm^3$ would have been suitable.

Our problem here is more complex because the beam is laterally unsupported. Because of this, it is expected that the resistance of that the resistance M_r will drop below $M_f = 52.8\times10^6 kNm$ due to lateral torsional buckling. Therefore, we should try a section with a larger Z_x; for instance a W200x36 with $Z_x = 376\times10^3 mm^3$

4. Section Classification

Now that we selected a section, we need to check whether it meets Class 2 requirements (as assumed under Step 2)

For the flange:

$$\frac{b_{el}}{t} = \frac{165}{2\times6.5} = 8.1 \le \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{350}} = 9.1$$ Flanges meet Class 2 requirement

For the web:

$$\frac{h}{t} = \frac{201-10.2\times2}{5.8} = 29.1 \le \frac{1,700}{\sqrt{F_y}} = 90.8$$ Web meets Class 2 requirements

As assumed under item 1, section meets Class 2 requirements. Thus, the following calculations will be based on the plastic moment

5. Calculate the plastic moment

$$M_p = Z_x F_y = 376,000\times0.35\times10^{-3} = 131.6kNm$$

6. Calculate the elastic lateral torsional buckling moment

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

$$= \frac{1.13\times\pi}{6,500}\sqrt{200,000\times\left(7.64\times10^6\right)\times77,000\times\left(1.39\times10^6\right)+\left(\frac{\pi\times200,000}{6,500}\right)^2\left(7.64\times10^6\right)\left(6.95\times10^{10}\right)}\times10^{-6}$$

$$= 79.7\ kNm$$

Note: For the detailed calculation of ω_2, refer to the lecture notes.

7. Calculate the flexural resistance of the beam

Since $M_u \le 0.67 M_p$,

$$M_r = \phi M_u = 0.90\times79.7 = 71.8 kNm$$

8. Evaluation

Since the flexural resistance of the beam is greater than the maximum factored moment determined in Step 1, section is judged safe.

Note: If $M_r < M_f$, the section would have been unsafe, one would then try a heavier section and repeat Steps 4-7.

Is there a more economic design? Students to verify that a lighter section such as a W200x31 or W150x30 would lead to unsafe design

9. Check the shear capacity of the section:

Maximum shear = reaction

$$V_f = \frac{wl}{2} = \frac{10\times6.5}{2} = 32.5kN$$

From the web classification in Step 4, we had

$$\frac{h}{w} = 29.7 .$$

This value is smaller than the limit value determined by

$$439\sqrt{\frac{k_v}{F_y}} = 439\sqrt{\frac{5.34}{350}} = 54.2$$

Therefore, one can use the following value of the shear stress:

$$F_s = 0.66 F_y$$

The corresponding shear resistance is

$$V_r = \phi A_w F_s = 0.9 \times (201 \times 6.2) \times (0.66 \times 0.350) = 259kN >> V_f = 32.5kN$$

The shear capacity is significantly larger than the reaction. This will be the case in most shallow sections.

Solution for Question 3

(a)

Classification:

Class 2 requirements

For the flange:

$$\frac{b_{el}}{t} = \frac{204}{2 \times 14.6} = 6.99 \leq \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{350}} = 9.09$$

Flanges meet Class 2 requirement

For the web:

$$\frac{h}{t} = \frac{306 - 14.6 \times 2}{8.5} = 32.6 \leq \frac{1,700}{\sqrt{F_y}} = 90.9$$

Web meets Class 2 requirements and the section meets Class 2 requirements

Beam Plastic Moment

$$M_p = Z_x F_y = 1.06 \times 10^6 \times 0.350 = 371kNm$$

Moment gradient factor:

Need to draw the bending moment diagram, from which we determine that

$$M_{\max} = M_b = \frac{PL}{3}$$

$$M_a = M_c = \frac{PL}{3} \times \frac{L/4}{L/3} = 0.75M_b$$

The moment gradient factor is

$$\omega_2 = \frac{4M_{\max}}{\sqrt{M_{\max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}}$$

$$\omega_2 = \frac{4}{\sqrt{1 + 4 \times 0.75^2 + 7 + 4 \times 0.75^2}} = 1.13$$

Elastic Critical Moment

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

$$= \frac{1.13\times\pi}{7,500}\sqrt{(200,000)\left(20.7\times10^6\right)(77,000)(522,000) + \left(\frac{\pi\times200,000}{7,500}\right)^2 \left(20.7\times10^6\right)\left(4.39\times10^{11}\right)}\times10^{-6}$$

$$= 1.13\times 201kNm = 227kNm$$

5. Flexural resistance:

Since $M_u < 0.67M_p$, elastic lateral torsional buckling will occur and the corresponding resistance is

$$M_r = \phi M_u$$

$$M_r = 0.9\times201 = 180.9kNm$$

6. Corresponding force:

$$\frac{P_f l}{3} = M_r \Rightarrow P_f = \frac{180.9\times3}{7.5} = 72.4kN$$

(b)

Bearing resistance based on a yield limit state

$$B_{r1} = \phi_{be} w\left(N + 4t\right)F_y = 0.60\times0.75\times8.5\times\left(200+4\times14.6\right)(350)\times10^{-3} = 576kN$$

Bearing resistance based on crippling limit state

$$B_{r2} = 0.60\times\phi_{be} w^2\sqrt{F_y E} = 0.60\times0.75\times8.5^2\sqrt{350\times200,000}\times10^{-3} = 272kN$$

Bearing resistance of the web

$$B_r = \min\left(B_r^1, B_r^2\right) = 272kN$$

Since the bearing resistance of the web is greater than the concentrate force (reaction), bearing stiffeners are not needed

(c) Shear resistance

$$\frac{h}{w} = 32.6$$

$$439\sqrt{\frac{k_v}{F_y}} = 439\sqrt{\frac{5.34}{350}} = 54.2$$

$$\frac{h}{w} < 439\sqrt{\frac{k_v}{F_y}} \Rightarrow V_r = \phi A_w F_s = 0.9\times(306\times 8.5)\times(0.66\times 0.350) = 540kN$$

Shear capacity of $540kN$ is significantly greater than the maximum shear force $P_f = 72.4kN$. OK