PHY2361 Test solutions 2019

1. a) $t' = \frac{D}{c}$ is time light signal reaches front & back of rocket, as seen in its rest frame S'.

b) Method (i):

Front: $t_1 = \gamma\left(t_1' + \frac{v x_1'}{c^2}\right) = \gamma\left(\frac{D}{c} + \frac{vD}{c^2}\right) = \frac{D}{c}\gamma\left(1+\frac{v}{c}\right) = \frac{D}{c}\sqrt{\frac{1+v/c}{1-v/c}}$

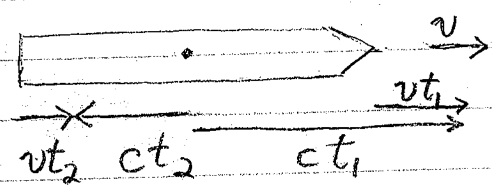
$x_1 = c\,t_1 = D\sqrt{\frac{1+v/c}{1-v/c}}$ $\quad$ (or $x_1 = \gamma(x_1' + v t_1') = \gamma(D + v\frac{D}{c})\ldots$)

back: $t_2 = \gamma\left(t_2' + \frac{v x_2'}{c^2}\right) = \gamma\left(\frac{D}{c} + \frac{v(-D)}{c^2}\right) = \frac{D}{c}\gamma\left(1-\frac{v}{c}\right) = \frac{D}{c}\sqrt{\frac{1-v/c}{1+v/c}}$

$x_2 = -c\,t_2 = -D\sqrt{\frac{1-v/c}{1+v/c}}$ $\quad$ (or $x_2 = \gamma(x_2' + v t_2') = \gamma(-D + v\frac{D}{c})$
$= -D\gamma(1-\frac{v}{c})$—

Method (ii):

As seen in S:



$\therefore\ vt_2 + ct_2 = D/\gamma$ and $ct_1 = D/\gamma + vt_1$

$\therefore\ t_1 = \frac{D}{\gamma(c-v)} = \frac{D/c}{\gamma(1-v/c)} = \frac{D}{c}\sqrt{\frac{1+v/c}{1-v/c}}$ and $x_1 = ct_1$

and $t_2 = \frac{D}{\gamma(c+v)} = \frac{D/c}{\gamma(1+v/c)} = \frac{D}{c}\sqrt{\frac{1-v/c}{1+v/c}}$ and $x_2 = -ct_2$

c) Yes, simultaneous in S'.

d) No, not simultaneous in S: $t_2 < t_1$

– light reaches back of rocket first

2. $\underset{m}{\circ} \xrightarrow{v_1 = \frac{c}{2}} \xleftarrow{v_2 = -3c/4} \overset{m}{\circ} \Rightarrow \underset{M}{\bigcirc} \xrightarrow{v_f} \longrightarrow x$ (v_1, v_2, v_f are x-components)

$p_1 = \gamma_1 m v_1$ where $\gamma_1 = \left(1 - (v_1/c)^2\right)^{-1/2} = \left(1 - (1/2)^2\right)^{-1/2} = \frac{2}{\sqrt{3}} = 1.155$

$= \frac{2}{\sqrt{3}} m \frac{1}{2} c = \frac{1}{\sqrt{3}} mc$ and $\gamma_2 = \left(1 - (v_2/c)^2\right)^{-1/2} = \left(1 - (3/4)^2\right)^{-1/2} = \left(\frac{7}{16}\right)^{-1/2} = \frac{4}{\sqrt{7}} = 1.512$

$\therefore p_2 = \gamma_2 m v_2 = \frac{4}{\sqrt{7}} m\left(-\frac{3}{4}c\right) = -\frac{3}{\sqrt{7}} mc$

$\therefore p_f = p_i = p_1 + p_2 = \left(\frac{1}{\sqrt{3}} - \frac{3}{\sqrt{7}}\right) mc = (0.577 - 1.134)\, mc = -0.557\, mc$

$E_1 = \gamma_1 mc^2 = \frac{2}{\sqrt{3}} mc^2 = 1.155\, mc^2$; $E_2 = \gamma_2 mc^2 = \frac{4}{\sqrt{7}} mc^2 = 1.512 mc^2$

$\therefore E_f = E_i = E_1 + E_2 = \frac{2}{\sqrt{3}} mc^2 + \frac{4}{\sqrt{7}} mc^2 = (1.155 + 1.512)\, mc^2 = 2.667 mc^2$

Note $p_f = \gamma_f M v_f$ and $E_f = \gamma_f M c^2$

$\therefore \frac{p_f}{E_f} = \frac{\gamma_f M v_f}{\gamma_f M c^2} = \frac{v_f}{c^2}$

$\therefore \frac{v_f}{c} = \frac{p_f c}{E_f} = \frac{-0.557 mc^2}{2.667 mc^2} = -0.209 \Rightarrow v_f = -0.209\, c$

$= -6.3 \cdot 10^7\ \text{m/s}$

4. a) $hf = E_{12} - E_{11} = -E_0\left(\frac{1}{12^2} - \frac{1}{11^2}\right)$ where $E_0 = 13.6\,\text{eV}$

$f = -\underbrace{\frac{13.6\,\text{eV} \cdot 1.6 \cdot 10^{-19}\,\text{J/eV}}{6.63 \cdot 10^{-34}\,\text{Js}}}_{3.28 \cdot 10^{15}\,\text{Hz}} \left(\frac{1}{12^2} - \frac{1}{11^2}\right) = 4.33 \cdot 10^{12}\ \text{Hz}$

b) $r_n = n^2 a_0$ $\quad n\hbar = L_n = m v_n r_n \Rightarrow v_n = \frac{n\hbar}{m r_n}$

$f_n^{-1} = T_n = \frac{2\pi r_n}{v_n}$

$f_n = \frac{v_n}{2\pi r_n} = \frac{n\hbar}{m r_n} \frac{1}{2\pi r_n} = \frac{n\hbar}{2\pi m r_n^2}$

$= \frac{12 \cdot 6.63 \cdot 10^{-34}\,\text{Js}\, (2\pi)^{-1}}{2\pi (9.11 \cdot 10^{-31}\,\text{kg})(7.62 \cdot 10^{-9}\,\text{m})^2}$

$= 3.81 \cdot 10^{12}\ \text{Hz}$

$r_n = n^2 a_0 = 12^2 \cdot 0.529 \cdot 10^{-10}\,\text{m}$

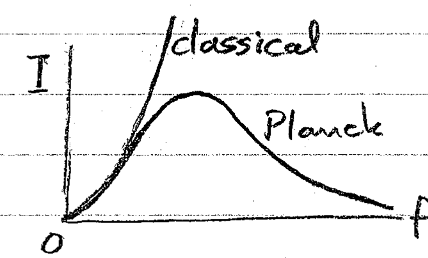
$= 7.62 \cdot 10^{-9}\,\text{m}$

$v_n = \frac{n\hbar}{m r_n} = \frac{12\,(1.055 \cdot 10^{-34}\,\text{Js})}{(9.11 \cdot 10^{-31}\,\text{kg})(7.62 \cdot 10^{-9}\,\text{m})}$

$= 1.82 \cdot 10^5\ \text{m/s}$

3. a) - electron described by deBroglie wave.
- diffraction of deBroglie wave from ordered rows of atoms in crystal predicts directions of scattered electrons, that was not predicted classically
- this supports $\lambda = h/p$ for electron

b) - energy of electromagnetic radiation of frequency f is quantized in units of hf
- gives thermal distribution of radiation $I(f)$ of Planck, in agreement with experiments



- no "ultraviolet divergence" which classical theory had predicted.