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From: ya boi

MCG 3306 System Dynamics

Final Exam

Dec. 08, 2018 at 14:00
GYM B

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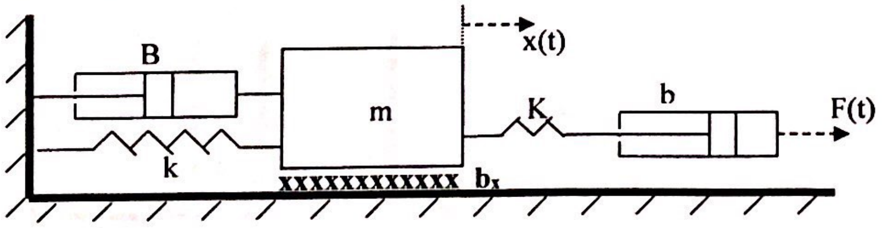
Closed book. Calculators permitted. All 10 problems of equal value. Duration 3h

1. For the mechanical system shown below F(t) [N] is the external force applied and x [m] is the position variable of the mass m [kg]. For the case of no friction between m and the fixed frame, and zero initial conditions, obtain for m=1.5 [kg], k=2.5 [N/m], K=4 [N/m], b=4 [N/ms^{-1}], b_x=3 [N/ms^{-1}]and B=5[N/ms^{-1}]

a) the equation of transfer function T(s)=x(s)/F(s).
b) obtain the state space representation of the system and the matrices A, B and C, assuming that the output y is the velocity of m.
c) derive analytically the formula for the transfer function from the state space representation matrices A, B and C. Verify the result of a)
d) for an unit impulse input, calculate x(t) for t $\rightarrow\infty$.



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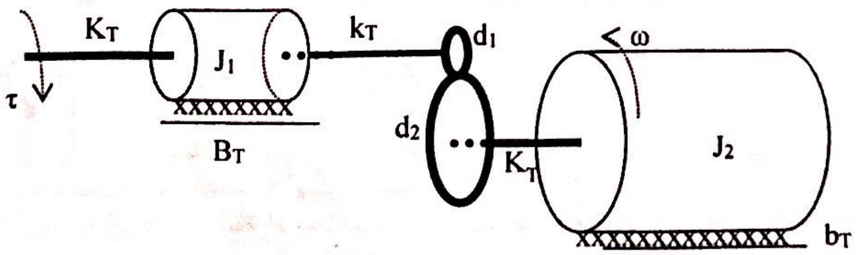
2. This system is subject to a torque τ [Nm] and has two discs with the moments of inertia J_1 and J_2 [Kg m^2], linked by a gear train with diameters d_1 and d_2 [m] . The flexible shafts have the coefficients K_T and k_T [Nm/rad]. Between the disks and the fixed frame there is viscous friction with coefficients B_T and b_T [Nm/rad s^{-1}].
Obtain the transfer function ω(s) / τ(s), where ω is angular velocity [rad/s].



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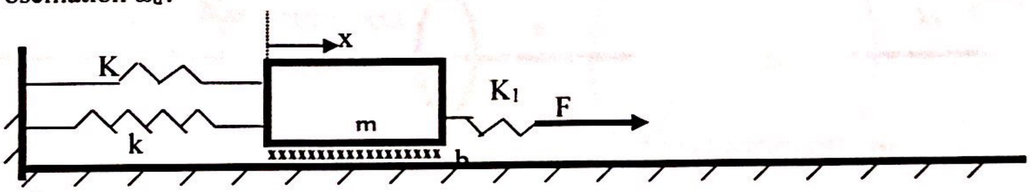
3. For the mechanical system bellow, F(t) is the external force applied and b is the viscous friction coefficient between the mass m and the ground. Initial conditions are

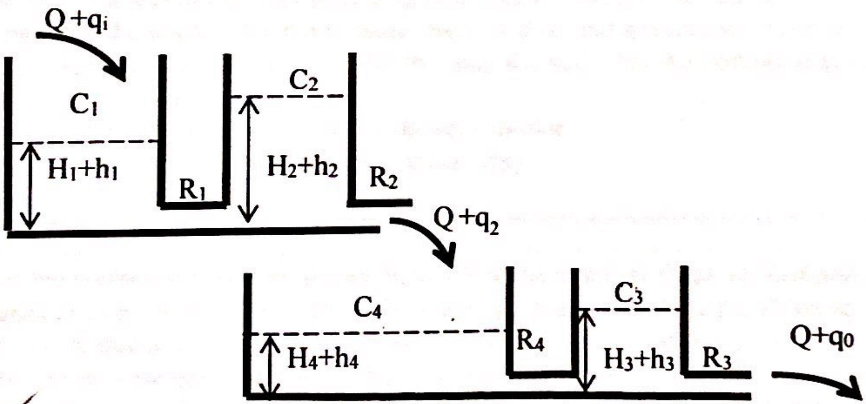
$$x(0) = 0.02 \ \ [m] \qquad \dot{x}(0) = 0.1 \ \ [m/s]$$

a) Obtain X(s) for a impulse force input F=3 $\delta(t)$, and m=1.5 [Kg], b =0.3 [N/ms^{-1}] k=15 [N/m], K_1=3 [N/m] and K=2 [N/m].

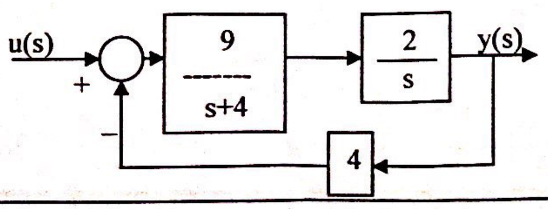
b) What is the damping ratio ζ, the natural frequency ω_n and the damped natural frequency of oscillation ω_d?



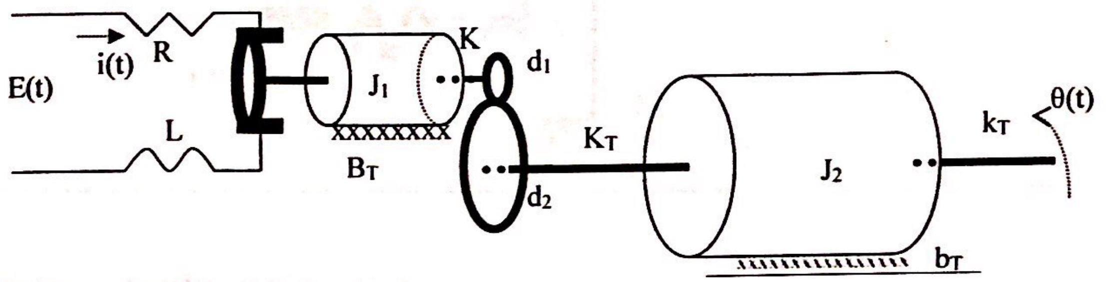
4. For this system, obtain the transfer function $q_o(s)/q_i(s)$.



5. Obtain the state space representation in matrix form for this system:



6. This system has an input voltage E [V] and a DC motor with rotor resitance R and inductance L, motor gain K, back emf gain k (E_b=kω, ω is motor angular velocity [rad/s]) and two disks with the moments of inertia J_1 et J_2 [Kg m^2], linked by gears with diameters d_1 and d_2. The disks have viscous friction with the fixed frame with the coefficients B_T [Nm /(rad s^{-1})] and b_T [Nm /(rad s^{-1})]. The flexible shafts have the coefficients K [Nm/rad], k_T [Nm/rad] and K_T [Nm/rad]. Obtain the transfer function θ(s) / E(s).



7. For the system from problem 6, consider that the transfer funtion θ(s) / E(s) is used for the design of a servomotor by attaching two potentiometers with the same gain k [V/ rad] to transform the angle θ(t) in a voltage $e_\theta(t)$ =k θ (t) and a reference input r(t) in a voltage $e_r(t)$ = k r(t), applied to an amplifier with the gain K_p, such that the voltage applied to the rotor is $E(t)= K_p [e_r(t) - e_\theta(t)]$

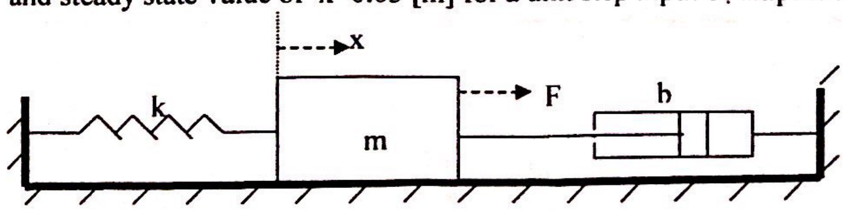
a. Draw the block diagram of the servomotor
b. Obtain the transfer function θ (s) / r(s).

8. For the mechanical system shown below F is the external force applied and x is the position variable of m. There is no friction between the mass m and the fixed frame.

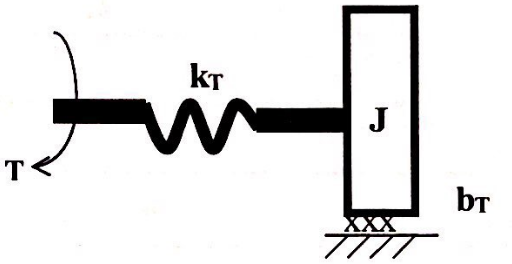
a) Obtain parameters : the natural frequency ω_n and the damping ratio ζ and the transfer function x(s)/F(s) in the canonic form using the parameters ω_n and ζ

b) Obtain the equations for the maximum overshoot and the peak time function of the natural frequency ω_n and the damping ratio ζ

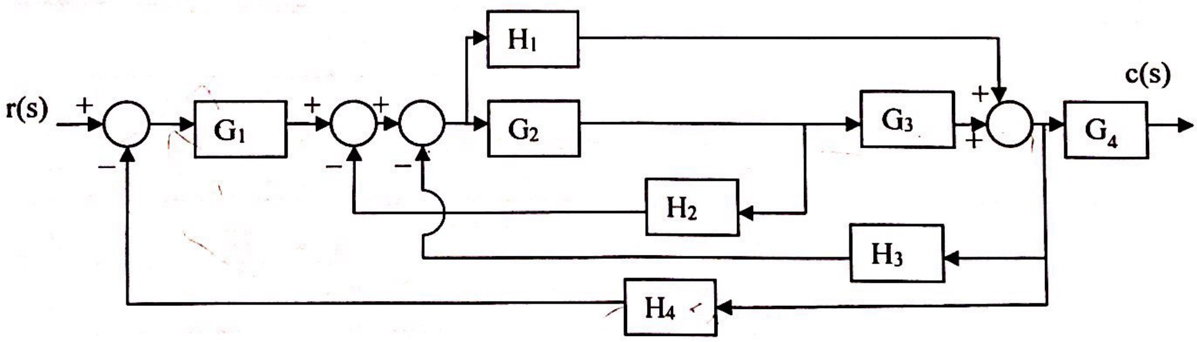
c) obtain the values of m, b and k that result in a maximum overshoot of 0.25 [m], peak time of 2 [s] and steady state value of x=0.03 [m] for a unit step input of amplitude of 1 [N].



9. For the underdamped system shown below, find k_T, b_T and J to yield maximum overshoot of 0.4 [m], a peak time of 0.25 [s] and a steady state value of x=0.15[rad] for a unit step input torque T(t) [Nm].



10. Obtain the transfer function c(s) / r(s).



HINTS $\exp\{-\pi\zeta/\sqrt{(1-\zeta^2)}\}$, $\pi/[\omega_n\sqrt{(1-\zeta^2)}$,