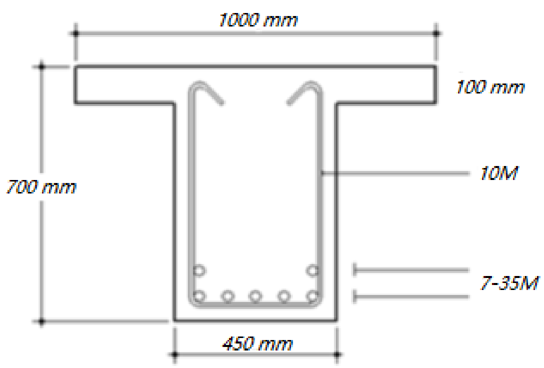
Solution Assignment 4

Problem 1 (20 marks)



NOTE:

* f'c = **30 MPa**, fy = **400 MPa**

* 30 mm clear cover

* 40 mm clear spacing between layers of steel

a) Moment Resistance without using the CDH tables

$A_s = 7\ 000\ mm^2$; $\alpha_1 = 0.81$; $\beta_1 = 0.90$

Check T-beam design

$$A_{s,ref} = \frac{\alpha_1 \phi_c f'_c b_f h_f}{\phi_s f_y} = 4645.6\ mm^2;\ A_s > A_{s,ref}\ \therefore a > h_f;\ T-beam\ design$$

Design as T-beam

$$A_c = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c} = 150\ 681\ mm^2$$

$$A_c = A_f + A_w = (b_f \times h_f) + \left(b_w(a - h_f)\right)$$

$$a = h_f + \frac{(A_c - A_f)}{b_w} = 212.6\ mm > h_f\ \ OK$$

$$A_w = (a - h_f)b_w = 50670\ mm^2$$

$$\bar{a} = \frac{A_f\left(\frac{h_f}{2}\right) + A_w\left(h_f + \frac{a - h_f}{2}\right)}{A_c} = 85.7\ mm$$

$$d_1 = 567.5\ mm;\ d_2 = 642.5\ mm;\ d = 621\ mm$$

$$z = d - \bar{a} = 535.3\ mm$$

Check yielding

$$\frac{c}{d} = \frac{a/\beta_1}{d} = 0.38 \leq \frac{700}{700 + f_y} = 0.64\ \therefore steel\ yields\ (under - reinforced)$$

Moment resistance

$$M_r = \phi_s A_s f_y (d - \bar{a}) = 1274\ kN \cdot m$$

$$\boldsymbol{M_r = 1274.0\ kN.m}$$

b) Moment Resistance using the CDH tables

$A_s = 7\ 000\ mm^2$; $A_{s,ref} = 4645.6\ \ mm^2$

$A_s > A_{s,ref}\ \ \therefore a >\ h_b;\ T - beam\ design$

Flange moment resistance

$M_{rf} = K_{rf}\left(b_f - b_w\right)d^2$ (Overhang)

$\frac{d}{h_f} = 6.21$

$\rho_f = 0.75\%$ (Table 2.3, CDH); Alternatively, calculate $\rho_f = \frac{\alpha_1 \phi_c f'_c h_f}{\phi_s f_y d} = 0.75\%$

$K_{rf} = 2.35$ (Table 2.1, CDH); $M_{rf} = K_{rf}\left(b_f - b_w\right)d^2 = 498.4\ kN.m$

$A_{sf} = 0.0075 \times \left(b_f - b_w\right) \times d = 2561.6\ mm^2$

Web moment resistance

$M_{rw} = K_{rw}(b_w)d^2$; $K_r = 4.475$ (Table 2.1, CDH); $M_{rw} = 776.6\ kN.m$

$A_{sw} = A_s - A_{sf} = 4438.4\ mm^2$

$\rho_w = \frac{A_{sw}}{b_w d} = 1.59\%$

Total moment resistance

$M_r = M_{rf} + M_{rw} = 1275\ kN.m$

$\boldsymbol{M_r = 1275\ kN.m}$

Problem 2 (20 marks)

$A_s = 5600\ mm^2$; $\alpha_1 = 0.8125$; $\beta_1 = 0.9075$

Factored moment at critical section (mid-span of beams)

$W_L = L_L \times 6 = 36\ kN/m$; $W_D = L_D \times 6 = 48\ kN/m$; $W_F = 1.25W_D + 1.5W_L = 114\ kN/m$

$M_F = \frac{W_f L^2}{8} = 912\ kN.m$

Effective depth and effective width

$d = 509\ mm$

$$Overhang < \begin{cases} 12\ h_f \\ 1/5\ \ Clear\ Span \\ 1/2\ \ clear\ dist.to\ next\ beam \end{cases}$$

$b_f = 2 \times overhang + b_w = 3\ 440\ mm$

Check T-beam design

$$A_{s,ref} = \frac{\alpha_1 \phi_c f'_c b_f h_f}{\phi_s f_y} = 32060.3\ mm^2$$

$A_s < A_{s,ref}\ \therefore a < h_f;\ Rectangular\ beam\ design$

Check yielding

$$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b_f} = 41.9\ mm$$

$$\frac{c}{d} = \frac{a/\beta_1}{d} = 0.077 \leq \frac{700}{700 + f_y} = 0.64\ \therefore steel\ yields\ (under - reinforced)$$

Moment resistance

$$M_r = \phi_s A_s f_y \left(d - \frac{a}{2}\right) = 929.2\ kN.m$$

$\boldsymbol{M_r = 929.2\ kN.m}$

Check design

$\boldsymbol{M_r = 929.2\ kN.m > M_f = 912\ kN.m}$ → **Acceptable design**

Problem 3 (20 marks)

a) Design critical beam section in positive bending

$M_F = \frac{W_f L^2}{24} = 266.7\ kN.m;\ \alpha_1 = 0.805;\ \beta_1 = 0.895$

$d = 550\ mm$

Assume rectangular design ($a < h_f$)

$$K_r = \frac{M_r}{b_f d^2} = 0.735$$

$\rho = 0.22\%$ (Table 2.1, CDH)

$A_s = \rho b_f d = 1452\ mm^2$ → 5-20M, 1 layer (A_{20M} = 300 mm^2)

$$A_{s,ref} = \frac{\alpha_1 \phi_c f'_c b_f h_f}{\phi_s f_y} = 11081\ mm^2$$

$A_s = 1500\ mm^2 < A_{s,ref} = 14\,774\ mm^2 \therefore a < h_f;\ Rectangular\ beam\ design$

Check yielding

$$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b_f} = 27.1\ mm$$

$$\frac{c}{d} = \frac{a/\beta_1}{d} = 0.055 \leq \frac{700}{700 + f_y} = 0.64\ \therefore steel\ yields\ (under - reinforced)$$

Check moment resistance

$M_r = \phi_s A_s f_y \left(d - \frac{a}{s}\right) = 273.6 \times 10^6\ N.mm = 273.6\ kN.m$

$\boldsymbol{M_r = 273.6\ kN.m > M_F = 266.7\ kN.m}$ → **OK**

Check spacing

$s = \frac{b_w - 2Cover - 2\phi_{stirrups} - nd_b}{n-1} = 105\ mm$

$s_{min} = max\begin{cases} 1.4d_b \\ 1.4a_{max} \\ 30\ mm \end{cases}$

$\boldsymbol{s = 105\ mm > s_{min} = 35\ mm}$ → **OK**

Check crack control

$f_s = 0.6f_y = 240\ MPa;\ d_c = 50\ mm;\ A = \frac{b\times 2y}{n} = 12000\ mm^2$

$z = f_s(d_c A)^{1/3} = 240(50 \times 12000)^{1/3} = 20242\ N/mm$

$\boldsymbol{z = 20242\ N/mm < 25\,000\ N/mm}$ → **OK for exterior exposure**

b) Design critical beam section in negative bending

$M_F = \frac{W_f L^2}{12} = 533.3\ kN.m$

$\alpha_1 = 0.805;\ \beta_1 = 0.895;\ d = 550\ mm$

Assume rectangular design ($a < h - h_f$)

$K_r = \frac{M_r}{b_f d^2} = 2.94$

$\rho = 0.97\%$ (Table 2.1, CDH)

$A_s = \rho b_w d = 3201\ mm^2$ → 11-20M, 1 layer (A_{20M} = 300 mm^2)

$A_{s,ref} = \frac{\alpha_1 \phi_c f'_c b_f h_f}{\phi_s f_y} = 11\,080\ mm^2$

$A_s = 3300\ mm^2 < A_{s,ref} = 11\,080\ mm^2 \therefore a < h - h_f;\ Rectangular\ beam\ design$

Check yielding

$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b_w} = 119.1\ mm$

$\frac{c}{d'} = \frac{a/\beta_1}{d'} = 0.242 \le \frac{700}{700 + f_y} = 0.64 \therefore steel\ yields\ (under-reinforced)$

Check moment resistance

$M_r = \phi_s A_s f_y \left(d' - \frac{a}{s}\right) = 550.2\ kN.m$

$\boldsymbol{M_r = 550.2\ kN.m > M_F = 533.3\ kN.m}$ **→ OK**

<u>Check spacing</u>

$s = \frac{b_w - 2Cover - 2\phi_{stirrups} - nd_b}{n-1} = 90\ mm$

$s_{min} = max \begin{cases} 1.4d_b \\ 1.4a_{max} \\ 30\ mm \end{cases}$

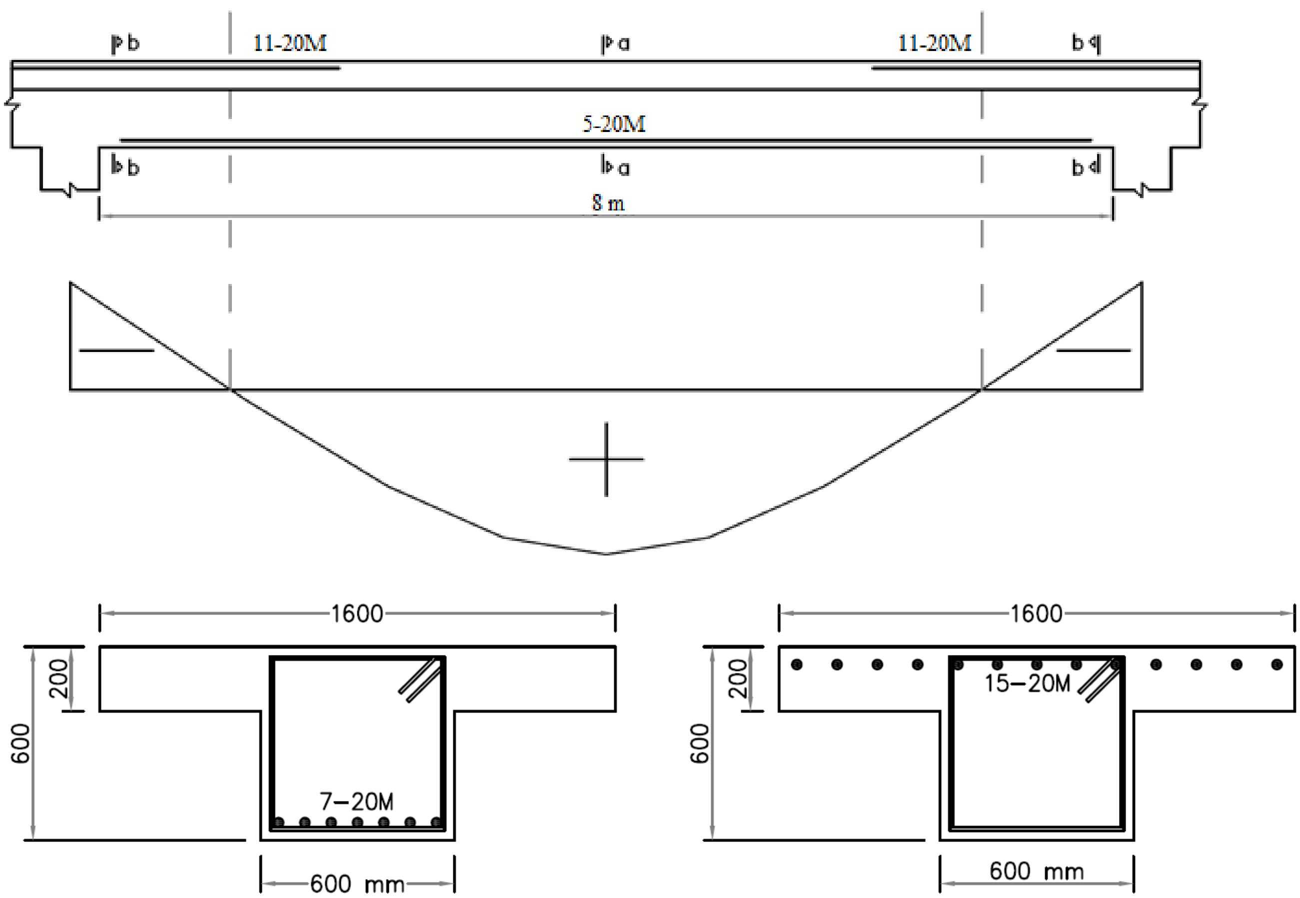
$\boldsymbol{s = 90\ mm > s_{min} = 35\ mm}$ **→ OK**

<u>Check crack control</u>

$f_s = 0.6f_y = 240\ MPa;\ d_c = 50\ mm;\ A = \frac{b \times 2y}{n} = 10909\ mm^2$

$z = f_s(d_c A)^{1/3} = 19609\ N/mm < \boldsymbol{25\ 000\ N/mm}$ **→ OK for exterior exposure**

c) Design summary drawing



Section a–a

Section b–b