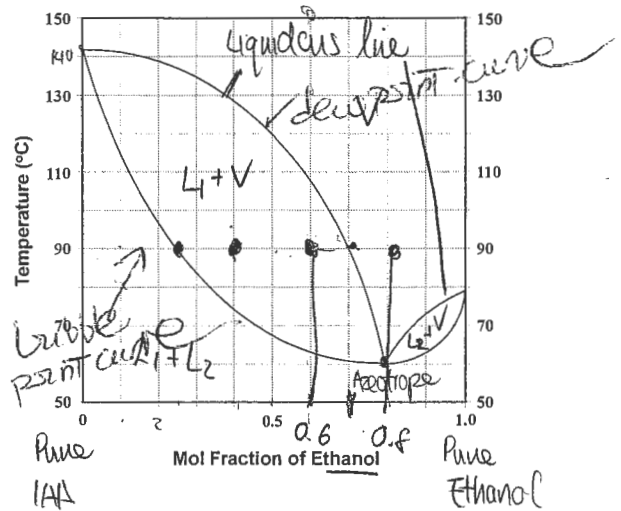




**QUESTION I (18 marks)**

Isoamyl acetate (IAA) is a chemical that is used in the food industry to provide the scent of bananas, and is called banana oil. IAA has  $M=130.19 \text{ kg/kmol}$  and  $T_c = 586.1 \text{ K}$ . When IAA is mixed with ethanol, the resulting mixture smells like pears, and is called pear oil. Ethanol has  $M=46.07 \text{ kg/kmol}$  and  $T_c = 514 \text{ K}$ . You carry out some experiments on a mixture of IAA and ethanol, and obtain the liquid-vapour phase diagram to the right at a constant pressure of 1 atm.



- (a) On the diagram, label all the regions and curves  
 (b) What is the normal boiling point (in °C) of pure IAA?

141°C

- (c) You are in a region of the graph that contains 40 mol% ethanol at 90°C. Calculate the degrees of freedom remaining in this region.

$F = C - P + 2$   
 $2 - 2 + 2 = 2$   
 BUT constant pressure  
 $2 - 1 = 1$

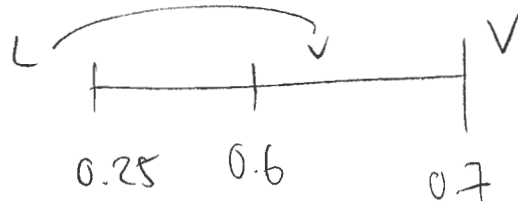
- (d) You mix together 52.08 kg of IAA with 27.64 kg of ethanol in a sealed container at a temperature of 45°C. The vessel is then heated to a final equilibrium temperature of 90°C and pressure of 101.325 kPa.

- (i) Determine the amount (in mol) of each phase present under these conditions.

$52.08 \text{ kg} \times \frac{\text{kmol}}{130.19 \text{ kg}} = 0.400 \text{ kmol IAA}$

$27.64 \text{ kg} \times \frac{\text{kmol}}{46.07 \text{ kg}} = 0.600 \text{ kmol ethanol}$

Total # mole: 1.00 kmol



$L = \frac{0.7 - 0.6}{0.7 - 0.25} = (0.2222 \dots) (1 \text{ kmol})$

$\approx 0.222 \text{ kmol of liquid}$

$1 - 0.222 \text{ kmol} = 0.7777$

$0.778 \text{ kmol of vapor}$

Use the other side of this page if you run out of space.

**QUESTION I (Contd.)**

(ii) Determine the mass (in kg) of IAA in each phase present under these conditions.

3

Liquid = 0.222 kmol  
 ~~$(0.25 \text{ mol fraction ethanol})(0.222 \text{ kmol}) = 0.0555 \text{ kmol ethanol}$~~   
 $(0.75 \text{ mol fraction IAA})(0.222 \text{ kmol}) = 0.1665 \text{ kmol}$   
 $0.1665 \text{ kmol} \times \frac{130.19 \text{ kg}}{\text{kmol}} = 21.6766 \text{ kg of liquid is IAA}$

Vapour:  ~~$(0.778 \text{ kmol}) \times 0.3 \text{ mol fraction IAA} = 0.2334 \text{ kmol}$~~   
 $0.2334 \text{ kmol} \times \frac{130.19 \text{ kg}}{\text{kmol}} = 30.386 \text{ kg of vapour is IAA}$

2

(e) How many moles of ethanol need to be added to the mixture in part (d) to reach the azeotrope composition?

$$0.2 = \frac{0.400 \text{ kmol}}{1.0 \text{ kmol} + x}$$

$$0.2 + 0.2x = 0.4 \text{ kmol}$$

$$x = 1$$

$\therefore$  1 kmol of ethanol needs to be added

(f) The mixture in (d) is isobarically heated in a variable volume container to a temperature of 609.3 K 336.15°C.

(i) Determine the partial pressure (in kPa) of IAA under these conditions

1

$$(101.325 \text{ kPa})(0.4 \text{ mole fraction IAA}) = 40.53 \text{ kPa}$$

**QUESTION I (Contd.)**

(ii) Determine the volume (in m<sup>3</sup>) of the mixture under these conditions.

1

$$\frac{PV}{P} = \frac{nRT}{P} = \frac{(1 \text{ kmol}) \left( \frac{8.314 \text{ kPa} \cdot \text{m}^3}{\text{K} \cdot \text{kmol}} \right) (609.3 \text{ K})}{101.325 \text{ kPa}}$$

$$V = 49.995 \text{ m}^3$$



(iii) Determine the rate of collisions per unit area (Note: use  $M_{avg}$  if the molar mass is required).

4

$$= P_N \frac{\bar{c}}{4} \quad M_{avg} = (0.4 \times 130.19 \frac{\text{kg}}{\text{kmol}}) + (0.6 \times 46.07 \frac{\text{kg}}{\text{kmol}})$$

$$= 79.718 \text{ kg/kmol}$$

$$\bar{c} = \sqrt{\frac{8RT}{\pi M}} = \frac{(8) \left( \frac{8.314 \text{ kPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}} \right) (609.3 \text{ K})}{\pi (0.079718)} = 402.265 \text{ m/s}$$

$$P_N = \frac{P}{kT} = \frac{101325 \text{ Pa}}{(1.3805 \times 10^{-23} \text{ J/K}) (609.3 \text{ K})}$$

$$P_N = 1.2046 \times 10^{26} \text{ molecules}$$

$$= (1.2046 \times 10^{26} \text{ molecules}) \left( \frac{402.265 \text{ m/s}}{4} \right)$$

$$= 1.21143 \times 10^{27} \text{ collisions/m}$$



$\frac{14}{15}$

**QUESTION II (15 marks)**

**PART A:** At  $20^\circ\text{C}$ , calcium ( $M = 40.08 \text{ kg/kmol}$ ) is known to exist in an FCC structure for which the closest interatomic distance is  $3.94 \text{ \AA}$ .

$\sigma = 3.94 \text{ \AA}$

(a) Calculate the density (in  $\text{kg/m}^3$ ) of a crystal of calcium.

2

$$\frac{40.08 \text{ g}}{\text{mol}} \times \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ molecules}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 6.657807 \times 10^{-26} \text{ kg}$$

$$\frac{(\sqrt{2})(6.657807 \times 10^{-26} \text{ kg})}{(3.94 \times 10^{-10} \text{ m})^3} = \boxed{\frac{1538.9 \text{ kg}}{\text{m}^3}}$$

(b) If a crystal of calcium occupies a volume of  $1.0 \text{ cm}^3$ , estimate the number of calcium atoms in that crystal.

3

$$1.0 \text{ cm}^3 \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1539.4 \text{ kg}}{\text{m}^3} \times \frac{1 \times 10^{-6} \text{ m}^3}{1 \text{ cm}^3} = 1.5394 \times 10^{-3} \text{ kg}$$

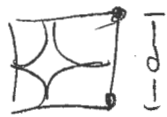
$$1 \text{ molecule} = \frac{6.657807 \times 10^{-26} \text{ kg}}{\text{molecule}}$$

$$\frac{1.5394 \times 10^{-3} \text{ kg}}{6.657807 \times 10^{-26} \text{ kg/mole}} = \boxed{2.31217 \times 10^{22} \text{ atoms}}$$

**QUESTION II (Contd.)**

**PART B:** Core samples taken from an oil reservoir are porous in nature and contain void spaces. To simulate a porous core in the laboratory, spherical glass beads with a diameter of 0.5 mm are packed into a container that has the dimensions 45 cm by 45 cm by 10 cm. The beads pack in a simple cubic structure.

- 3 (c) Determine the number of beads that can fit in the container.  $d = 0.5 \text{ mm} \Rightarrow 5 \times 10^{-4} \text{ m}$



Volume of unit cell =  $(5 \times 10^{-4} \text{ m})^3 = 1.25 \times 10^{-10} \text{ m}^3$

$a = d$       b) 1 unit cell holds 1 atom/molecule.  
 $(0.45 \text{ m}) \times (0.45 \text{ m}) \times (0.10 \text{ m})$

$= \frac{0.02025 \text{ m}^3}{1.25 \times 10^{-10} \text{ m}^3} \Rightarrow \underline{\underline{162\,000\,000 \text{ beads}}}$

- 3 (d) Calculate the total mass (in kg) of the glass bead pack in the container if the density of each glass bead is 2650 kg/m<sup>3</sup>.

$\frac{m}{d^3} = \text{density}$

$\frac{m}{(5 \times 10^{-4})^3} = 2650 \text{ kg/m}^3$   
 beads unit unit cell  
 $m = (3.3125 \times 10^{-7} \text{ kg molecule}) (162\,000\,000 \text{ beads})$

$= 53.6625 \text{ kg}$

**QUESTION II (Contd.)**

2

(e) Determine the void fraction of the glass bead pack.

$$V = \frac{1.25 \times 10^{-10} \text{ m}^3 - \left( \frac{4}{3} \pi (2.5 \times 10^{-4} \text{ m})^3 \right)}{1.25 \times 10^{-10} \text{ m}^3}$$

$-0.4764$

(f) Oil with a density of 800 kg/m<sup>3</sup> is injected into the container until the void space between the packed beads is totally filled. Calculate the mass of oil (in kg) that can fit in the container.

$$2 \quad (0.4764) (1.25 \times 10^{-10} \text{ m}^3) (800 \frac{\text{kg}}{\text{m}^3}) = 4.76401 \times 10^{-8} \text{ kg per unit cell}$$

There 162 000 000 "unit cells"

$$/ 162 \text{ 000 000} (4.76401 \times 10^{-8} \text{ kg})$$

$= 7.71769 \text{ kg of oil can fit}$

**QUESTION III (18 marks)**

Molecules of octane (C<sub>8</sub>H<sub>18</sub>) and nonane (C<sub>9</sub>H<sub>20</sub>) are similar in size and shape. These two substances are completely miscible.

Parameter	Octane	Nonane
Molar Mass	114.23 kg/kmol	128.26 kg/kmol
C <sub>PL</sub>	262.2 J/mol·K	322.2 J/mol·K
Normal Boiling Point	398.82 K	
Vapour Pressure at 415 K	155.79 kPa	76.235 kPa
ΔH <sub>v</sub>	39,400 J/mol	46,500 J/mol

3

(a) Develop a correlation for nonane in the form  $\ln(P_v) = -A/T + B$  where A and B are constants.

$$\Delta H_v = \frac{46500 \text{ J}}{\text{mol}} \times \frac{1000 \text{ mol}}{1 \text{ kmol}} \times \frac{1 \text{ kJ}}{1000 \text{ J}} = 46500 \text{ kJ/kmol}$$

$$-A = \frac{-\Delta H_v}{R} = \frac{46500 \text{ kJ/kmol}}{(8.314 \text{ kPa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K})} = 5592.9$$

$$\ln(P_v) = \frac{-5592.9}{T} + C$$

$$\ln(76.235 \text{ kPa}) = \frac{-5592.9}{415 \text{ K}} + C$$

$$C = 17.8106 \dots$$

$$\ln(P_v) = \frac{-5592.9}{T} + 17.81$$

Where P<sub>v</sub> is in kPa and T is in kelvin.

**QUESTION III (Contd.)**

(b) Determine the normal boiling point (in K) of nonane.

2

$$\ln(101.325 \text{ kPa}) = \frac{-5592.9}{T} + 17.81$$

$$T = 423.9 \text{ K}$$

(c) 20 mol of nonane is mixed with 80 mol of octane in a rigid, sealed vessel, and the mixture is allowed to come to equilibrium at a final temperature of 415 K and pressure of 135.9 kPa. Under these equilibrium conditions, a vapour and liquid co-exist within the vessel.

(i) Determine the composition (mol fraction) of the liquid phase at equilibrium.

2.5

Total pressure = 135.9 kPa

$$135.9 \text{ kPa} = (x \times 155.79 \text{ kPa}) + (1-x) (76.235 \text{ kPa})$$

$$135.9 \text{ kPa} = 155.79x + 76.235 \text{ kPa} - 76.235x$$

$$59.665 = 79.555x$$

$$x = 0.75$$

0.75 mol fraction octane  
0.25 mol fraction nonane

(ii) Determine the composition (in mol fraction) of the vapour phase at equilibrium

1.5

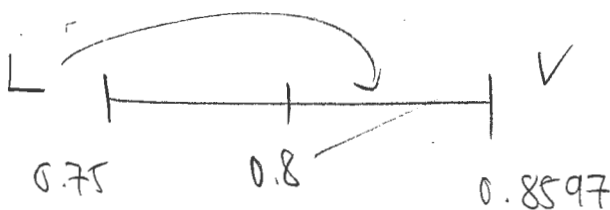
$$y_i = \frac{P_{vi} x_i}{P}$$

$$\frac{(155.79 \text{ kPa})(0.75)}{135.9 \text{ kPa}} = 0.8597 \text{ mol fraction octane}$$

$$0.1402 \text{ mole fraction nonane}$$

(iii) Determine the amount (in mol) of liquid in the vessel at equilibrium.

3



$$\frac{0.8597 - 0.8}{0.8597 - 0.75} = (0.544211 \dots) (100 \text{ mol})$$

$$(0.8597 - 0.75)$$

$$= 54.42 \text{ mol of liquid}$$

**QUESTION III (Contd.)**

$$\Delta E_{\text{gain}} = -\Delta E_{\text{lost}}$$

(d) If 10 mol of liquid octane at 20°C is mixed with 10 mol of liquid nonane at 40°C, determine the final equilibrium temperature of the mixture (in °C).

3

$$-(10 \text{ mol})(262.2 \text{ J/mol K})(T_2 - 20^\circ\text{C}) = (10 \text{ mol})(322.2 \text{ J/mol K})(T_2 - 40^\circ\text{C})$$

$$-(2622 T_2 - 52440) = 3222 T_2 - 128880$$

$$52440 - 2622 T_2 = 3222 T_2 - 128880$$

$$T_2 = 31.019^\circ\text{C}$$

∴ The final equilibrium temperature

$$\boxed{31.019^\circ\text{C}}$$

(e) You have a new vessel that can be heated at a constant rate of 20 kW. You place 2 kmol of pure liquid octane into this vessel at a starting temperature of 20°C. What is the minimum time required (in min) to completely vaporize the octane?

3

$$20000 \frac{\text{J}}{\text{s}} \quad 2 \text{ kmol } \text{C}_8\text{H}_{16} \quad 293.15 \text{ K}$$

Assume normal boiling point

$$(2000 \text{ mol})(262.2 \text{ J/mol K})(398.82 \text{ K} - 293.15 \text{ K}) + (2000 \text{ mol})(39400 \text{ J/mol})$$

$$= 55413348 + 78800000 \text{ J}$$

$$= 134213348 \text{ J} / 20000 \text{ J/s}$$

$$= 6710.6674 \text{ s}$$

$$\boxed{111.8 \text{ minutes}}$$

$$V_m = \frac{kmol}{m^3}$$

$$PV = nRT$$

$$V_m = \frac{V}{n}$$

**QUESTION IV (18 marks)**

18  
18

Three pure gases are combined in a laboratory to test the effectiveness of this mixture as a miscible solvent for an enhanced oil recovery project. Because of the hazardous properties of one of the components (H<sub>2</sub>S), the gas mixture is prepared in a fixed volume cell, and then transferred to a variable volume cell from which it is injected into an apparatus that contains a reservoir core. Properties of the three gases are provided in the following table.

Compound	M (kg/kmol)	T <sub>c</sub> (K)	P <sub>c</sub> (kPa)	a (kPa(m <sup>3</sup> /kmol) <sup>2</sup> )	b (m <sup>3</sup> /kmol)
Methane	16	190.6	4600	228.0	0.0428
Carbon Dioxide	44	304.2	7376	364.8	0.0428
Hydrogen Sulfide	34	373.2	8940	454.	

(a) The gas mixture is prepared by weighing the fixed volume cell (volume of 5 litres) as the individual components are injected. The masses of the components are 26 grams of CH<sub>4</sub>, 23 grams of CO<sub>2</sub> and 16 grams of H<sub>2</sub>S. The cell is then maintained at an equilibrium temperature of 20°C.

(i) Determine the composition of the gas mixture in mole percent.

3 
$$\frac{26g \text{ CH}_4}{16.0g} \times \frac{\text{mol}}{\text{mol}} = 1.625 \text{ mol} \rightarrow 62.0 \text{ mole percent CH}_4$$

$$\frac{23g \text{ CO}_2}{44g} \times \frac{\text{mol}}{\text{mol}} = 0.5227 \text{ mol} \rightarrow 20.0 \text{ mole percent CO}_2$$

$$\frac{16g}{34g} \times \frac{\text{mol}}{\text{mol}} = 0.4705 \text{ mol} \rightarrow 18.0 \text{ mole percent H}_2\text{S}$$

$$\underline{\hspace{10em}} \\ 2.6182 \text{ mol}$$

(ii) Calculate the average molecular mass (in kg/kmol) of the gas mixture.

1 
$$(0.62 \times 16g/\text{mol}) + (0.20 \times 44g/\text{mol}) + (0.18 \times 34g/\text{mol})$$

$$= 24.84 \text{ kg/kmol}$$

**QUESTION IV (Contd.)**

(iii) Determine the van der Waals constants  $\bar{a}$  (in  $\text{kPa} \cdot (\frac{\text{m}^3}{\text{kmol}})^2$ ) and  $\bar{b}$  (in  $\frac{\text{m}^3}{\text{kmol}}$ ) for the mixture.

4  $a$  for  $\text{H}_2\text{S} = \bar{a} = \frac{27}{64} \frac{R^2 T_c^2}{P_c} = \frac{(8.314 \frac{\text{kPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}})^2 (373.2 \text{ K})^2}{8940 \text{ kPa}} \left( \frac{27}{64} \right) = 454.307 \frac{\text{kPa} \cdot \text{m}^6}{\text{kmol}^2}$

$b$  for  $\text{H}_2\text{S} = 0.04283 \text{ m}^3/\text{kmol}$

$\bar{a} = \left[ (0.52 \times \sqrt{228.0 \text{ kPa} \cdot \text{m}^3/\text{kmol}}) + (0.2 \times \sqrt{264.8 \text{ kPa} \cdot (\text{m}^3/\text{kmol})^2}) + (0.18 \times \sqrt{454.307 \text{ kPa} \cdot (\text{m}^3/\text{kmol})^2}) \right]^2$

$\bar{a} = 289.624 \text{ kPa} \cdot (\frac{\text{m}^3}{\text{kmol}})^2$

$\bar{b} = (0.52 \times 0.0428) + (0.20 \times 0.0428) + (0.18 \times 0.0433)$

$\bar{b} = 0.04289 \text{ m}^3/\text{kmol}$

4 (iv) The fixed volume cell containing the gas mixture is placed in a constant temperature oven and heated until the pressure within the cell reaches 2000 kPa. Estimate the temperature (in °C) in the cell at this high pressure.

$V = 5 \text{ L} \times \frac{1 \text{ m}^3}{1000 \text{ L}} = 5 \times 10^{-3} \text{ m}^3$

$2.6182 \times 10^{-3} \text{ kmol}$

$V_m = 1.909 \frac{\text{m}^3}{\text{kmol}}$

~~$T = 488.9609 \text{ K}$   
 $T = 756.11^\circ\text{C}$~~

Use  $\left( P + \frac{a}{V_m^2} \right) (V_m - b) = \frac{RT}{R}$

$T = \frac{(2000 \text{ kPa} + 289.624 \text{ kPa} \cdot (\frac{\text{m}^3}{\text{kmol}})^2)}{1.909 \text{ m}^3/\text{kmol}} \left( 1.909 \text{ m}^3/\text{kmol} - 0.04289 \text{ m}^3/\text{kmol} \right)$   
Use the other side of this page if you run out of space.  
 $\approx 211 \text{ kPa} \cdot \text{m}^3$

**QUESTION IV (Contd.)**

(b) 30 grams of gas mixture from the fixed volume cell is transferred to a variable volume cell. The temperature of this cell is set at 105°C and the volume of the gas mixture is reduced until the pressure reaches 10,500 kPa. Using the pseudocritical method (Kay's rule) with the generalized compressibility chart, estimate the volume (in litres) occupied by the gas mixture.

6

$$30g \times \frac{\text{mol}}{24.84g} = 1.208 \text{ mol}$$

$$T_{pc} = (0.62 \times 190.6 \text{ K}) + (0.20 \times 304.2 \text{ K}) + (0.18 \times 373.2 \text{ K})$$

$$= 246.188 \text{ K}$$

$$P_{pc} = (0.62 \times 4600 \text{ kPa}) + (0.20 \times 7376 \text{ kPa}) + (0.18 \times 8940 \text{ kPa})$$

$$= 5936.4 \text{ kPa}$$

$$T_{RC} = \frac{378.15 \text{ K}}{246.188 \text{ K}} = 1.536$$

$$P_{RC} = \frac{10500 \text{ kPa}}{5936.4 \text{ kPa}} = 1.768$$

$$Z = 0.88$$

$$\frac{PV}{P} = \frac{ZnRT}{D}$$

$$V = \frac{(0.88)(0.001208 \text{ kmol})(8.314)(378.15 \text{ K})}{10500 \text{ kPa}}$$

$$V = 3.18 \times 10^{-4} \text{ m}^3$$

$$\boxed{0.3183 \text{ L}}$$

**QUESTION V (14 marks)**

A company discovers an oil reservoir located below the floor of a sea, and drills a pipe from the sea floor (which is located at a water depth of 1550 m) down to the reservoir, which is 4050 m below the sea floor. The absolute pressure in the producing reservoir is 59.1 MPa. The drill pipe, which allows oil to flow from the reservoir up to the level of the sea floor, has an inside diameter of 70 mm, and a roughness corresponding to commercial steel pipe. After drilling, a blowout occurs, and all of the flowing oil accidentally escapes from the drill pipe into the sea water.

You are given the following data:

- Density of sea water: 1020 kg/m<sup>3</sup>
- Atmospheric pressure: 101.325 kPa
- Density of oil: 850 kg/m<sup>3</sup>
- Acceleration due to gravity: 9.81 m/s<sup>2</sup>
- Viscosity of the oil: 0.16 mPa·s

Use the information provided to answer the questions below (Note: if you ever need to iterate, start with  $f = 0.0047$ , and perform a maximum of 2 iterations).

(a) Determine the pressure (in MPa) of the water at the sea floor.

2

$$101325 \text{ Pa} + (9.81 \text{ m/s}^2)(1550 \text{ m})(1020 \text{ kg/m}^3)$$

$$= 15.5096 \text{ MPa}$$

(b) The oil pressure at the exit of the drill pipe is 0.49 MPa greater than the water pressure at the sea floor. Calculate the pressure drop (in MPa) in the drill pipe between the reservoir and the exit.

1

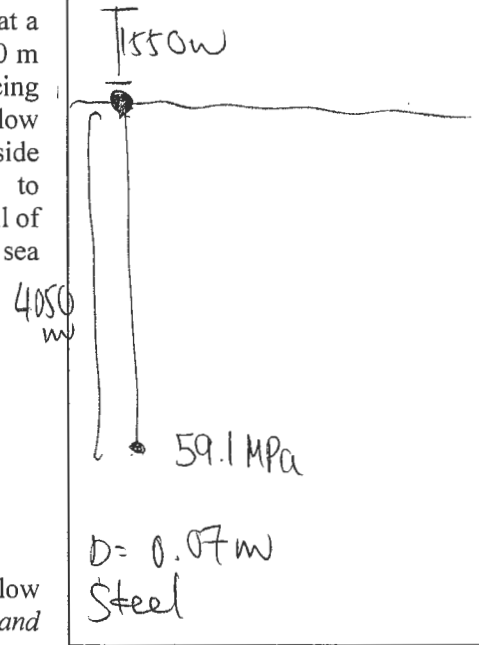
$$0.49 \text{ MPa} + 15.5096 \text{ MPa} = 16 \text{ MPa}$$

$$\Delta P = \text{Outlet} - \text{Inlet}$$

$$= 16 \text{ MPa} - 59.1 \text{ MPa}$$

$$= -43 \text{ MPa}$$

Space to draw a figure if needed  
(no marks associated with this)



**QUESTION V (Contd.)**

(c) Estimate the average velocity (in m/s) of the oil flowing in the drill pipe.

6

$$Re = \frac{D \bar{u} \rho}{\mu} \Rightarrow \frac{(0.07 \text{ m})(u)(850 \text{ kg/m}^3)}{(0.16 \times 10^{-3} \text{ Pa}\cdot\text{s})}$$

$Re = 371875 u$   
Assume: Turbulent flow!

$$\left[ \frac{\Delta P}{L} + \rho g h \frac{\Delta h}{L} \right] = \frac{2 f \rho (\bar{u})^2}{D}$$

Assume  $\Delta h = L$

$$\left[ \frac{43100000 \text{ Pa}}{4050 \text{ m}} + \frac{(850 \text{ kg/m}^3)(9.81 \text{ m})(4050 \text{ m})}{(4050 \text{ m})} \right] = \frac{2 f \bar{u}^2 (850 \text{ kg/m}^3)}{0.07 \text{ m}}$$

$$+2303.475 = 24285.71 f u^2$$

$$u = \sqrt{\frac{0.09484}{f}}$$

$f_{\text{guess}} = 0.0047$

$u = 4.49 \text{ m/s}$  (See next page)

$Re = 1670568.943$

$f_{\text{guess}} \neq f_{\text{result}}$

$f_{\text{result}} = 0.0040$

Average velocity = 4.869 m/s

(d) Estimate the daily volumetric flow rate (in m<sup>3</sup>/day) of the oil escaping into the sea.

2

$Q = uA$

$A = \pi \left( \frac{0.07}{2} \right)^2 = (1.225 \times 10^{-3} \text{ m}^2) (4.869 \text{ m/s})$  0.5

$Q = (5.9845 \times 10^{-3} \text{ m}^3/\text{s}) \left( \frac{3600 \text{ s}}{1 \text{ hr}} \right) \left( \frac{24 \text{ hr}}{\text{day}} \right)$

$Q = 515.33 \text{ m}^3/\text{day}$

$$f_{\text{guess } 2} = 0.0040$$

$$u = 4.869 \text{ m/s}$$

$$Re = 1810767$$

$$f_{\text{result}} = 0.0040$$

$\therefore$  Average velocity is 4.869 m/s

**QUESTION V (Contd.)**

(e) An experiment was performed using a Fann viscometer in order to determine the viscosity of the oil. The data shown in the table to the right were obtained at 50°C.

Trial	shear rate (s <sup>-1</sup> )	shear stress (Pa)
1	25	50
2	60	100

(i) In 20 words or less, explain why this oil is not a Newtonian fluid.

1

Ratio between shear rate & shear stress are not constant between trials.

(ii) If this oil is a power law fluid, calculate the value of the fluid behavior index (n).

2

$$\ln \tau = \ln K + n \ln \left( \frac{du}{dy} \right)$$

$$\ln(100) = \ln K + n \ln(60)$$

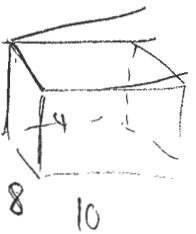
$$\ln(50) = \ln K + n \ln(25)$$

$$0.69314 = 0.87546 n$$

$$n = 0.7917$$

**QUESTION VI (17 marks)**

A detached garage located in the town of Watson Lake, Yukon, has the dimensions 10 m long, 8 m wide and 4 m tall. The four walls and the flat roof are made of a three layer composite structure. The inner layer is 1 cm thick drywall ( $\kappa = 0.18 \text{ W/m}\cdot\text{K}$ ), the middle layer is 3 cm thick mineral wool ( $0.03 \text{ W/m}\cdot\text{K}$ ) and the outer layer is 1 cm thick pine plywood ( $\kappa = 0.15 \text{ W/m}\cdot\text{K}$ ). One of the 8 m x 4 m walls contains a maple wood ( $\kappa = 0.13 \text{ W/m}\cdot\text{K}$ ) garage door that is 7 m x 3.5 m with a thickness of 2 cm. The garage must maintain a constant temperature of 20°C using an electrical heater. On a particularly cold day, the air temperature outside is -45°C. Using the information provided, answer the following questions:



(a) Calculate the area (in m<sup>2</sup>) of:

(i) The roof

$$8\text{m} \times 10\text{m} = 80\text{m}^2$$

(ii) The garage door

$$7\text{m} \times 3.5\text{m} = 24.5\text{m}^2$$

(iii) The composite walls

$$2(4 \times 10) + 1(4 \times 8) + 1((4 \times 8) - 24.5\text{m}^2) = 119.5\text{m}^2$$

(b) Calculate the rate of heat loss (in kW) through the roof of the garage.

2

$$\frac{-(80\text{m}^2)(-45^\circ\text{C} - 20^\circ\text{C})}{\frac{0.01\text{m}}{0.18\frac{\text{W}}{\text{m}\cdot\text{K}}} + \frac{0.03\text{m}}{0.03\frac{\text{W}}{\text{m}\cdot\text{K}}} + \frac{0.01\text{m}}{0.15\frac{\text{W}}{\text{m}\cdot\text{K}}}} = \frac{5200\text{m}^2\text{K}}{1.1222\frac{\text{W}}{\text{m}^2\cdot\text{K}}} = 4633.66\text{W}$$

$$\boxed{= 4.633\text{ kW}}$$

**QUESTION VI (Contd.)**

(c) Calculate the total rate of heat loss (in kW) through the walls and the garage door.

3

$$\frac{-24.5 \text{ m}^2 (-45^\circ\text{C} - 20^\circ\text{C})}{0.02 \text{ m}} + \frac{-(119.5 \text{ m}^2) (-45^\circ\text{C} - 20^\circ\text{C})}{\frac{0.01 \text{ m}}{8.18 \text{ W/mK}} + \frac{0.03 \text{ m}}{0.03 \text{ W/mK}} + \frac{0.01 \text{ m}}{0.15 \text{ W/mK}}$$

$$\frac{1592.5 \text{ m}^2\text{K}}{0.153846 \frac{\text{m}^2\text{K}}{\text{W}}} + \frac{7767.5}{1.12222 \frac{\text{m}^2\text{K}}{\text{W}}}$$

$$= 10348 \text{ W} + 6921.534 \text{ W}$$

$$\boxed{= 17.272 \text{ kW}}$$

(d) If energy costs \$0.04 per kWh, determine how much it will cost to heat the garage on a day where the average outside temperature is  $-45^\circ\text{C}$ .

2

$$4.633 \text{ kW} + 17.272 \text{ kW}$$

$$= 21.905 \text{ kW}$$

$$(21.905 \text{ kW}) \left( \frac{0.04}{\text{kWh}} \right) \left( \frac{24 \text{ hr}}{1 \text{ day}} \right)$$

$$\boxed{= \$21.03}$$

**QUESTION VI (Contd.)**

- (e) Snow ( $\kappa = 0.3 \text{ W/m}\cdot\text{K}$ ) that settles on the roof of the garage acts as an additional layer of insulation. Determine the depth of snow (in cm) required on the roof to reduce the cost of heating the garage by 5%.

4

$$(\$21.03) - (0.05 \times 21.03) = \$19.98$$

$$\begin{array}{r} \text{Total energy lost} = 20.8125 \text{ kW} \\ - 17.272 \text{ kW} \\ \hline 3.5405 \end{array}$$

$$\frac{-(80 \text{ m}^2)(-45^\circ\text{C} - 20^\circ\text{C})}{1.222 \frac{\text{W}}{\text{m}^2\cdot\text{K}} + \frac{\Delta x}{0.3}} = 3.5405 \text{ kW}$$

$$1.222 \frac{\text{W}}{\text{m}^2\cdot\text{K}} + \frac{\Delta x}{0.3}$$

$$1.222 \frac{\text{W}}{\text{m}^2\cdot\text{K}} + \frac{\Delta x}{0.3} = 1.4687$$

$$\Delta x = 0.0740 \text{ m}$$

$\therefore$  7cm of snow is needed

out

- (f) In 20 words or less, describe one thing that you could do to significantly reduce the heat loss from the garage.

$$V_m = \frac{\text{m}^3}{\text{kmol}}$$

Add pink insulation inbetween the drywood and the wood walls.

- (g) The air in the garage has a density of  $1.205 \text{ kg/m}^3$ . Assuming that air acts as an ideal gas, determine the density of the air outside.

$$\rho = \frac{1.205 \text{ kg}}{\text{m}^3} \times \frac{1 \text{ kmol}}{28.97 \text{ kg}} = 0.0416 \text{ kmol/m}^3$$

$$V_m = 24.041 \text{ kmol/m}^3$$

Use volume of garage = 320 m<sup>3</sup>

$$P V_m = RT$$

Set a ratio.

$$\frac{V_m}{V_m} = \frac{RT}{P} \Rightarrow$$

$$\frac{24.041 \text{ kmol/m}^3}{0.0416 \text{ kmol/m}^3} = \frac{293.15 \text{ K}}{228.15 \text{ K}}$$

P

$$V_m = \frac{RT}{P}$$

$$V_m = 18.7104 \text{ kmol/m}^3 \times \frac{1 \text{ kmol}}{28.97 \text{ kg}} \Rightarrow \rho = 1.548 \text{ kg/m}^3$$

Use the other side of this page if you run out of space.