ELG 3155

Lab Assignment 5: Compensator Design

a)

L(s) = K, K > 0):

$LG(s) = \frac{K}{s(s+3)(s+6)}$ H(s)=1

$1 + k\frac{1}{s(s+3)(s+6)}$ (Open loop Transfer function)

3 poles at 0,-3 and -6. No zeros.

Figure 1 Shows the Root locus for the open loop transfer function.

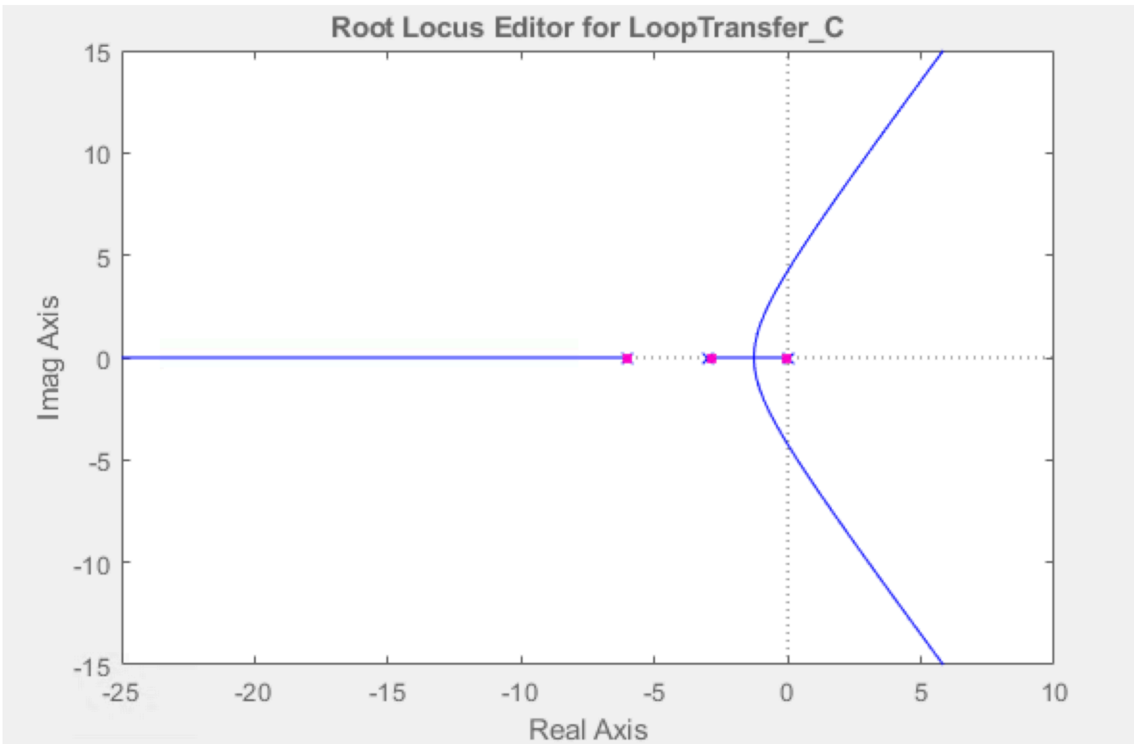


Figure 1: Root Locus of the OLTF.

From figure 2 we see the value of K that produces an OS% of 20%. We notice that at K=32.3 the operation point Is -0.947+j1.91 and an

OS% of 20% is produced.

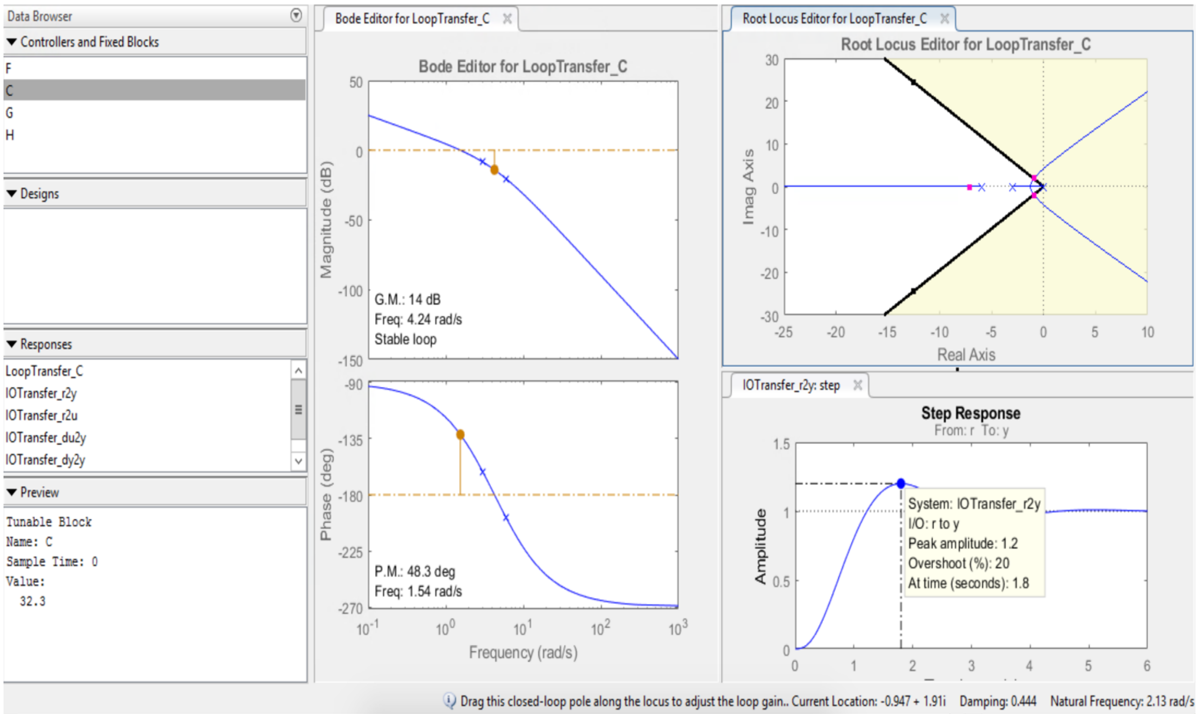


Figure 2: Closed loop Root Locus showing Gain K that produces 20% OS% for the uncompensated system.

b)

From Figure 3 we see that at K=32.3 (-0.947+j1.91 operating point)

T_p(Peak Time) = 1.8sec

T_r(Rise Time)= 0.766sec

T_s(Settling time)= 4.06sec

OS%(Overshoot %)= 20%

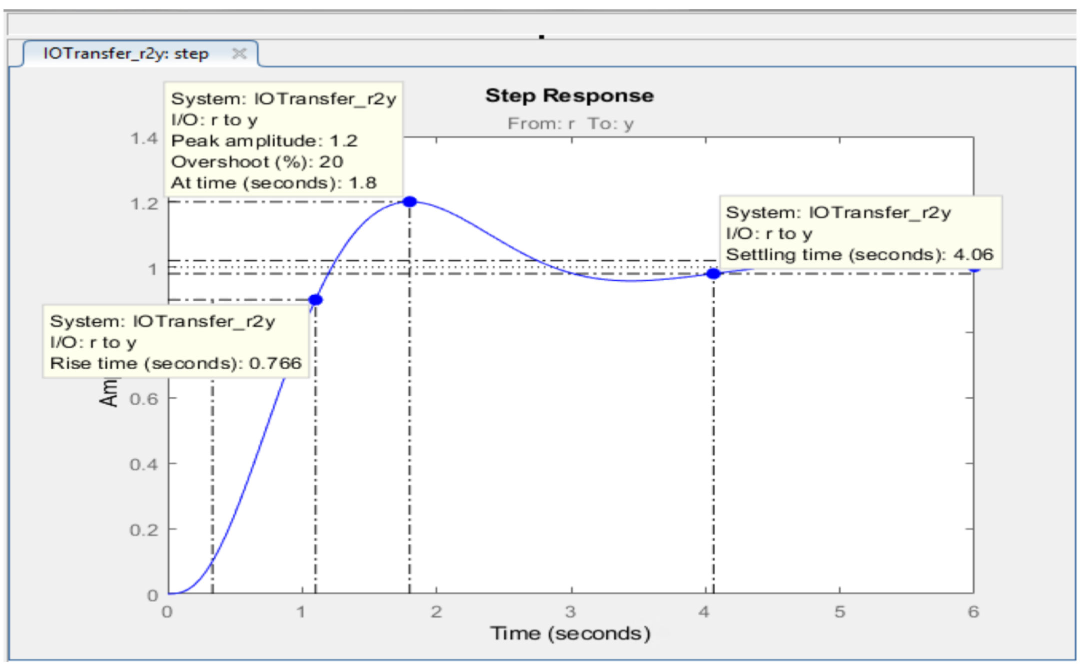


Figure 3: Step response of the uncompensated system at gain K=32.3.

The system is of type 1 therefore

K_p(position constant) $= \infty$

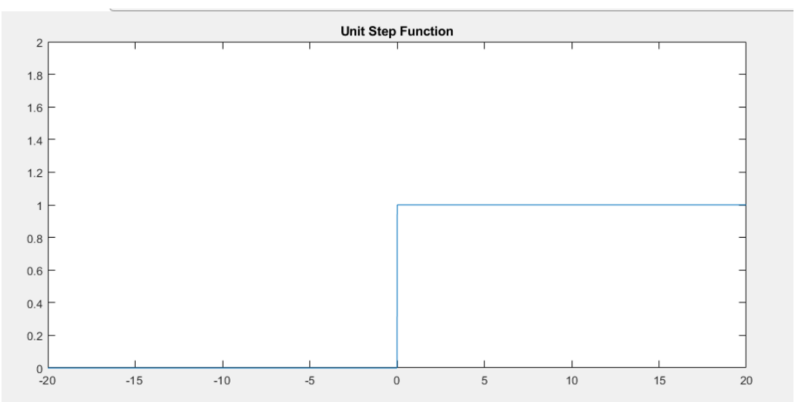
$$K_v = \lim_{s \to 0} s\,G(s)$$

$K_a = 0$

$$K_v = \lim_{s \to 0} s \frac{32.3}{s(s+3)(s+6)}$$

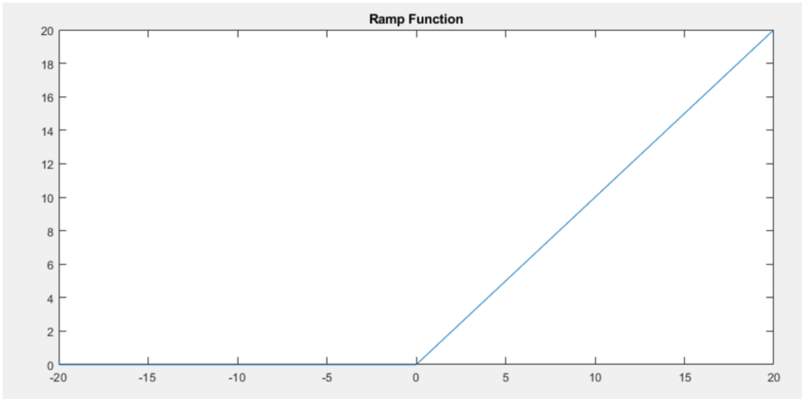
$K_v = 1.794$

<u>Step Input</u>



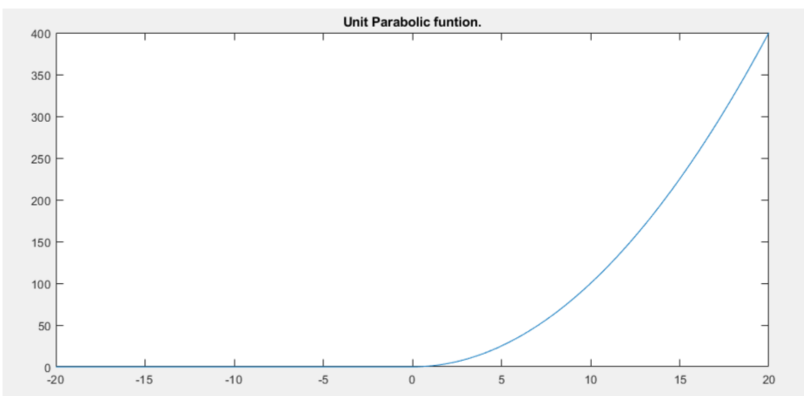
Error=0

<u>Ramp Input</u>



Error= $\frac{1}{1.794}$ =0.5572

<u>Parabolic Input.</u>



Error = ∞

c)

$L_1(s)=G_{PI}(s)=K_1+\frac{K2}{s}=\frac{K1(s+ZcI)}{s}$

$L_1G(s)=\frac{K1(s+ZCI)}{s\text{^}2(s+3)(s+6)}$ H(s)=1

$$1 + k1\frac{(s+ZCI)}{s\text{^}2(s+3)(s+6)}$$ (Open loop Transfer function)

3 poles at 0,-3 and -6. 1 zero at Z_{CI}

Z_{CI}= one tenth of the real part of the dominant poles of the uncompensated System.

$Z_{CI}= \frac{1}{10}$x -0.947

Z_{CI} is located at -0.0947.

$$1 + k1\frac{(s+0.0947)}{s(s+3)(s+6)}$$

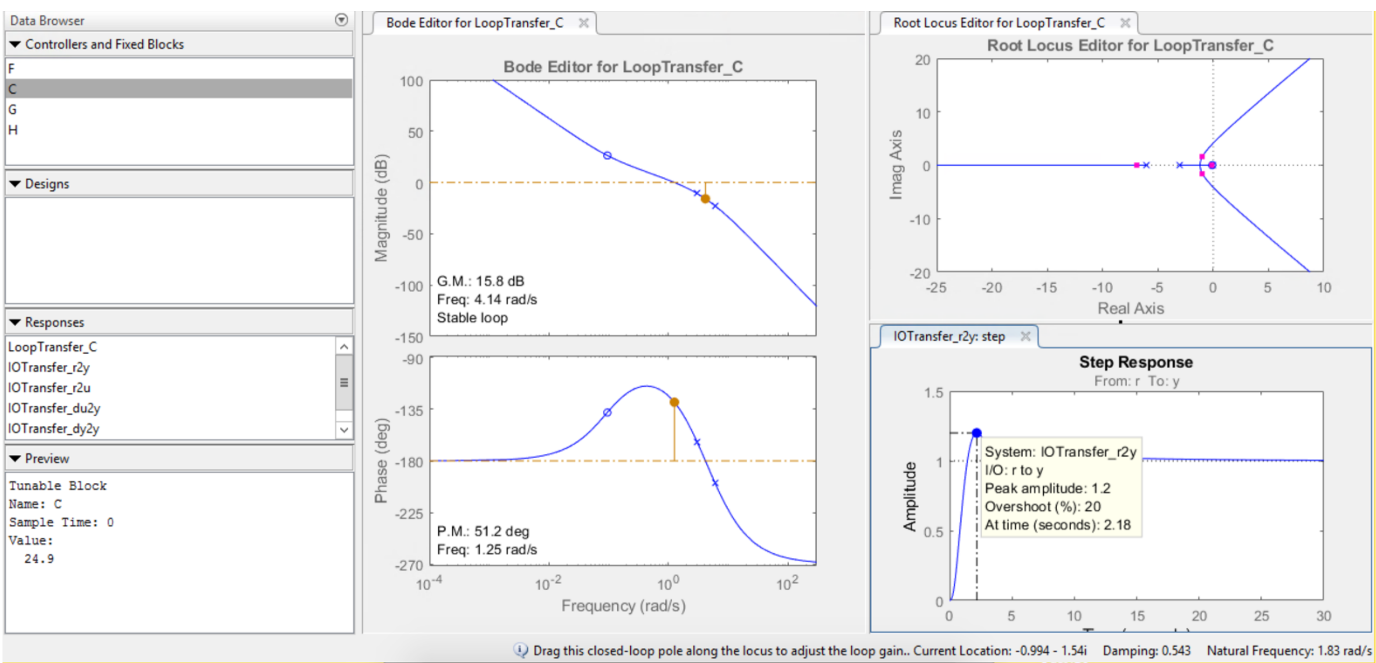


Figure 4: Gain K_1 of the PI controller for OS 20%

From figure 4 we notice that at gain K_1=24.9

(Operating point -0.994+J1.54) the system has an

OS% of 20%.

$L_1(s)=K_1+\frac{K2}{s}=\frac{K1(s+ZcI)}{s}$

$L_1(s)=24.9+\frac{K2}{s}=\frac{24.9(s+0.0947)}{s}$

K_2= 2.358.

From Figure 5 we see that at K_1=32.3 -

0.994+j1.54operating point)

T_p(Peak Time) = 2.18sec

T_r(Rise Time)= 0.9066sec

T_s(Settling time)= 13.7sec

OS%(Overshoot %)= 20%

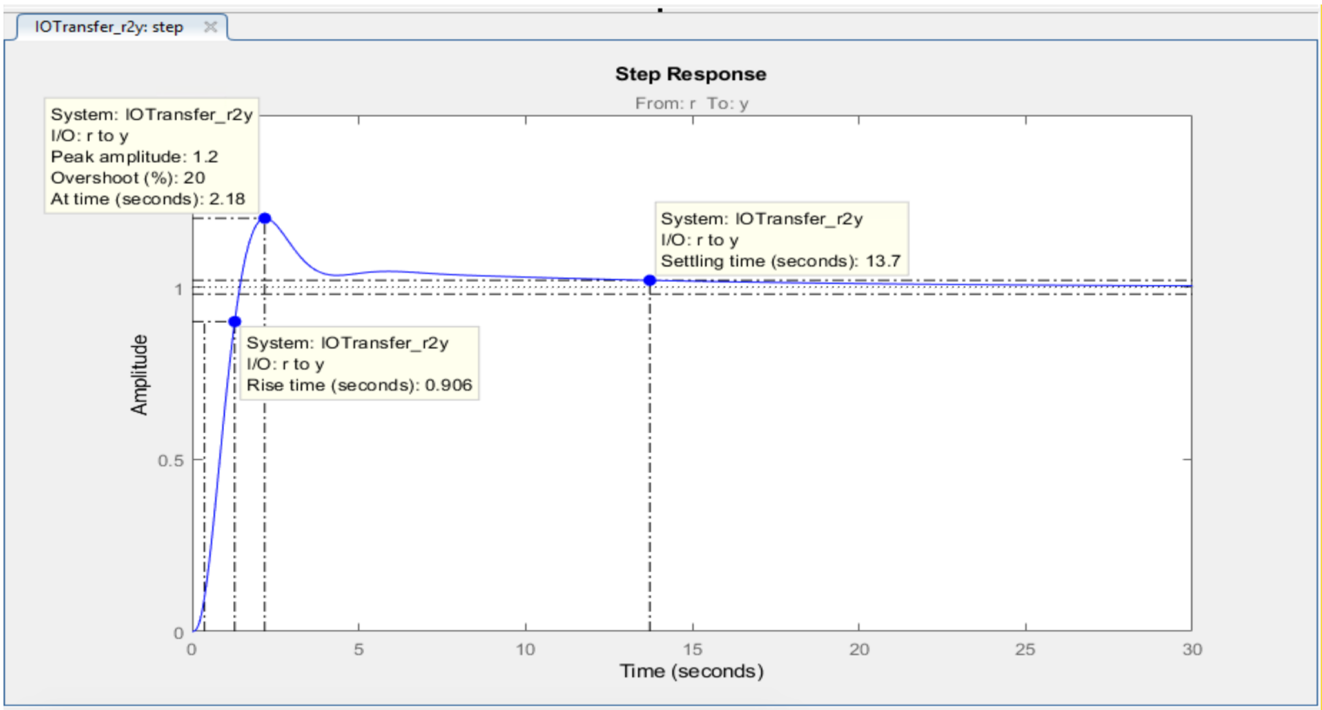


Figure 5: Step response of the PI controller

system at gain K_1=24.9

The system is of type 2 therefore

K_p(position constant) = ∞

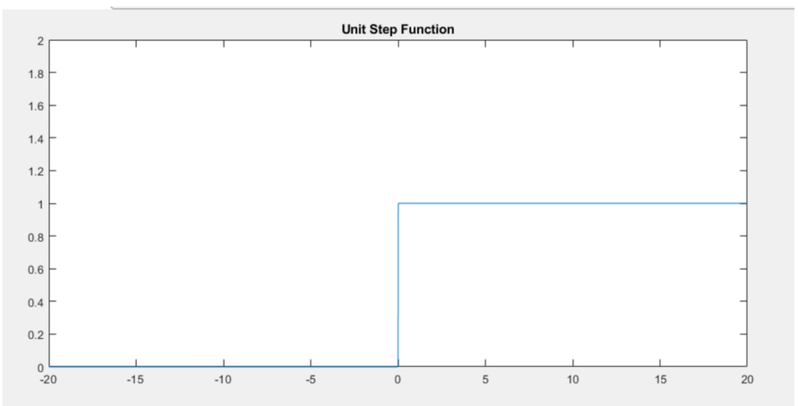
K_v= ∞

$K_a= \lim_{s \to 0} s^2 LG(s)$

$$K_a = \lim_{s \to 0} s^2 \frac{24.9(s+0.0947)}{s^2(s+3)(s+6)}$$

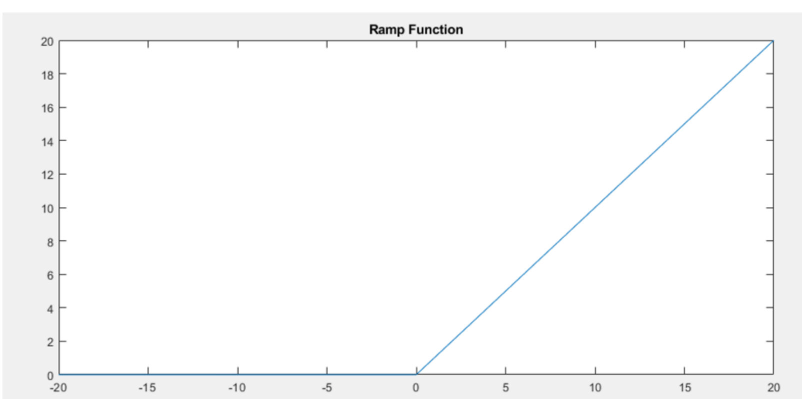
$$K_a = \frac{24.9 \times 0.0947}{18} = 0.131$$

Step Input



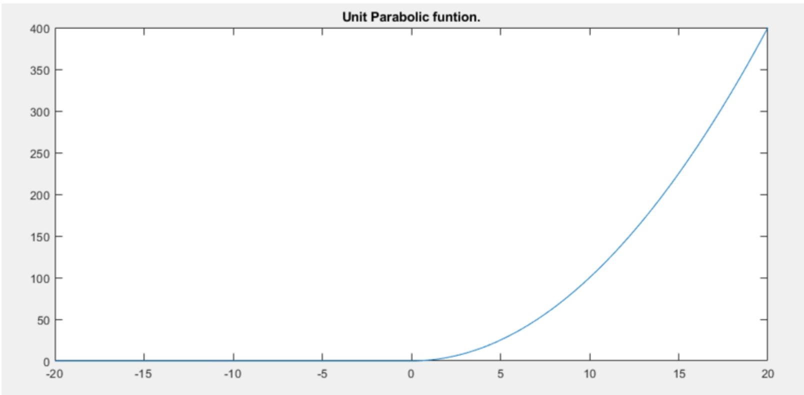
Error=0

Ramp Input



Error= 0

Parabolic Input.



Error $= \frac{1}{Ka} = \frac{1}{0.131} = 7.63$

	Uncompensated system	PI Controller system
Gain	32.3	24.9
T_p	1.8sec	2.18sec
T_r	0.766sec	0.9066sec
T_s	4.06sec	13.7sec

OS%	20%	20%
K_p | step error	∞ | 0	∞ | 0
K_v | ramp error	1.794 | 0.5572	∞ |0
K_a | parabolic error	0 | ∞	0.131| 7.63

The Goal of the PI compensator was to eliminate the error of the system, and from the values shown in the table above. The Error was decreased for all inputs(Step, Ramp and parabolic).

d)

$L_2(s)=G_{PD}(s)=K_4s+K_3 = K_4(s+Z_{CD})$

$L_2G(s)=\frac{K4(S+ZCD)}{s(s+3)(s+6)}$ $H(s)=1$

$1 + k4\frac{(s+ZCD)}{s(s+3)(s+6)}$ (Open loop Transfer function)

$OS\%=20$ & $T_s=2sec$

$$OS\% = e^{-\frac{\pi}{\tan\theta}} \times 100\% = 20\%$$

$$\ln(0.2)=\frac{-\pi}{\tan\theta}$$

$$\tan\theta = \frac{-\pi}{-1.609}$$

$$\tan\theta = 1.952$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{|Re(poles)|}$$

$$2sec = \frac{4}{|Re(poles)|}$$

$$|Re(poles)| = 2$$

$$\tan\theta = \frac{|Im(poles)|}{|Re(poles)|}$$

$1.952 = \frac{|Im(poles)|}{2}$

$|Im(poles)| = 3.90$

Operatring point $p_1 \equiv -2 + j3.90$

⇨ $[\angle s + Z_{CD}) - [\angle(s) + \angle(s+3) + \angle(s+6)]] = \angle \frac{-1}{K_4} = 180°$

$\angle s + Z_{CD}) = -= 180° + [\angle(s) + \angle(s+3) + \angle(s+6)]]$

$\angle(s) = \angle p_1 = 117.15°$

$\angle(s+3)\,|_{s=p1} = 75.62°$

$\angle(s+6)\,|_{s=p1} = 44.27°$

$\angle s + Z_{CD} = 180° + [117.15° + 75.62° + 44.27°]$

$\angle s + Z_{CD}\,|_{s=p1} = 57.04°$

$\tan(180° - 57.04°) = \frac{3.90}{2 - |Z_{CD}|}$

$Z_{CD} = 4.53$

Using Rule 10 of the RL to find K_4

$$1 + k4\frac{(s+ZCD)}{s(s+3)(s+6)} = 0$$

$$K_4 = \frac{-s(s+3)(s+6)}{(s+4.53)}$$

$S = p_1 = -2 + j3.90$

$K_4 = 21.20$

$K_3 = K_4 \times Z_{CD} = 21.20 \times 4.53$

$= 96.036$

$L_2(s) = G_{PD}(s) = K_4 s + K_3 = K_4(s + Z_{CD})$

$L_2(s) = G_{PD}(s) = 21.20s + 96.036K_3 = 21.20(s+4.53)$

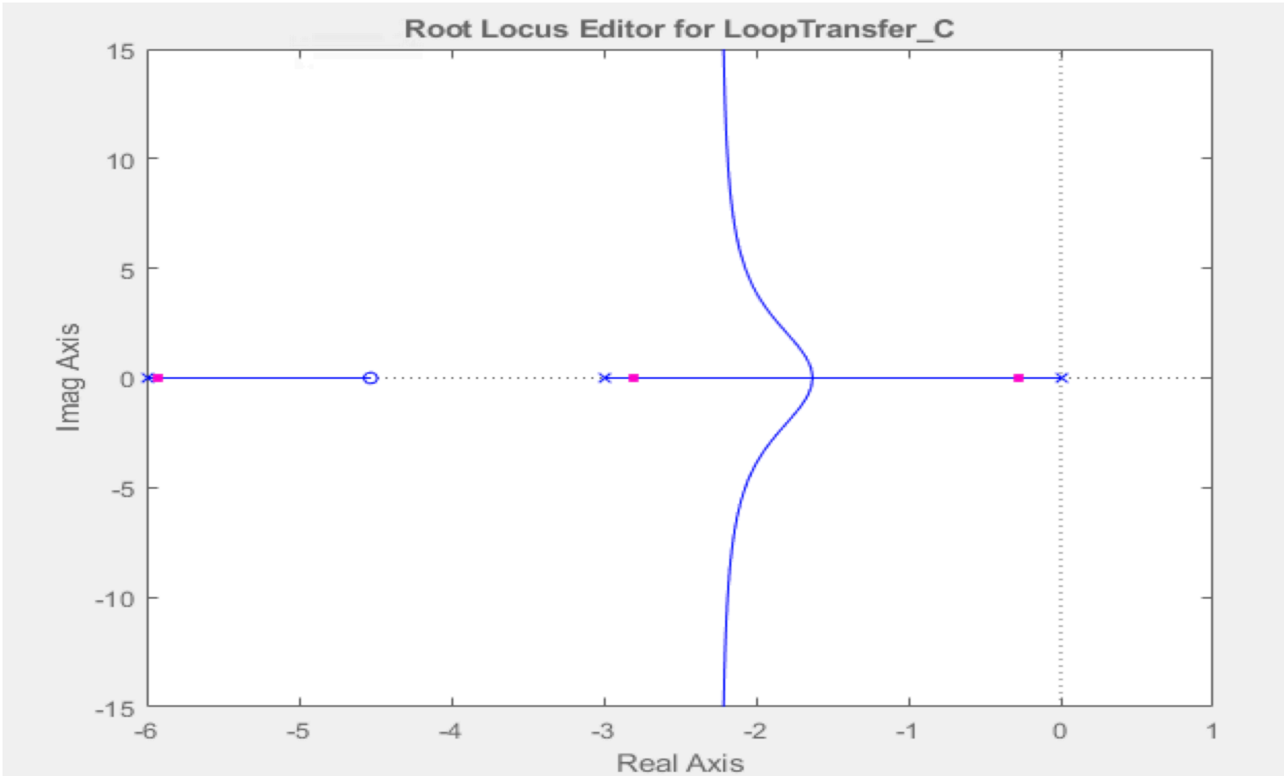


Figure 6: The root locus of the PD compensated system with K_4 as the free parameter.

From figure 7 we see at K_4 =21.20 (-2+j3.9) the PD compensated system has a settling time of 1.9sec and an over shoot% of 21.9%

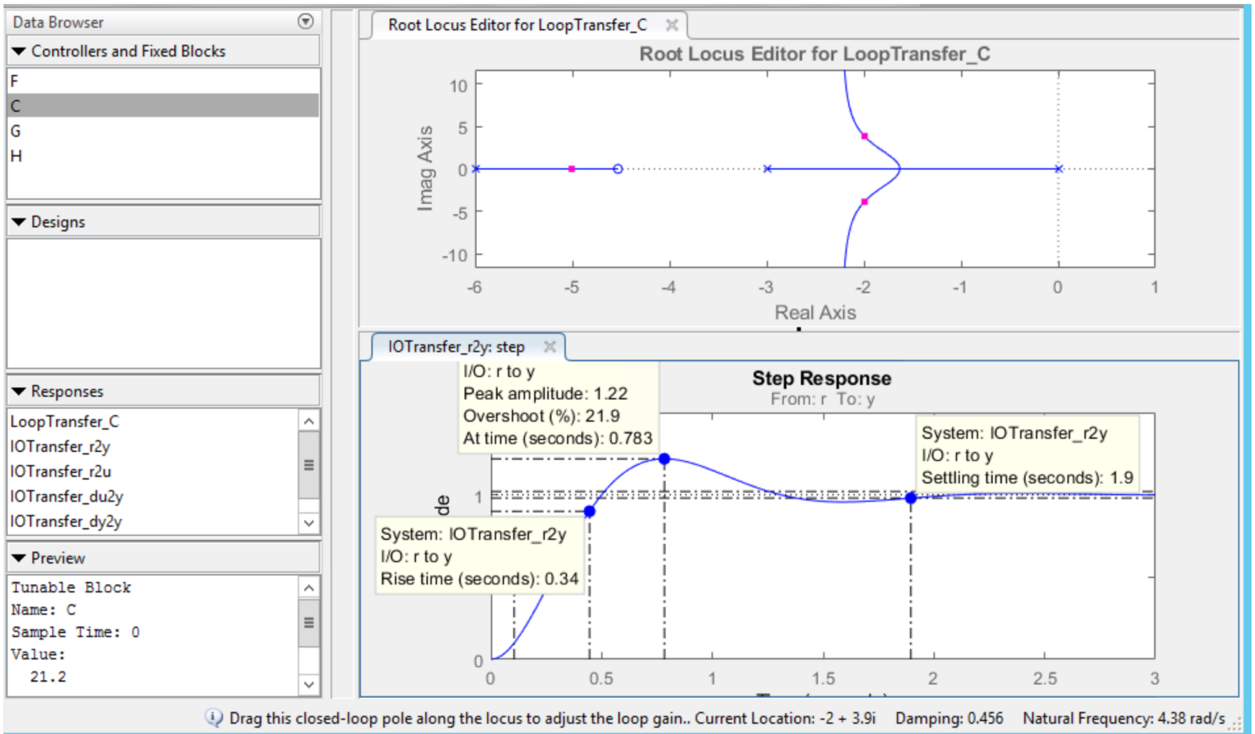


Figure 7: The root locus of the PD compensated system at K_4 =21.20 (-2+j3.9)

From Figure 7 we see that at K_4=21.20. -2+j3.9(operating point)

T_p(Peak Time) = 0.783sec

T_r(Rise Time)= 0.34sec

T_s(Settling time)= 1.9sec

OS%(Overshoot %)= 21.9%

	Analytical design	Numerical validation (sisotool)	Numerical Design (sisotool)
Compensator	$K_4(s+4.53)$	$K_4(s+4.53)$	$K_4(s+4.53)$
K_4	21.20	21.20	19.2
Conjugate poles	-2±j3.9	-2±j3.9	-.198±j3.65
ζ	0.456	0.456	0.477

ω_n(rad/s)	4.39	4.39	4.15
OS%	20%	21.9%	20%
T_p (s)	1.02	0.783s	0.838
T_s (s)	2s	1.9s	1.98

The system is of type 1 therefore

K_p(position constant) = ∞

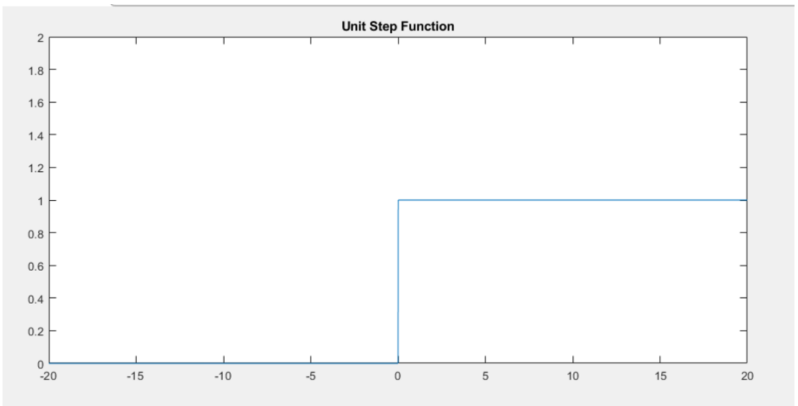
$K_v = \lim_{s \to 0} s\, LG(s)$

$K_a = 0$

$K_v = \lim_{s \to 0} s \frac{21.20(s+4.53)}{s(s+3)(s+6)}$

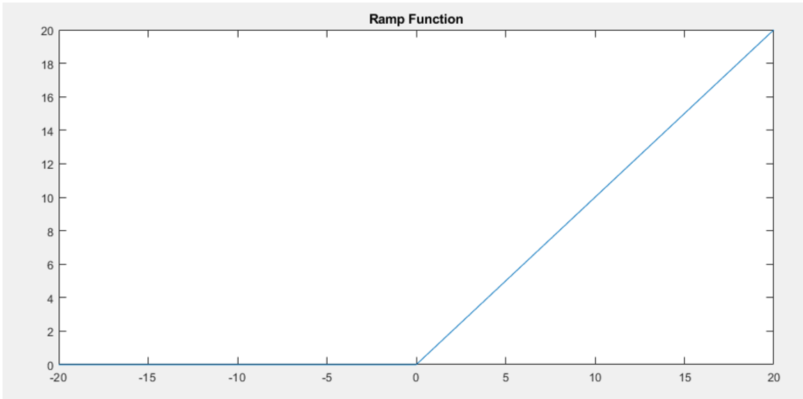
$K_v = 5.34$

<u>Step Input</u>



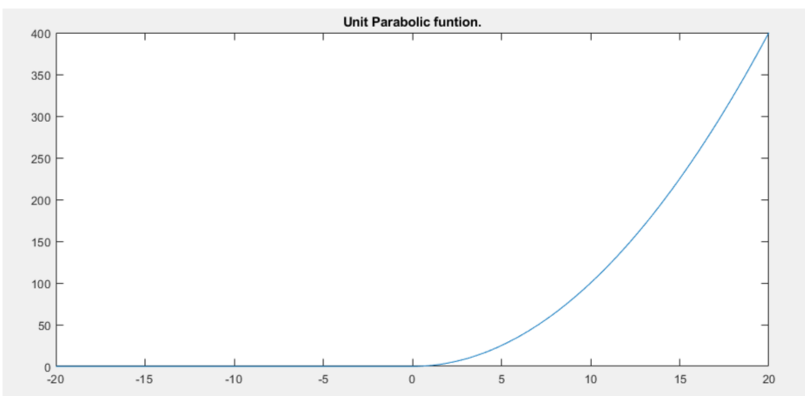
Error=0

Ramp Input



Error= $\frac{1}{5.34}$ =0.1873

<u>Parabolic Input.</u>



Error $= \infty$

	Uncompensated system	PD Controller system
Gain	32.3	21.20
T_p	1.8sec	0.783sec
T_r	0.766sec	-
T_s	4.06sec	1.8s
OS%	20%	20%
K_p | step error	∞ |0	∞ |0
K_v | ramp error	1.794 | 0.5572	5.34 |0.1873
K_a | parabolic error	0 | ∞	0 | ∞

From the table above we notice that the transient properties of the system change when we keep the OS% at 20%, therefore achieving the results we wanted. The Rise, Peak and settling time were all shorter.

e)

$L_1(s)=24.9+\frac{K2}{s}=\frac{24.9(s+0.0947)}{s}$

$L_2(s)=G_{PD}(s)=21.20s+96.036K_3 = 21.20(s+4.53)$

$L_3(s)= L_1L_2(s) = \frac{(24.9\times 21.20)(s+0.0947)(s+4.53)}{s}$

$L_3(s) = \frac{527.88(s^2 4.625s+0.429)}{s}$

From Observation

$K = 527.88$

$K_5 = 4.625$

$K_6 = 0.429$

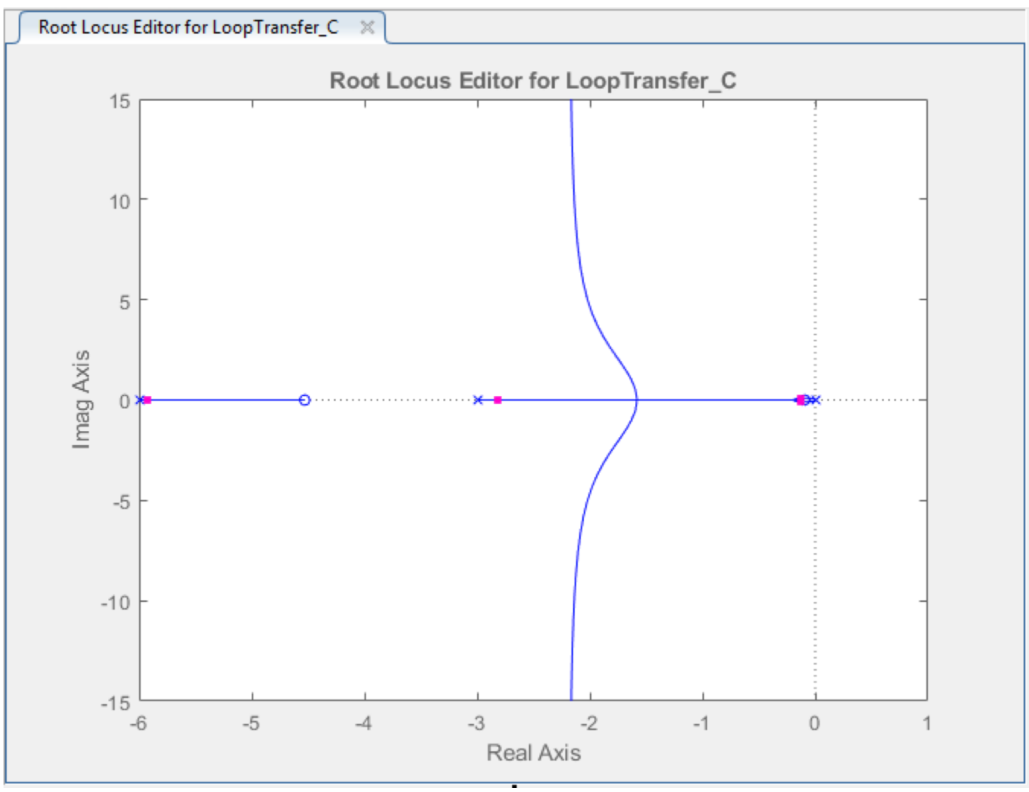


Figure 8: the root locus of the compensated system with K being the free parameter

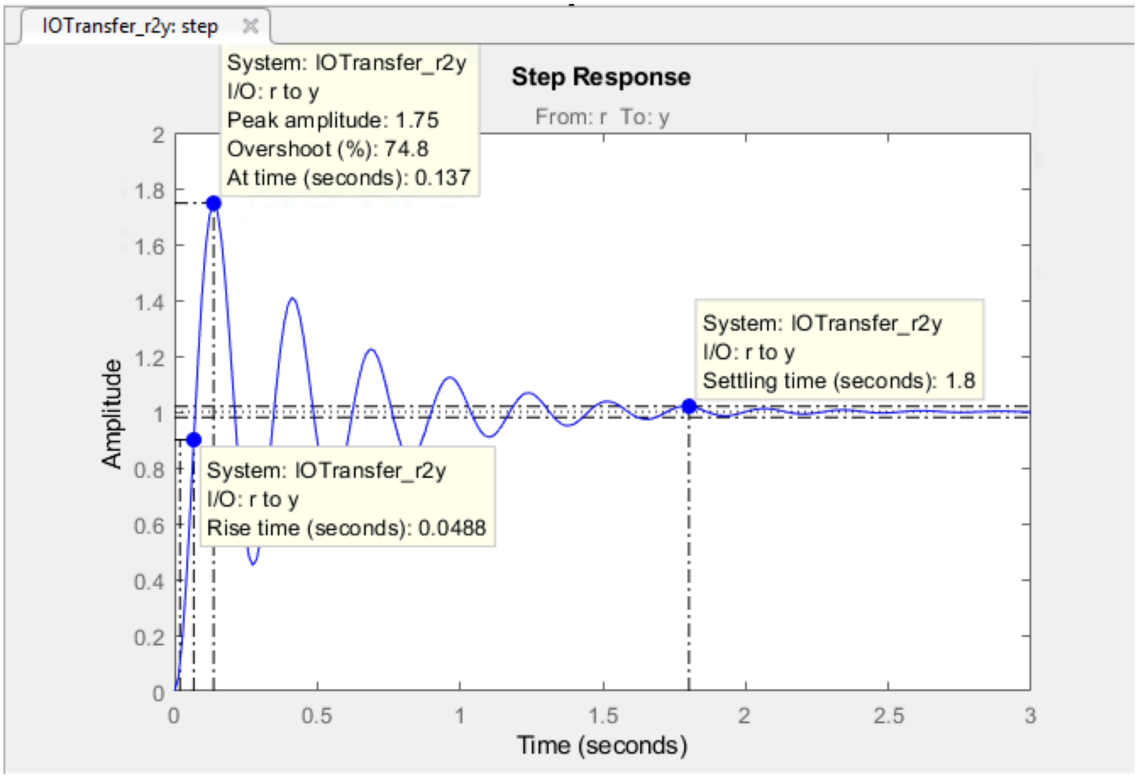


Figure 9: Step response of this PID compensated system with K= 527.88.

T_p(Peak Time) = 0.137sec

T_r(Rise Time)= 0.0488sec

T_s(Settling time)= 1.8sec

OS%(Overshoot %)= 74.8%

The system is of type 2 therefore

K_p(position constant) = ∞

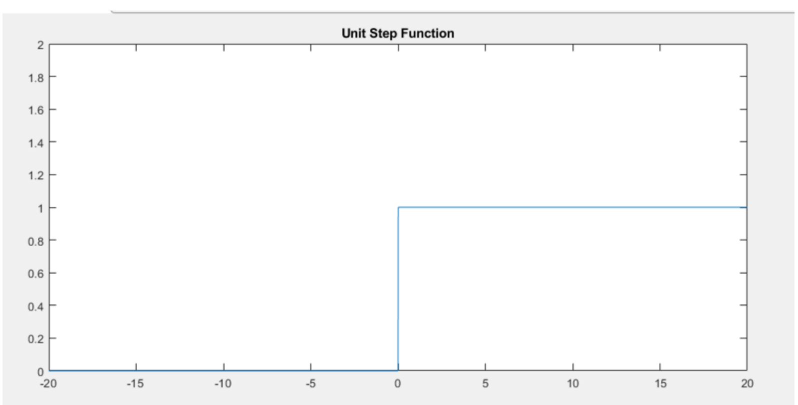
K_v= ∞

$K_a= \lim_{s \to 0} s^2 LG(s)$

$K_a= \lim_{s \to 0} s^2 \frac{527.88(s+4.53)(s+0.0947)}{s^2(s+3)(s+6)}$

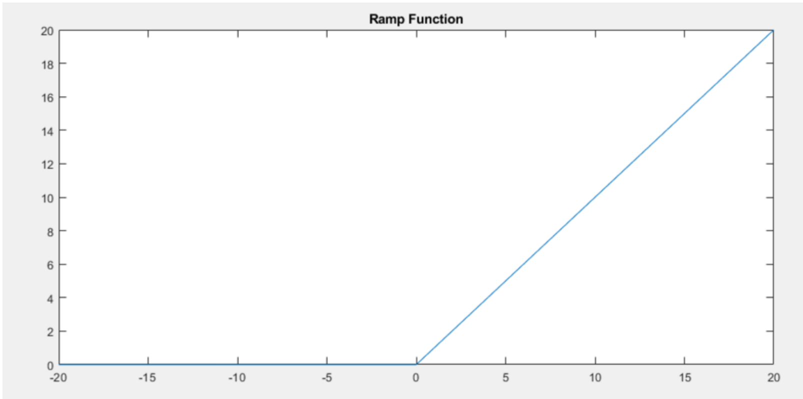
$K_a= \frac{527.88 \times 4.53 \times 0.0947}{18} = 12.58$

<u>Step Input</u>



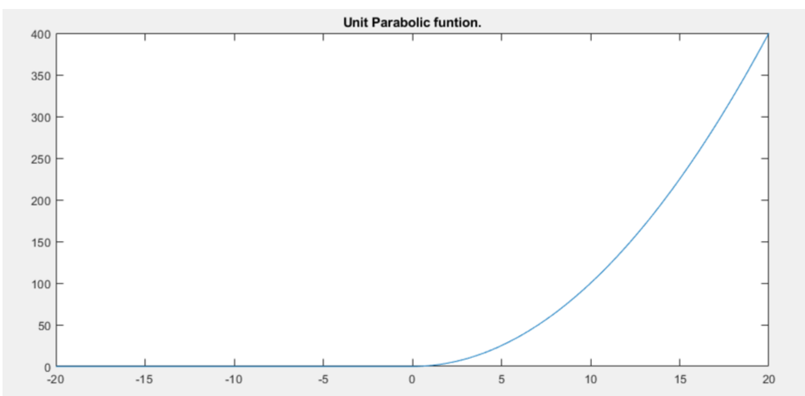
Error=0

<u>Ramp Input</u>



Error= 0

Parabolic Input.



$$\text{Error} = \frac{1}{Ka} = \frac{1}{12.58} = 0.079$$

	Uncompensated system	This PID Controller system
Gain	32.3	527.88
T_p	1.8sec	0.137sec
T_r	0.766sec	0.0488sec
T_s	4.06sec	1.8s
OS%	20%	74.8%
K_p | step error	∞ |0	∞ |0
K_v | ramp error	1.794 | 0.5572	∞ |0
K_a | parabolic error	0 | ∞	12.58| 0.079

From observation we see that the compensator succeeded in eliminating the error but the OS% was too large.

f)

By Tuning the gain manually it is noticed that when the gain is 16 the OS% is 20 but the settling time is 3.27s. This method of designing a PID controller is in efficient because multiplying the gains creates too much oscillations and a high overshoot.

g)

From question d we already designed the PD controller to satisfy the transient properties of 20% OS% and 2s Settling time.

$L_2(s)=G_{PD}(s)=21.20s+96.036K_3 = 21.20(s+4.53)$

We put a pole at the origin and another zero really close to the origin at 0.1.

$L_4(s)= L_1L_2(s) = k\frac{(s+0.1)(s+4.53)}{s}$

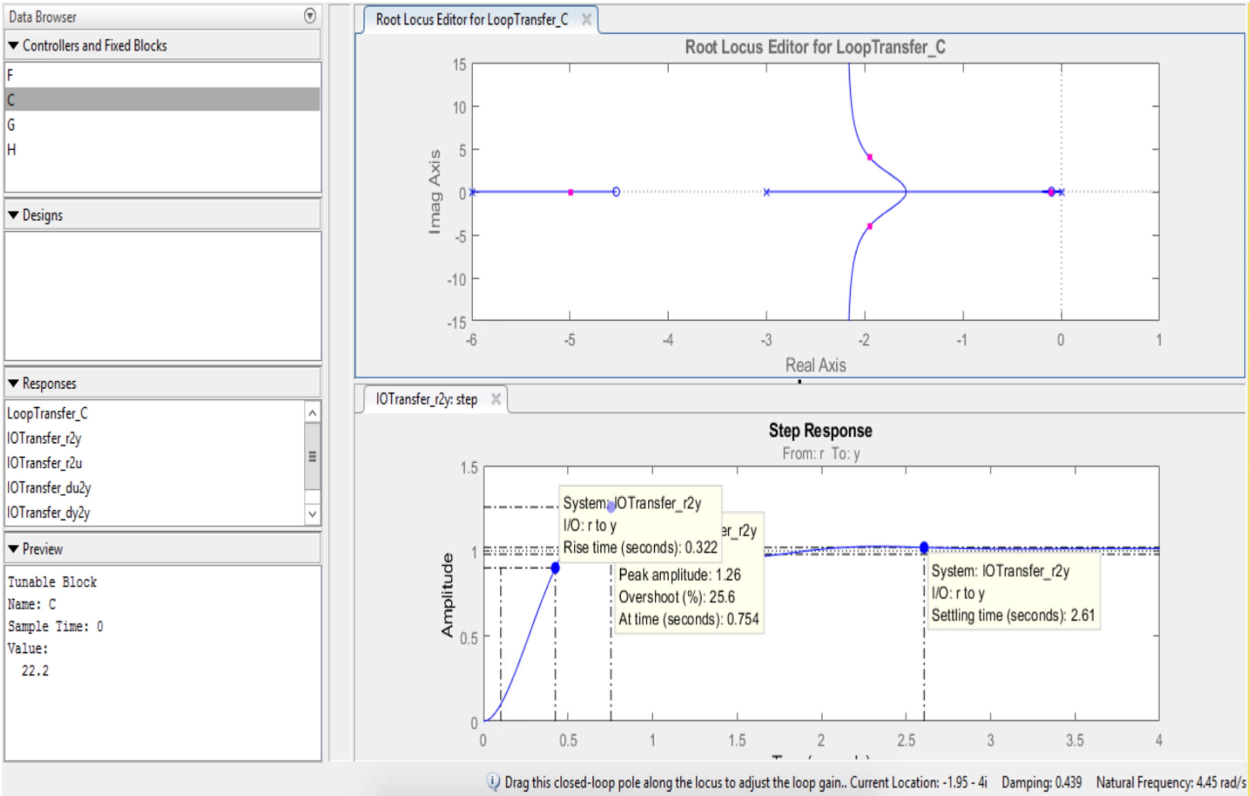


Figure 10: Root Locus and Step response at K= 22.2

When k= 22.2 -1.95+j4 operating point.

$$L_4(s) = L_1L_2(s) = 22.2\frac{(s+0.1)(s+4.53)}{s}$$

The OS% is 25.6% and the settling time is 2.61 % which is as close to our design requirements as possible.

To get more accurate results we need to place our zero at a more accurate position.

T_p(Peak Time) = 0.754sec

T_r(Rise Time)= 0.322sec

T_s(Settling time)= 2.61sec

OS%(Overshoot %)= 25.6%

The system is of type 2 therefore

K_p(position constant) = ∞

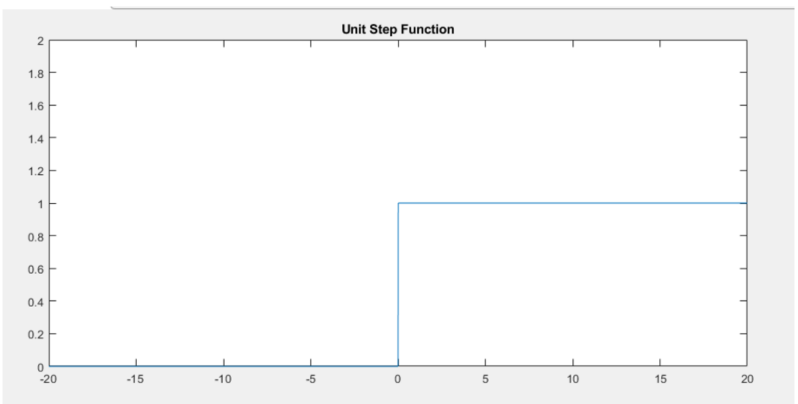
K_v= ∞

$$K_a = \lim_{s \to 0} s^2 \, LG(s)$$

$$K_a = \lim_{s \to 0} s^2 \frac{22.2(s+4.53)(s+0.1)}{s^2(s+3)(s+6)}$$

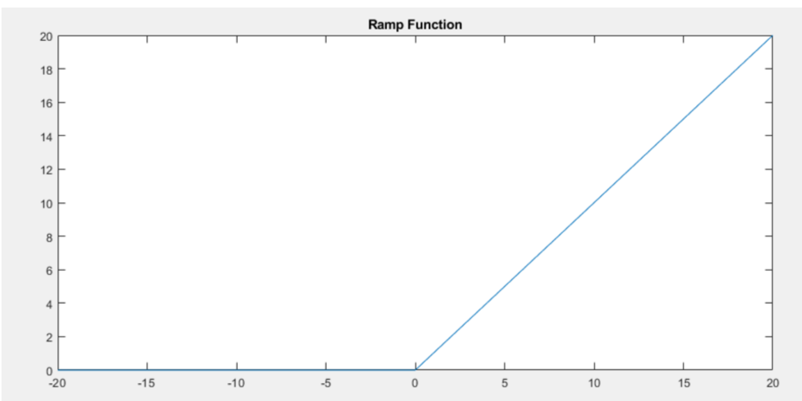
$$K_a = \frac{22.2 \times 4.53 \times 0.1}{18} = 0.5587$$

Step Input



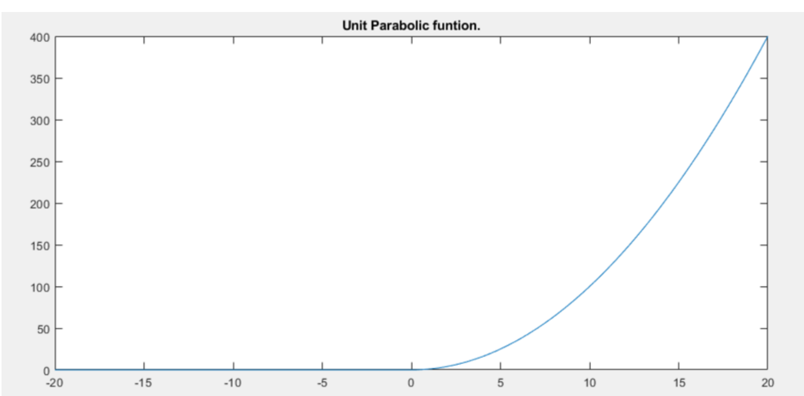
Error=0

Ramp Input



Error= 0

Parabolic Input.



$$\text{Error} = \frac{1}{Ka} = \frac{1}{0.5587} = 1.79$$

	Uncompensated system	This PID Controller system
Gain	32.3	22.2
T_p	1.8sec	0.754sec
T_r	0.766sec	0.322sec
T_s	4.06sec	2.61s
OS%	20%	25.6%
K_p | step error	∞ |0	∞ |0

K_v | ramp error	1.794 | 0.5572	∞ | 0
K_a | parabolic error	0 | ∞	0.5587| 1.79

From the table above we notice that this controller is able to reduce the transient response times and eliminate the steady state error of the system while maintaining minimal oscillation and close to the required OS%.

This form of designing a PID controller is more efficient than the previous form of multiplying the gains, because we are able to reduce the times of our transient response with lower oscillations while also eliminating the steady state error.

h)

We are going to design a Phase Lead compensator.

Where $L_5(s) = \frac{(s-Z_C)}{(s-P_C)}$

Where $|P_C| > |Z_C|$

We are going to place Z_C close to the origin but not at 0.

$Z_C = 0.1$

P_C close to Z_C

$P_C = 0.2$

$$L_5(s) = k\frac{(s+0.1)}{(s+0.2)}$$

$$L_5G(s) = K\frac{(s+0.1)}{(s+0.2)(s+3)(s+6)}$$

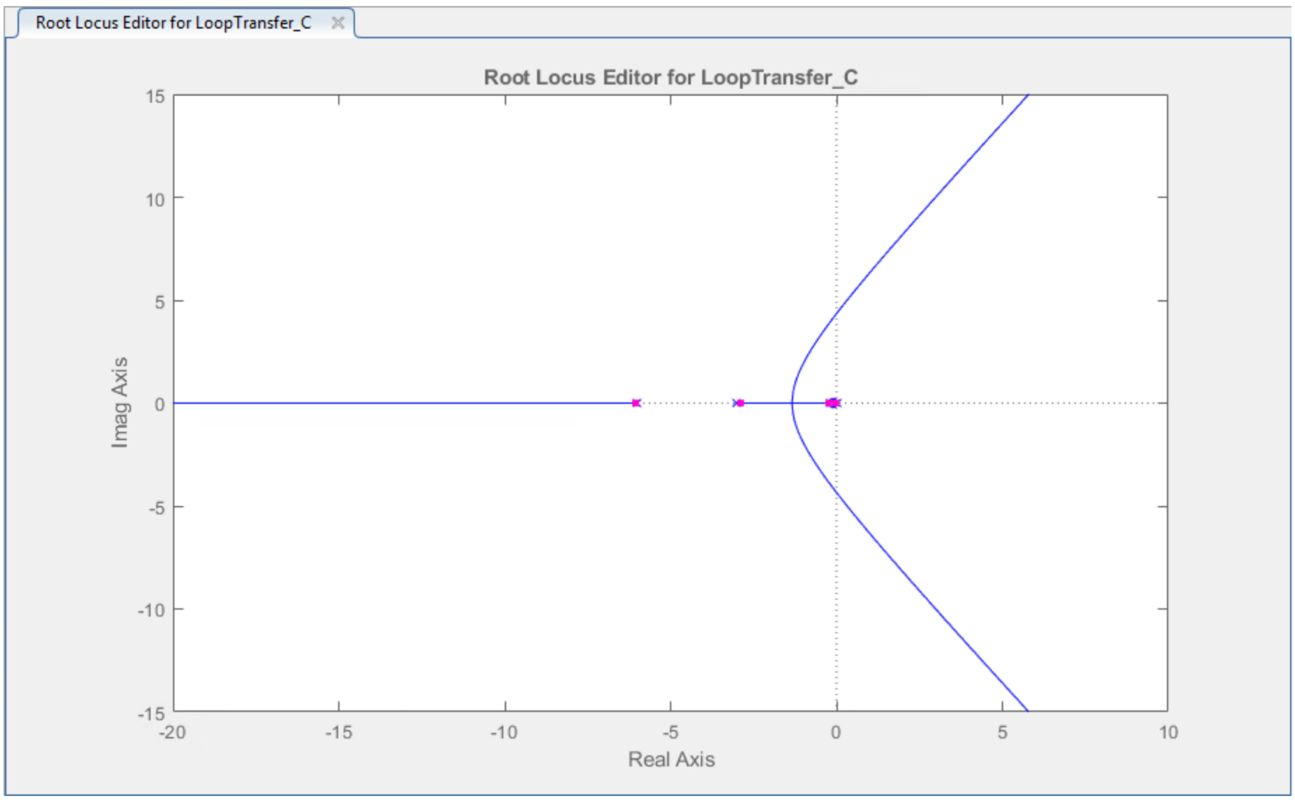


Figure 11: The Root Locus of $L_5G(s)$ for K as a free parameter.

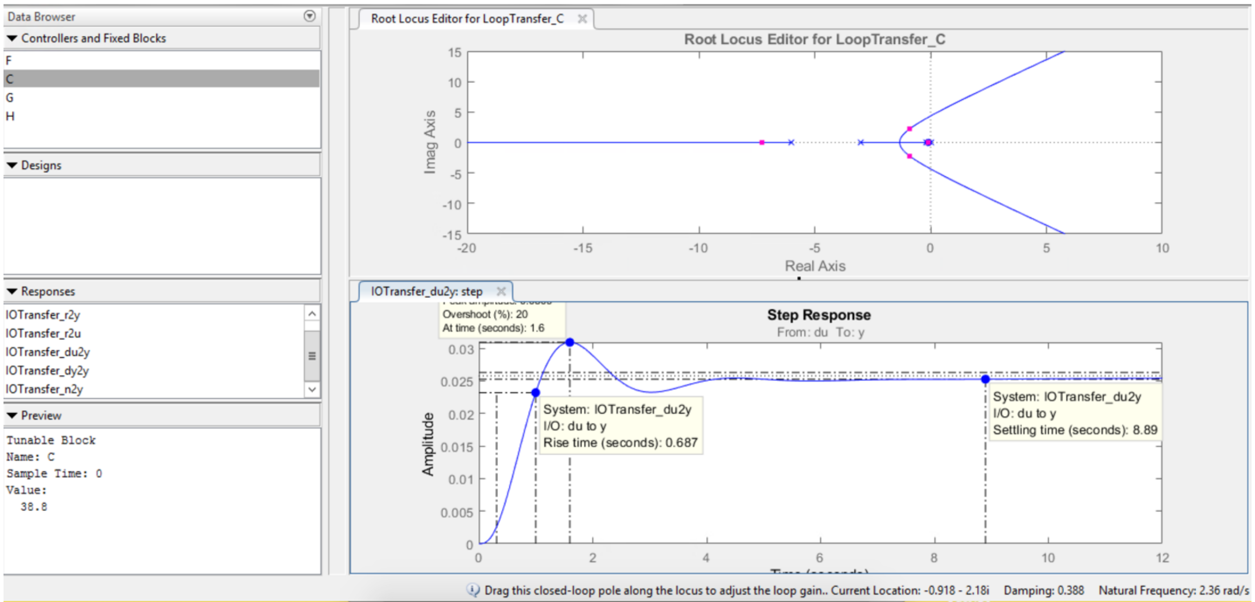


Figure 12: The root locus when K=38.8 to create an OS% of 20%

T_p(Peak Time) = 1.6sec

T_r(Rise Time)= 0.687sec

T_s(Settling time)= 8.89sec

OS%(Overshoot %)= 20%

The system is of type 1 therefore

K_p(position constant) = ∞

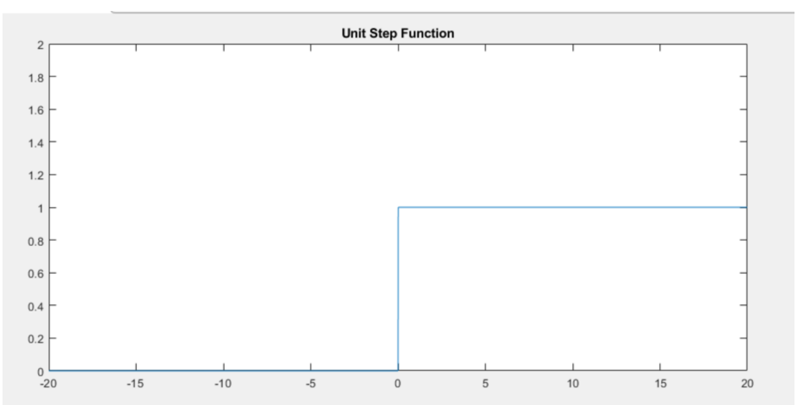
$K_v = \lim_{s \to 0} s\,G(s)$

$K_a = 0$

$K_v = \lim_{s \to 0} s \frac{38.8(s+0.1)}{s(s+0.2)(s+3)(s+6)}$

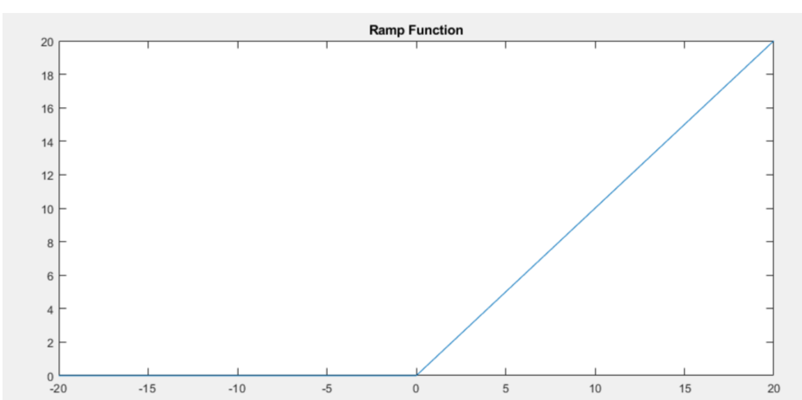
$K_v = 1.0778$

<u>Step Input</u>



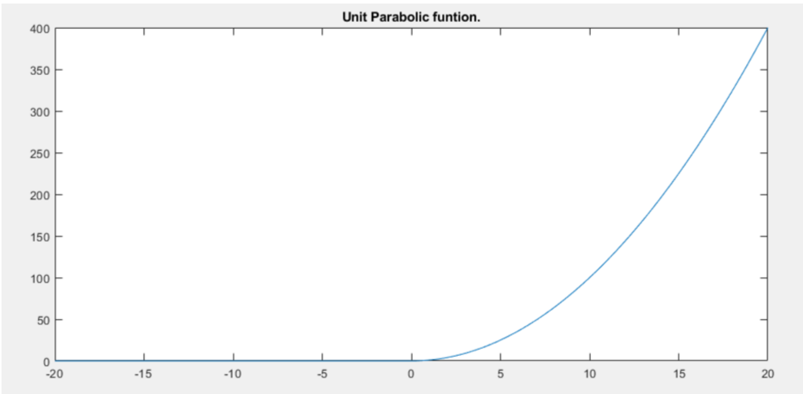
Error=0

Ramp Input



$$\text{Error} = \frac{1}{1.0778} = 0.927$$

Parabolic Input.



Error = ∞

	Uncompensated system	This Phase Lead Controller system
Gain	32.3	38.8

T_p	1.8sec	1.6sec
T_r	0.766sec	0.687ec
T_s	4.06sec	8.89
OS%	20%	20%
K_p | step error	∞ |0	∞ |0
K_v | ramp error	1.794|0.5572	1.0778|0.927
K_a | parabolic error	0| ∞	0| ∞

This compensator reduced the Rise time and Peak time but had a longer settling time so it was partially successful in in reducing the transient response. As for the steady state error it was difficult to design a controller to reduce the error without using a controller in the PID family.

The PID controller we designed in question (g) is better than this because if fulfills more parameters we were after.