

CVG3148 - ASSIGNMENT 5 - Solutions

Problem 1 (30 marks):

a) Original design:

$\rho = 3.14\% > \rho_b = 2.27\% \Rightarrow$ the actual design is an over – reinforced beam

b) Revised design

Since $\rho = 3.14\% < \rho_{\max} = 2.27\% + 1.25\% = 3.52\%$ the beam reinforcement is **OK**.

c) M_r without using CDH Tables:

$$T = \phi_s f_y A_s$$

$$C_s = \phi_s f_y' A_s'$$

$$C_c = \alpha_1 \phi_c f_c' b a$$

Compression Reinforcement at the plastic stage ($f_s' = f_y = 400$ MPa)

At The equilibrium: $C = T \Rightarrow T = C_s + C_c = C_s + \alpha_1 \phi_c f_c' b a$ (To check)

- $a = 275$ mm

$$T = 2380 \text{ kN}$$

$$C_s = 952 \text{ kN}$$

$$C_c = 1430 \text{ kN}$$

$$\Rightarrow T = C_s + C_c$$

- $c = 305$ mm
- check if A_s' and A_s are in the plastic range

$$A_s (\varepsilon_s \geq \varepsilon_y); \varepsilon_s = \mathbf{0.0029} \geq \varepsilon_y = 0.002 \quad \text{OK}$$

$$A'_s (\varepsilon'_s \geq \varepsilon_y); \varepsilon'_s = \mathbf{0.0029} \geq \varepsilon_y = 0.002 \quad \text{OK}$$

$$M_r = 1079 \text{ kN.m} > M_f = 850 \text{ kN.m} \quad \text{OK}$$

d) M_r Using CDH Tables:

$$\rho' = \mathbf{1.25\%} \rightarrow \text{By linear Interpolation (Table 2.2): For } \rho' \% = 1.25\% \text{ and } \frac{d'}{d} = 0.1, k'_r = 3.83 \text{ MPa}$$

$$\rho = \mathbf{1.88\%} \rightarrow \text{By linear Interpolation (Table 2.1): For } \rho = \mathbf{1.88\%}, k_r = 4.85 \text{ MPa}$$

$$M_r = 1081.06 \text{ kN.m}$$

The result is close to what is found by analytical method.

Problem 2 **before update** (30 marks):

$$d = 341.5 \text{ mm}$$

$$d' = 54 \text{ mm}$$

We should keep $M_r \geq M_f$ (Assume $M_r = M_f = 650 \text{ kN.m}$)

$$k_r = \mathbf{9.29 \text{ MPa}}$$

In Table 2.1 in CSA A23.3: No value for $\rho\%$ corresponds to this case ($k_r = 9.29 \text{ MPa}$ and $f'_c = 40 \text{ MPa}$).

Provide maximum tension reinforcement given in table 2.1

$$k_{r,\max}(f'_c = 40 \text{ MPa}) = 8.2 \text{ MPa} \rightarrow \text{correspond to } \rho_b = 3.33\%$$

$$k'_r = 9.29 \text{ MPa} - 8.2 \text{ MPa} = 1.09 \text{ MPa}$$

We must use compression reinforcement too.

$$\rho' = \mathbf{0.38\%}$$

$$A'_S = 778.62 \text{ mm}^2$$

Use 2-25M $\rightarrow A'_S = 1000 \text{ mm}^2 > 778.62 \text{ mm}^2$.

Recalculate: $\rho' = 0.00488$

Recalculate: $k_r = 0.00488(1 - 0.158) * 0.85 * 400 = 1.4 \text{ MPa}$

$$M_{r1} = 573.78 \text{ kN.m}$$

$$M_{r2} = 97.96 \text{ kN.m}$$

$$M_r = 573.78 + 97.96 = 671.74 \text{ kN.m} > M_f = 650 \text{ kN.m} \quad \text{OK}$$

Required Tension Reinforcement:

$$\rho = 3.33\% + 0.38\% = 3.71\%$$

$$A_S = 7601.79 \text{ mm}^2$$

We can use 16 – 25M distributed in two layers (8-25M in each)

Check Bar spacing:

$$s = 45 \text{ mm}$$

Bar spacing:

$$s = 45 \text{ mm} \geq \begin{cases} 35 \text{ mm} \\ 28 \text{ mm} \\ 30 \text{ mm} \end{cases} \quad \text{OK}$$

Check $A_S > A_{Smin}$:

$$A_S = 8000 \text{ mm}^2 > A_{Smin} = 815.87 \text{ mm}^2 \quad \text{OK}$$

Maximum Reinforcement:

$$\rho = 3.9\%$$

$$\rho' = 0.488\%$$

$$\rho_b = 3.33\%$$

$$\rho = 3.9\% > \rho_{max} = 3.33\% + 0.488\% = 3.818\% \quad \text{NOT OK}$$

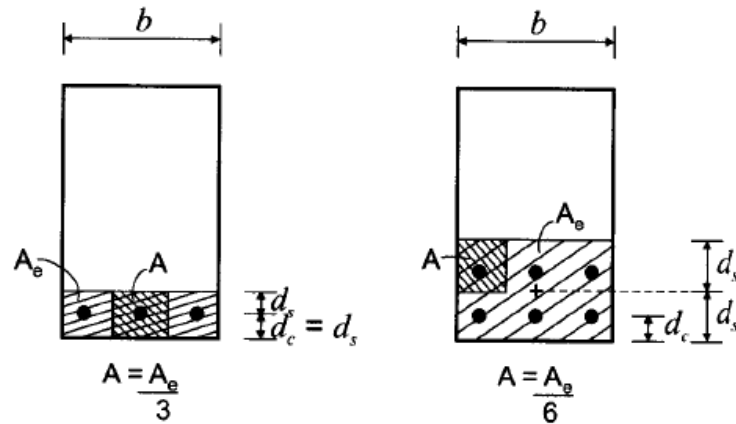
Increase A'_S and use 3-25M $\rightarrow A_S = 1500 \text{ mm}^2 > 778.62 \text{ mm}^2$

$$\rho' = 0.732\%$$

$$\rho_b = 3.33\%$$

$$\rho = 3.9\% > \rho_{\max} = 3.33\% + 0.732\% = 4.062\% \quad \text{OK}$$

Crack controlling (Z factor):



For Interior exposure: $z = f_s \sqrt[3]{d_c A} < 30000 \text{ N/mm}$

$$f_s = 240 \text{ MPa}$$

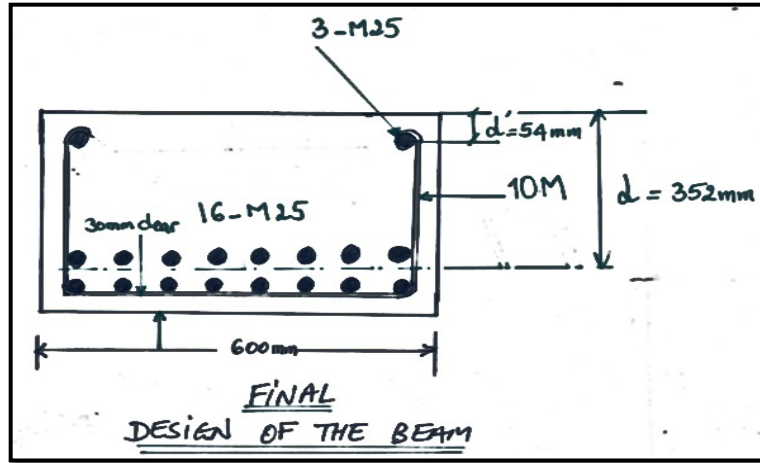
- $d_c = 54 \text{ mm}$
- $d_s = 89 \text{ mm}$

$N = \text{number of bars}$

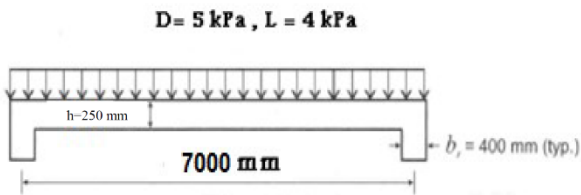
$$A_e = 106800 \text{ mm}^2$$

$$A = 6675 \text{ mm}^2$$

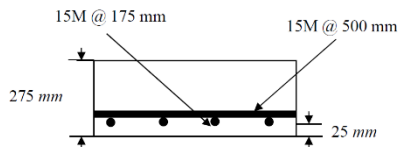
$$z = 17080 \text{ N/mm} < 30000 \text{ N/mm} \quad \text{OK.}$$



Problem 2 (30 marks):



(a) slab



Section @ midspan

(b) slab design details

$$L_n = 6600 \text{ mm}$$

$$q_f = 12.3 \text{ kPa}$$

$$W_f = 12.3 \text{ kN/m} \quad \text{Per (m) width of slab}$$

$$M_f = 67 \text{ kN/m} \quad \text{Per (m) width of slab}$$

$$d = 275 - 25 = 250 \text{ mm}$$

a)

$$A_s \rightarrow 15\text{M} @ 175 \text{ mm}$$

$$A_s = 1143 \text{ mm}^2/\text{m}$$

$$a = 21.35 \text{ mm} M_r = 93 \text{ kN.m} > M_f = 67 \text{ kN.m}$$

→ Strength requirement is met.

Check s_{\max} and $A_{s_{\min}}$ requirements of CSA standard.

$$A_{s_{\min}} = 550 \text{ mm}^2/\text{m}, \quad 1143 > 550 \quad \text{OK}$$

$$s_{\max} \rightarrow \min \left\{ \begin{array}{l} 500 \\ 825 \rightarrow \text{control} \end{array} \right. \quad s = 175 < 500 \quad \text{OK}$$

b)

$$A_s = 400 \text{ mm}^2/\text{m}, \quad A_{s_{\min}} = 550 \text{ mm}^2/\text{m}, \quad A_s < A_{s_{\min}} \quad \text{Not OK!!}$$

$$s_{\max} \rightarrow \min \left\{ \begin{array}{l} 500 \\ 1375 \rightarrow \text{control} \end{array} \right. \quad s = 500 \leq 500 \quad \text{OK}$$

c)

$$h \leq 350 \rightarrow \beta = 0.214$$

$$d_v \rightarrow \max \left\{ \begin{array}{l} 0.9d = 225 \text{ mm} \rightarrow \text{control} \\ 0.72h = 198 \end{array} \right.$$

$$V_C = 181.7 \text{ kN} \quad (\text{Per m width})$$

$$V_R = V_C = 181.7 \text{ kN}$$

$$V_f = 41 \text{ kN} \quad (\text{Per m width})$$

$$\text{Since } V_R > V_f \quad \text{OK}$$